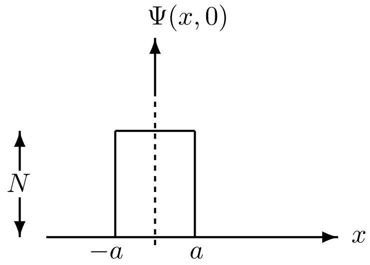
QUANTUM MECHANICS B **PHY413** **PROBLEMSET 1**

ISSUE: Thu, 27th September, 2012 **HAND–IN: 4.00 pm. Thu, 4th October, 2012.**

Post your solutions in the box next to the Departmental Offices on the 1st floor of the GO Jones Bldg.

QUESTION 1. At $t = 0$ a particle moving in one-dimension has the rather idealised wave function (which is in fact a wave packet):



(a) By normalising the wave function $\Psi(x, 0)$ determine N. [2]
(b) Evaluate $\langle x\rangle$ and $\langle x^2\rangle$ and hence the uncertainty in position, $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$. [4]
(c) A single measurement is performed of the position of the particle. What are the possible outcomes and their *relative* probabilities? What is the probability of finding the particle between $x = -\Delta x$ and $+\Delta x$? What is the probability of finding it outside this range? [4]

QUESTION 2. A one-dimension system is in a state described by the *normalisable* wave function $\Psi(x, t)$ *i.e.* $\Psi \to 0$ for $x \to \pm\infty$.
(a) Show that the expectation value of the position $\langle x\rangle$ is a real quantity. [1]
(b) Show that the expectation value of the momentum in the x-direction $\langle \hat{p}_x\rangle$ is a real quantity, too. Hint: using integration by parts and normalisability show that $\langle \hat{p}_x\rangle = \langle \hat{p}_x\rangle^*$. [4]
(c) From earlier QM courses you know that boundstate solutions of the TISE are *real functions* $\psi_n(x)$ (states of particles trapped in a potential well). Hence, show that $\langle \hat{p}_x\rangle$ vanishes for *stationary* energy eigenstates $\Psi_n(x, t) = \psi_n(x)e^{-iE_nt/\hbar}$. [4]

QUESTION 3. Consider a one-dimensional system with Hamiltonian $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{m}{2}\omega^2 x^2$ (Simple Harmonic Oscillator(SHO)). Show that the function

$$\phi(x) = e^{+ax^2/2} \quad \text{with} \quad a = \frac{m\omega}{\hbar}$$

is an eigenfunction of the Hamiltonian with eigenvalue $-\frac{1}{2}\hbar\omega$. Compared to the facts you know about the SHO in previous courses what is strange about this result? What is wrong with this wavefunction and why is it physically unacceptable? It helps to actually draw this function recalling that $a > 0$. [6]