QUANTUM MECHANICS OF SPIN
(LECTURE NOTES SET 9)

- Experimental evidence for quantisation of angular momentum in Q.M. and Spin

- Periodic table: Schrödinger equation in Coulomb potential

$\Rightarrow \psi_{nlm} = R_{nl} Y_{lm} \Rightarrow$ 3 quantum numbers $\{n, l, m\}$

$l = n-1, n-2, \ldots, 0$
$m = l, l-1, \ldots, -l$

$E_n \sim -\frac{13.6\,eV}{n^2}$, $\hbar^2 l(l+1)$, $\hbar m$

- to explain electron configurations of atoms need another quantum number m_s related to z-component of spin of e^-: (PAULI)

S_z eigenvalues: $m_s \hbar$, $m_s = \pm\frac{1}{2}$

combined with Pauli exclusion principle

no two electrons agree in all 4 quantum numbers (n, l, m, m_s)

ONLY TRUE FOR SPIN $= \frac{1}{2}, \frac{3}{2}, \ldots$ PARTICLES $\Rightarrow$ FERMIONS (FERMI STATISTICS)

$n=1$: $l=0=m$, $m_s = \pm\frac{1}{2}$ 2

$n=2$: $l=0, m=0, m_s = \pm\frac{1}{2}$ 2
$l=1, m=1,0,-1, m_s = \pm\frac{1}{2}$ 6 $\Big\}$ 8

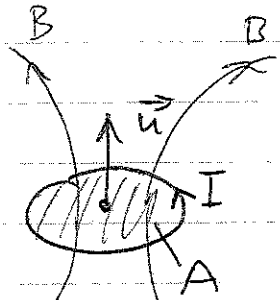
$n=3$: $l=0, m=0, m_s = \pm\frac{1}{2}$ 2
$l=1, m=1,0,-1, m_s = \pm\frac{1}{2}$ 6
$l=2, m=2,\ldots,-2, m_s = \pm\frac{1}{2}$ 10 $\Big\}$ 18

Ex: Ne: $Z=10 = \underbrace{\underset{n=1}{2} + \underset{n=2}{8}}_{\text{"closed shells"}}$, Na: $Z=11 = \underbrace{\underset{n=1}{2} + \underset{n=2}{8}}_{\text{closed shells}} + \underset{n=3,\, l=0}{1}$

Valence electron

INTERACTION WITH MAGNETIC FIELD

CLASSICALLY : ROTATING CHARGE PRODUCES MAGNETIC DIPOLE MOMENT $\vec{\mu}$



< magnetic field produced by current

$$\vec{\mu} = I A \vec{n}$$

electron orbiting on circle of radius r with velocity:

$$I = \frac{(-e)v}{2\pi r} \quad (\text{charge/sec.})$$

$$A = \pi r^2$$

$$IA = \frac{(-e)vr}{2} = \frac{(-e)}{2m_e} \underbrace{m_e v r}_{\text{A.M.}} = \frac{(-e)}{2m_e} L$$

$$\vec{\mu} = \frac{(-e)}{2m_e} \vec{L} = -\frac{\mu_B}{\hbar}\vec{L} \ , \mu_B \equiv \frac{e\hbar}{2m_e} \quad \text{BOHR magneton}$$

POTENTIAL ENERGY:

$$E_{mag} = -\vec{B}\cdot\vec{\mu} = +\frac{\mu_B}{\hbar}\underbrace{\vec{B}}_{\text{EXTERNAL MAGN. FIELD}}\cdot\vec{L}$$

$\vec{B}$ and $\vec{\mu}$ parallel is energetically preferred

Q.M. $\boxed{\hat{H}_{mag} = +\frac{\mu_B}{\hbar}\vec{B}\cdot\hat{\vec{L}}}$ (assuming e^- has no spin)

$$\text{for } \vec{B} = \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix}$$

$$\hat{H}_{mag} = \frac{\mu_B}{\hbar} B_z \hat{L}_z$$

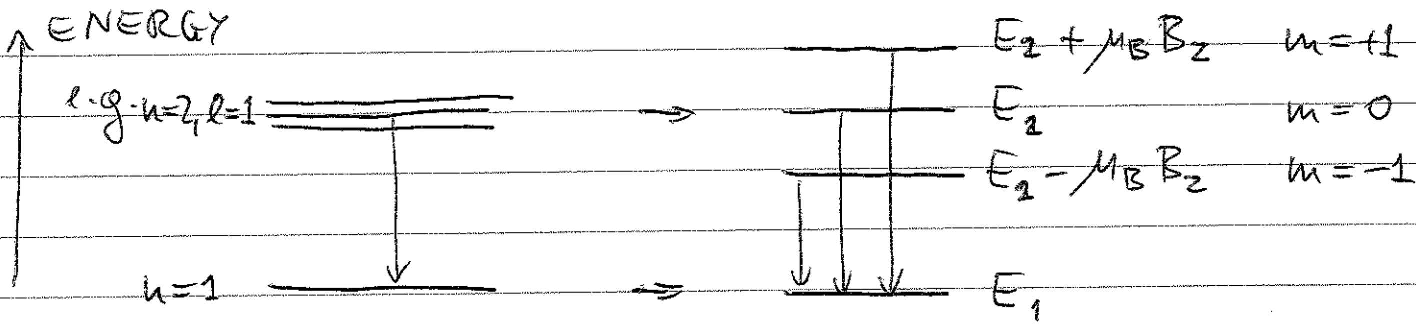
IF e^- is in an A.M. eigenstate

$\hat{L}_z$ eigenvalue $= \hbar m$

$$\underbrace{\langle \hat{H}_{mag} \rangle}_{\text{SHIFT OF ENERGY EIGENVALUES}} = \frac{\mu_B}{\hbar} B_z \hbar m = \mu_B B_z m$$

$m = -l, -l+1, \dots, +l$

DEGENERACY LIFTED! **ZEEMAN EFFECT**



- in general expect $(2l+1)$ (<u>ODD NUMBER</u>)

  OF LINES WHEN $\vec{B} \neq 0$

- IN PRACTISE PEOPLE FOUND <u>EVEN</u> <u>NUMBER</u> OF LINES

<u>PUZZLE</u> !? EVEN FOR $l=0 \rightarrow$ <u>2 LINES</u>

$$(2j+1) = 2 \Rightarrow j = \frac{1}{2} \quad \text{SPIN } \frac{1}{2} \text{ FOUND}$$

<u>ALSO</u>

- <u>DOUBLING OF LINESPECTRA</u> <u>OF ALKALI METALS + HYDROGEN</u>

Li: $Z=3$: $n=1$, $l=m=0$, $S_z = \pm\frac{1}{2} \rightarrow 2e^-$ (closed shell)

$n=2$, $l=0$, $S_z = \pm\frac{1}{2} \rightarrow e^-$ (Valence electron)

Na: $Z=11$: $11 = \underset{n=1;\, l=0}{2} + \underset{\substack{n=2:\, l=0,\, m=0 \\ l=1,\, m=1,0,-1}}{8} + 1 \rightarrow n=3, l=0$ <u>Valence electron</u>

STERN-GERLACH INHOMOGENEOUS $\vec{B}$

$\vec{B} = (0, 0, B_z)$ BUT B_z VARIES WITH z!

$$E_{mag} = -B_z \mu_z = -\vec{B} \cdot \vec{\mu}$$

$$\vec{F} = -\vec{\nabla} E_{mag} = +\vec{\nabla}(B_z \mu_z)$$

$$= +\left(\underbrace{\frac{\partial B_z}{\partial x}}_{\text{very small!}}, \underbrace{\frac{\partial B_z}{\partial y}}_{}, \frac{\partial B_z}{\partial z} \right) \mu_z \qquad \frac{\partial B_z}{\partial x}, \frac{\partial B_z}{\partial y} \ll \frac{\partial B_z}{\partial z}$$

$$= +\mu_z \left(0, 0, \frac{\partial B_z}{\partial z}\right)$$

ASSUMING THAT e^- HAS SPIN A.M.

$$\boxed{\vec{\mu} = -g_s \mu_B \frac{\vec{S}}{\hbar}}$$

FOR e^- WITH $\ell = 0$

$g_s \cong 2$ (PREDICTION OF DIRAC EQN.)

IN general for orbiting e^-

$$\vec{\mu}_{TOTAL} = -\frac{\mu_B}{\hbar}(\vec{L} + \underbrace{2}_{g_s}\vec{S})$$

$$\boxed{\hat{H}_{mag} = -\vec{B} \cdot \vec{\mu}_{TOTAL} = \frac{\mu_B}{\hbar} \vec{B} \cdot (\hat{\vec{L}} + 2\hat{\vec{S}})}$$

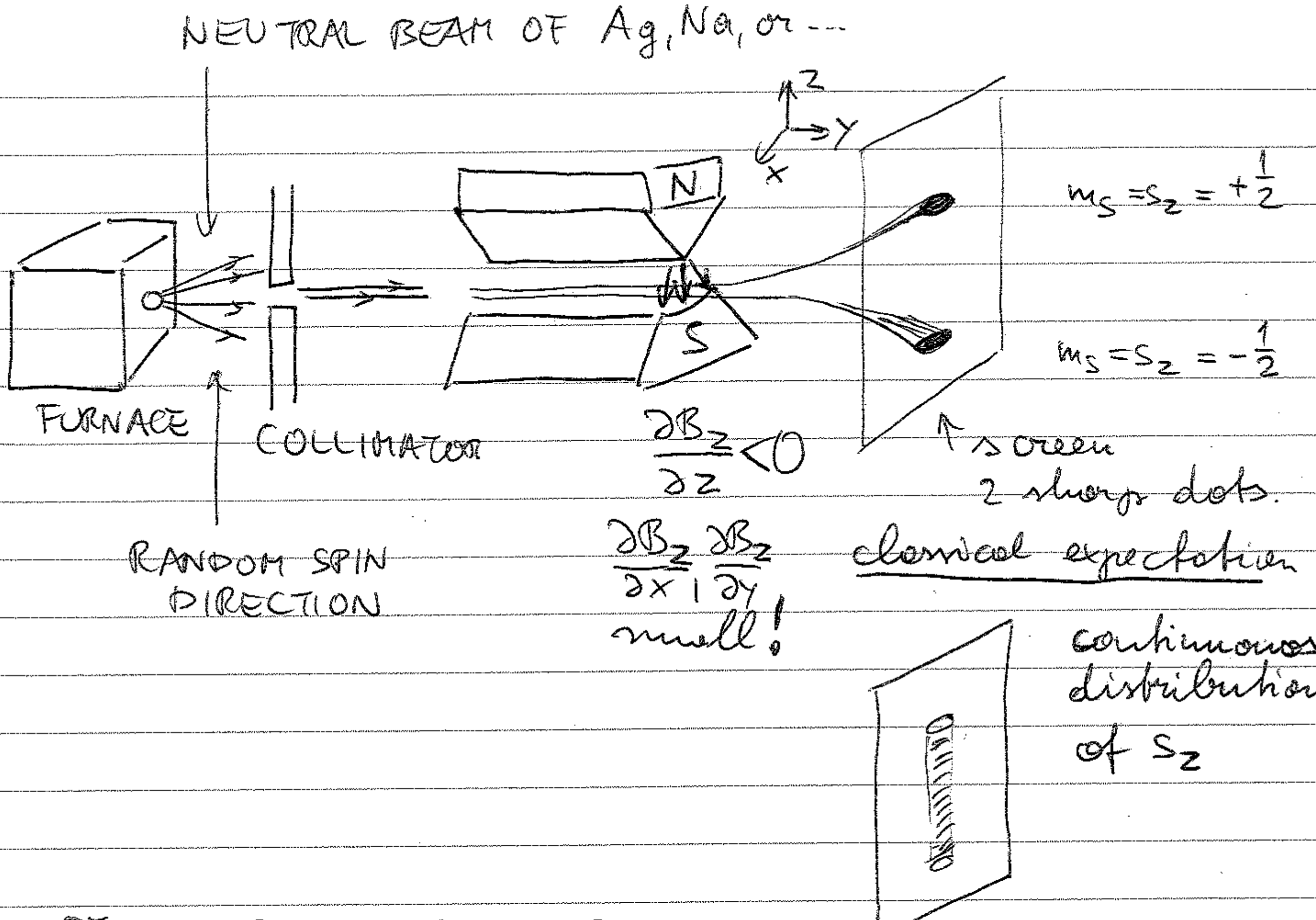
IF SPIN $= \frac{1}{2}$ IS QUANTIZED

$$S_z = m_s \hbar = s_z \hbar, \quad m_s = s_z = \pm \tfrac{1}{2}$$

$$\mu_z = -2\mu_B \frac{1}{\cancel{\hbar}} m_s \cancel{\hbar} = -2\mu_B m_s$$

$$F_z = -\mu_B m_s 2 \frac{\partial B_z}{\partial z} \quad \xrightarrow{m_s = +\frac{1}{2}} -\mu_B \partial B_z/\partial z, \quad \xrightarrow[m_s = -\frac{1}{2}]{} +\mu_B \partial B_z/\partial z$$

STERNGERLACH EXPERIMENT



STERN-GERLACH EXP. (HAS 2 USES)

- STATE PREPARATION: PREPARES ~~AN~~ TWO ENSEMBLES OF ATOMS (BEAMS) IN A DEFINITE STATE

$\Rightarrow$ UPPER BEAM: $s_z = m_s = +\frac{1}{2}$, $S_z = \frac{\hbar}{2}$

LOWER BEAM: $s_z = m_s = -\frac{1}{2}$, $S_z = -\frac{\hbar}{2}$

- MEASUREMENT OF SPIN (WHEN WE PUT A SCREEN)

• USE NEUTRAL ATOMS: NO DEFLECTION IN $\vec{B}$ DUE TO ELECTRIC CHARGE = LORENTZ-FORCE

• USE ATOMS WITH A SINGLE VALENCE ELECTRON (e^-)

e.g. Na: $Z = 11 = \underbrace{\underset{n=1,\, l=0}{2} + \underset{n=2,\, l=0,1}{8}}_{\text{closed shells}} + 1$

SODIUM

$1 \rightarrow n=3,\ l=0,\ m=0$, $m_s = +\frac{1}{2}$ OR $-\frac{1}{2}$

$\Rightarrow$ NO NET A.M.

$\Rightarrow$ A.M. OF ATOM ONLY DUE TO SPIN OF VALENCE e^-

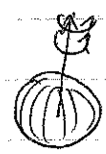
WHY SPIN IS NOT A CLASSICAL CONCEPT

e^-: $m_e \sim 0.5\,\mathrm{MeV}/c^2$ $\quad r_e < 10^{-20}\,\mathrm{m}$ (FROM EXPERIMENT)

e^- as a spinning ball:

$$J \approx m_e r_e V \approx \frac{\hbar}{2} \Rightarrow r_e \approx \frac{\hbar}{2 m_e V} \geq \frac{\hbar}{2 m_e c} \approx 2 \times 10^{-13}\,\mathrm{m}$$

OR $\quad V \approx \dfrac{\hbar}{2 m_e r_e} > 2 \times 10^{7} c$



e^- as a finite size ball of constant mass and charge density does not work

SPIN CAN ONLY BE DERIVED BY UNIFYING

SPECIAL RELATIVITY + Q.M.

$\rightarrow$ DIRAC EQN.

NON-RELATIVISTIC LIMIT $\rightarrow$ PAULI EQN.

$\Downarrow$

TWO COMPONENT WAVE FNCT. OF e^-

$$\Psi(\vec{x},t) = \psi_+(\vec{x},t)\begin{pmatrix}1\\0\end{pmatrix} + \psi_-(\vec{x},t)\begin{pmatrix}0\\1\end{pmatrix}$$

S_z eigenvalue $\frac{\hbar}{2}$, $\uparrow$

S_z eigenvalue $-\frac{\hbar}{2}$, $\downarrow$

MAIN LESSONS

ZEEMAN + STERN-GERLACH EXPERIMENTS MEASURE (QUANTISATION OF) A. M. + SPIN INDIRECTLY

$$\vec{\mu} = -\frac{\mu_B}{\hbar}\left(\vec{L} + g_s \vec{S}\right) = -\frac{\mu_B}{\hbar}\left(\vec{L} + 2\vec{S}\right)$$

$$E_{mag} = -\vec{\mu}\cdot\vec{B}$$

HOMOGENEOUS $\vec{B}$ (ZEEMAN) $\Rightarrow$ SHIFT OF ENERGY LEVELS

CONFIRMS QUANTISATION OF $\vec{L}$ (AND $\vec{S}$)

INHOMOGENEOUS $\vec{B}$ (S.G.) Ag, ALKALI METALS

VALENCE ELECTRON: $l=0$, $\vec{L}=0$

$\Rightarrow$ SHOWS QUANTISATION OF $\vec{S}$

$$\vec{B} \simeq \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix}, \quad F_z = -\mu_B \underbrace{S_z}_{\hbar m_s} \frac{\partial B_z}{\partial z} \quad \text{FORCE}$$

$\Rightarrow$ 2 dots $\Rightarrow m_s = \pm\frac{1}{2} \Rightarrow j = \frac{1}{2}$ SPIN $\frac{1}{2}$

S.G MEASURES z COMPONENT OF SPIN

$\Rightarrow$ W.F. COLLAPSES TO ONE OF TWO POSSIBLE OUTCOMES

AND PREPARES TWO SPATIALLY SEPARATED BEAMS (ENSEMBLES) IN DEFINITE S_z EIGENSTATES

$\Rightarrow$ TESTS OF MEASUREMENT POSTULATE, COLLAPSE OF W.F., EXPANSION THEOREM E.T.C. BY COUPLING SEVERAL S.G. EXPERIMENTS

SPIN ANGULAR MOMENTUM AND MATRIX MECHANICS

FROM EARLIER $\hat{H}\psi_n = E_n\psi_n \qquad \langle\psi_n|\psi_m\rangle = \delta_{nm}$

MATRIX OPERATOR CORRESPONDING TO OPERATOR $\hat{A}$

IS $\quad A_{mn} = \int \psi_m^* \hat{A}\psi_n = \langle m|\hat{A}\,n|\rangle$

$$\Psi = \sum c_m\psi_m \quad \Leftrightarrow \quad C = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ \vdots \end{pmatrix} = c_1\begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix} + c_2\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \dots$$

Expansion theorem

$$\langle\hat{A}\rangle = \int \Psi^*\hat{A}\,\Psi \quad = \quad C^\dagger A\, C$$

FOR A.M. $\quad \hat{\vec{J}}^2\psi_{jm} = \hbar^2 j(j+1)\,, \quad \hat{J}_z\psi_{jm} = \hbar m\,\psi_{jm}$

$[\hat{\vec{J}}^2, \hat{J}_z] = 0$ SIMULTANEOUS EIGENSTATES

HERMITIAN $\Rightarrow \langle\psi_{j'm'}|\psi_{jm}\rangle = \delta_{jj'}\delta_{mm'}$

$\boxed{|jm\rangle \equiv \psi_{jm}}$ ORTHONORMALITY

- THE SPIN j OF PARTICLES IS ALWAYS CONSTANT

e^-, quarks, .. $\quad j = \frac{1}{2}$

Higgs $\quad j = 0$

Photon, gluons, $\quad j = 1$

FOR FIXED j a general state with total A.M. j

$$\Psi_j = \sum_{m=-j}^{+j} c_{jm}\psi_{jm} \qquad \text{EXPANSION TH.}$$

$|c_{jm}|^2$ .. PROBABILITY TO FIND IN MEASUREMENT OF $\hat{J}_z$ THE VALUE $\underline{\underline{m\hbar}}$

MATRIX FORM OF STATE: $\Psi_j \Rightarrow$

$$C = \begin{pmatrix} c_j \\ c_{j-1} \\ \vdots \\ c_{-j} \end{pmatrix} = c_j \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\psi_{jj}} + c_{j-1} \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\psi_{jj-1}} + \dots + c_{-j} \underbrace{\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}}_{\psi_{j,-j}}$$

A.M. EIGENSTATES $\Leftrightarrow$ UNIT VECTORS IN $(2j+1)$ DIM'L VECTOR SPACE

<u>A.M. OPERATORS</u> (j - fixed) $= (2j+1) \times (2j+1)$ matrices

$$\left(\vec{J}^2\right)_{m'm} = \langle j, m' | \underbrace{\hat{\vec{J}}^2 | j, m \rangle}_{\hbar^2 j(j+1)|j,m\rangle} = \hbar^2 j(j+1) \underbrace{\langle j, m' | j, m \rangle}_{=\delta_{m'm}}$$

$$= \hbar^2 j(j+1) \begin{array}{c|c} & \rightarrow m = j, j-1, \dots \quad -j \\ \hline \begin{matrix} \downarrow m' = j \\ j-1, \dots \end{matrix} & \begin{pmatrix} 1 & & & \emptyset \\ & 1 & & \\ & & 1 & \\ \emptyset & & & \ddots \\ & & & & 1 \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} \left(\hat{\vec{J}}^2\right)_{j,j} & \left(\vec{J}^2\right)_{j,j-1} & \dots & \left(\vec{J}^2\right)_{j,-j} \\ \left(\vec{J}^2\right)_{j-1,j} & \left(\vec{J}^2\right)_{j-1,j-1} & \dots & \\ \vdots & & & \\ \left(\vec{J}^2\right)_{-j,j} & & & \end{pmatrix}$$

$$(\hat{J}_z)_{m'm} = \langle j,m'|\hat{J}_z|j,m\rangle =$$

$$= \hbar m \langle j,m'|j,m\rangle = \hbar m\, \delta_{m'm}$$

$$= \hbar \begin{pmatrix} +j & & & \\ & j-1 & & \emptyset \\ & & \ddots & \\ & \emptyset & & \ddots & \\ & & & & -j \end{pmatrix}$$

$$\boxed{\hat{J}_+ = \hat{J}_x + i\hat{J}_y} \Rightarrow J_x = \frac{1}{2}(J_+ + J_-)$$

$$\hat{J}_- = (\hat{J}_+)^\dagger = \hat{J}_x - i\hat{J}_y \qquad J_y = \frac{1}{2i}(J_+ - J_-)$$

$$(J_+)_{m'm} = \langle j,m'|\underbrace{\hat{J}_+|j,m\rangle} = \hbar\sqrt{j(j+1)-m(m+1)}\,\delta_{m',m+1}$$

$$= \hbar\sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle$$

<u>EX</u> $j = \frac{1}{2} = s$, $\vec{J} \equiv \vec{S}$, $\underline{\underline{\text{SPIN } \frac{1}{2}}}$

$$\vec{S}^2 = \hbar^2 \frac{1}{2}\left(\frac{1}{2}+1\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$S_+ = (S_+)_{m'm} = \begin{pmatrix} (S_+)_{\frac{1}{2},\frac{1}{2}} & (S_+)_{\frac{1}{2},-\frac{1}{2}} \\ (S_+)_{-\frac{1}{2},\frac{1}{2}} & (S_+)_{-\frac{1}{2},-\frac{1}{2}} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \hbar \\ 0 & 0 \end{pmatrix} = \hbar\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$$

$$S_x = \frac{1}{2}\left(S_+ + \underset{\shortparallel}{S_-}\right) = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(S_+)^\dagger = \begin{pmatrix} 0 & 0 \\ \hbar & 0 \end{pmatrix}$$

$$S_y = \frac{1}{2i}\left(S_+ - S_-\right) = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$\underline{\underline{j=1}}$

$$\vec{J}^2 = 2\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$J_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{array}{cc} & \overset{m}{\Rightarrow} \\ J_+ = \underset{m'}{\Downarrow} & \begin{pmatrix} (J_+)_{1,1} & (J_+)_{1,0}\,\hbar & (J_+)_{1,-1} \\ (J_+)_{0,1} & (J_+)_{0,0} & (J_+)_{0,-1} \\ (J_+)_{-1,1} & (J_+)_{-1,0} & (J_+)_{-1,-1} \end{pmatrix} \end{array} = \hbar\sqrt{2 - m(m+1)}\;\delta_{m',m+1}$$

$$= \begin{pmatrix} 0 & \hbar\sqrt{2} & 0 \\ 0 & 0 & \hbar\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$J_x = \frac{1}{2}\left(J_+ + (J_+)^\dagger\right) = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$J_y = \frac{1}{2i}\left(J_+ - (J_+)^\dagger\right) = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

SPIN OPERATORS FOR SPIN $= \frac{1}{2}$

$$S_{x,y,z} = \frac{\hbar}{2} \sigma_{x,y,z}$$

↑ PAULI MATRICES

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

SPIN WAVE FUNCTIONS

IN ORDER TO INCORPORATE THE ADDITIONAL DEGREES OF FREEDOM DUE TO SPIN THE WAVE FUNCTION Ψ BECOMES A $2j+1$ COMPONENT VECTOR!

FOR $\underline{\underline{j = \frac{1}{2}}}$:
$$\Psi = \begin{pmatrix} \Psi_+(\vec{x},t) \\ \Psi_-(\vec{x},t) \end{pmatrix}$$

$$= \Psi_+(\vec{x},t) \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\equiv \chi_+} + \Psi_-(\vec{x},t) \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{\equiv \chi_-}$$

χ_+, χ_- Eigenstates of S_z with eigenvalues $\pm \hbar/2$

= <u>POSSIBLE OUTCOMES OF S.G EXPERIMENT IF $\vec{B}$ FIELD IS ORIENTED IN z-DIRECTION</u>

PROOF

$$S_z \chi = \frac{\hbar}{2}\lambda\chi \qquad \chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\cancel{S_z}\ \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2}\lambda\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\Rightarrow \quad a = \lambda a$$
$$b = -\lambda b$$

EITHER $\lambda = 1,\ b = 0 \Rightarrow \chi = \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ normalised

$\lambda = -1,\ a = 0 \Rightarrow \chi = \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ normalised

EXPANSION THEOREM

GENERAL STATE: $\chi = a\chi_+ + b\chi_- = \begin{pmatrix} a \\ b \end{pmatrix}$

normalisation: $\chi^\dagger\chi = 1 = |a|^2 + |b|^2$

$|a|^2$ .. PROB. TO FIND $+\frac{\hbar}{2}$ IN MEAS. OF S_z

$|b|^2$ .. — " ——— $-\frac{\hbar}{2}$ — " ——

↓

MEASUREMENT: COLLAPSE OF WAVEFUNCTION ...
TO χ_+ OR χ_-

~~(S. G. DOES EXACTLY THAT)~~

SIMILARLY S_x, S_y have eigenvalues $\pm\frac{\hbar}{2}$

↓

QUANTISATION ALONG ~~x~~-AXIS

EX ROTATE S. G. EXPERIMENT BY 90°
AROUND y-AXIS $\Rightarrow \vec{B}$ FIELD IN x DIRECTION
$\Rightarrow$ Measurement of S_x
still expect eigenvalues $\pm\frac{\hbar}{2}$

$$S_x \chi' = \frac{\hbar}{2} \lambda \chi' \qquad \chi' = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2} \lambda \begin{pmatrix} a \\ b \end{pmatrix}$$

$$b = \lambda a$$

$$a = \lambda b \Rightarrow a = \lambda^2 a \Rightarrow \lambda = \pm 1$$

$\lambda = 1$: $a = b$ $\qquad \chi'_+ = \begin{pmatrix} a \\ a \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ normalised

$b = -a \qquad \chi'_- = \begin{pmatrix} a \\ -a \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ —"—

EXP. TH.

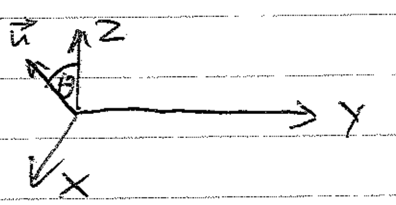
$$\chi'_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\chi_+ + \chi_-)$$

S_x EIGENSTATE $\qquad$ S_z EIGENSTATES χ_+, χ_-

$$\chi'_- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\chi_+ - \chi_-)$$

GENERAL ROTATED $\quad$ S. G EXPERIMENT IN DIR. $\vec{u}$, $|\vec{u}|^2 = 1$

$$\vec{u}\cdot\vec{S} = u_x S_x + u_y S_y + u_z S_z$$



e.g. $\vec{u} = \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix}$

$$\vec{u}\cdot\vec{S} = \sin\theta\, S_x + 0 + \cos\theta\, S_z$$

$$= \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

EIGEN VALUES $\pm \hbar/2$