THE MAGNETIC FIELD

This handout covers:

- The magnetic force between two moving charges
- The magnetic field, $\overline{\mathbf{B}}$, and magnetic field lines
- Magnetic flux and Gauss's Law for $\overline{\mathbf{B}}$
- Motion of a charged particle in $\overline{\mathbf{E}}$ and $\overline{\mathbf{B}}$: the Lorentz force

  Important special cases:

  - Motion perpendicular to uniform $\overline{\mathbf{B}}$
  - The velocity selector
  - The Hall effect

Note:

1. You do not need to remember the full vector treatment of the magnetic force between moving charges.

2. For exam purposes, the important relationship defining the magnetic field, $\overline{\mathbf{B}}$ is

$$\overline{\mathbf{F}}_{\mathbf{M}} = Q(\overline{\mathbf{v}} \times \overline{\mathbf{B}})$$

3. However, it is important to understand the vector treatment so that you can gain a good conceptual grasp of the subject.

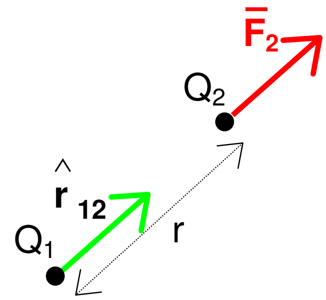
4. The key to understanding the magnetic field is to be able to use the vector cross product $\Rightarrow$ revise this if you are not sure of it.

The magnetic force

Up to now, we have considered the **ELECTROSTATIC** force, due to charges at rest .

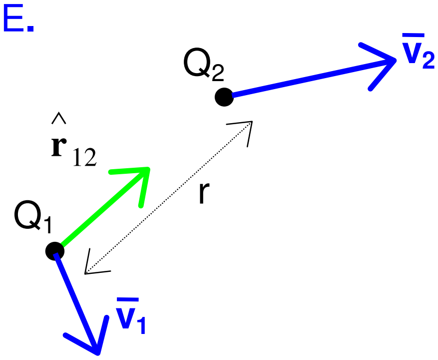
Coulombs Law:

$$\overline{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}\,\hat{\mathbf{r}}_{12}$$



If the charges are BOTH moving, another force exists between them: the MAGNETIC FORCE.

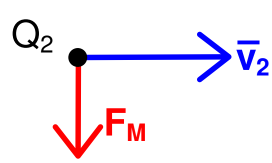
$$\overline{F}_M = \frac{\mu_o}{4\pi}\frac{Q_1 Q_2}{r^2}\left[\overline{\mathbf{v}}_2 \times (\overline{\mathbf{v}}_1 \times \hat{\mathbf{r}}_{12})\right]$$



Simpler case: $\overline{\mathbf{v}}_1$ and $\overline{\mathbf{v}}_2$ parallel

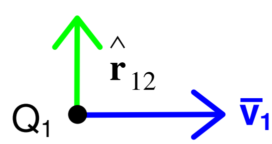
$\overline{\mathbf{v}}_1 \times \hat{\mathbf{r}}_{12}$ Magnitude = $v_1 r_{12} = v_1$

Direction = out of page



$\overline{\mathbf{v}}_2 \times (\overline{\mathbf{v}}_1 \times \hat{\mathbf{r}}_{12})$ Magnitude = $v_1 v_2$

Direction = Towards Q_1



$$\Rightarrow \quad \overline{F}_M = -\frac{\mu_o}{4\pi}\frac{Q_1 Q_2}{r^2} v_1 v_2\,\hat{\mathbf{r}}_{12}$$

Note:

1. $F_M \propto Q_1Q_2$ as for the electrostatic force

2. $F_M \propto v_1v_2$ $\Rightarrow$ no force unless BOTH charges are moving

3. $\overline{\mathbf{F}}_{\mathbf{M}}$ exists in addition to the electric force

4. The constant μ_o is called the

   PERMEABILITY CONSTANT or the
   PERMEABILITY OF FREE SPACE

   SI System: $\mu_o = 4\pi \times 10^{-7}$ N s^2 C^{-2}

Relative magnitudes of the electric and magnetic forces

$$F_M = \frac{\mu_o}{4\pi}\frac{Q_1Q_2}{r^2}v_1v_2 \qquad F_E = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$

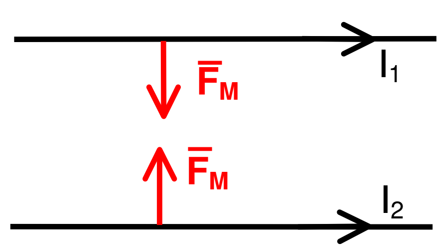
$$\Rightarrow \quad \frac{F_M}{F_E} = \mu_o\varepsilon_o v_1v_2 = \frac{v_1v_2}{c^2} \quad \text{where} \quad c = \left[\frac{1}{\mu_o\varepsilon_o}\right]^{1/2}$$

c = SPEED OF LIGHT [To be explained later]

$$\frac{F_M}{F_E} = \frac{v_1v_2}{c^2} \Rightarrow F_M << F_E$$ unless speeds are close to speed of light.

BUT: The magnetic force can still dominate even at very low speed because F_E tends to be cancelled out due to the overall charge neutrality of matter.

Example: Two wires carrying currents

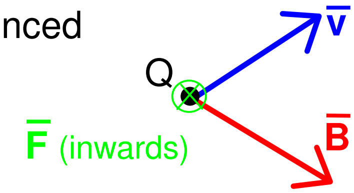


$\overline{\mathbf{F}}_{\mathbf{E}} = 0$ (neither wire is charged)

$\overline{\mathbf{F}}_{\mathbf{M}} \neq 0$ (and can be quite large)

The magnetic field

DEFINITION: The magnetic force experienced by a charge Q moving with velocity $\overline{\mathbf{v}}$ is

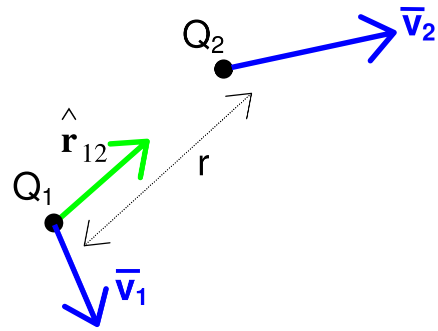


$$\overline{\mathbf{F}}_{\mathbf{mag}} = Q(\overline{\mathbf{v}} \times \overline{\mathbf{B}})$$

This equation defines $\overline{\mathbf{B}}$, the MAGNETIC FIELD

Special case: $\overline{\mathbf{v}} \perp \overline{\mathbf{B}} \quad \Rightarrow \quad F = QvB$

Recall:



$$\overline{\mathbf{F}}_{\mathbf{M}} = \frac{\mu_o}{4\pi}\frac{Q_1 Q_2}{r^2}\left[\overline{\mathbf{v}}_2 \times (\overline{\mathbf{v}}_1 \times \hat{\mathbf{r}}_{12})\right]$$

But, by definition of $\overline{\mathbf{B}}$,

$$\overline{\mathbf{F}}_2 = Q(\overline{\mathbf{v}}_2 \times \overline{\mathbf{B}}_1)$$

where $\overline{\mathbf{B}}_1$ = magnetic field at Q_2 due to Q_1.

So $$\overline{\mathbf{B}}_1 = \frac{\mu_o Q_1}{4\pi r^2}\left[\overline{\mathbf{v}}_1 \times \hat{\mathbf{r}}_{12}\right]$$

Magnetic field of a moving point charge

SI Units for Magnetic Field:

1 T (Tesla) = Field which exerts a force of 1 N on a 1-C charge moving with velocity 1 m s^{-1} perpendicular to $\overline{\mathbf{B}}$.

$$B = \frac{F}{Qv} \quad \Rightarrow \quad 1\ T \equiv 1\ N\ C^{-1}\ m^{-1}\ s$$

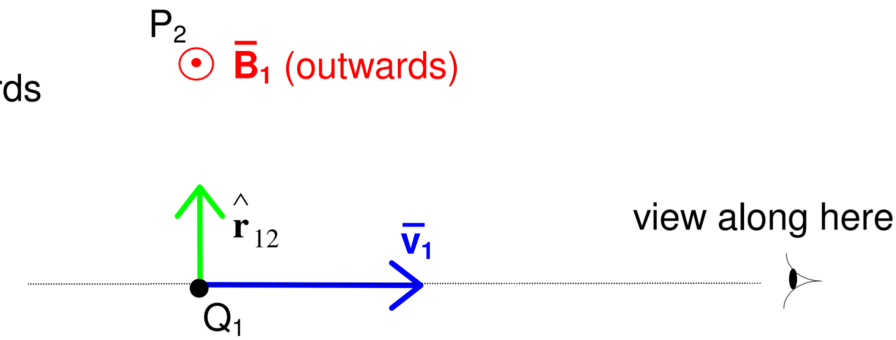
Magnetic field lines

Recall: Electric field lines point in the direction of $\overline{\mathbf{E}}$

Similarly, magnetic field lines point along $\overline{\mathbf{B}}$

Recall: $\overline{\mathbf{B}}_1 = \frac{\mu_o Q_1}{4\pi r^2}\left[\overline{\mathbf{v}}_1 \times \hat{\mathbf{r}}_{12}\right]$ = Magnetic field at P_2 due to Q_1

Direction is outwards by right hand rule



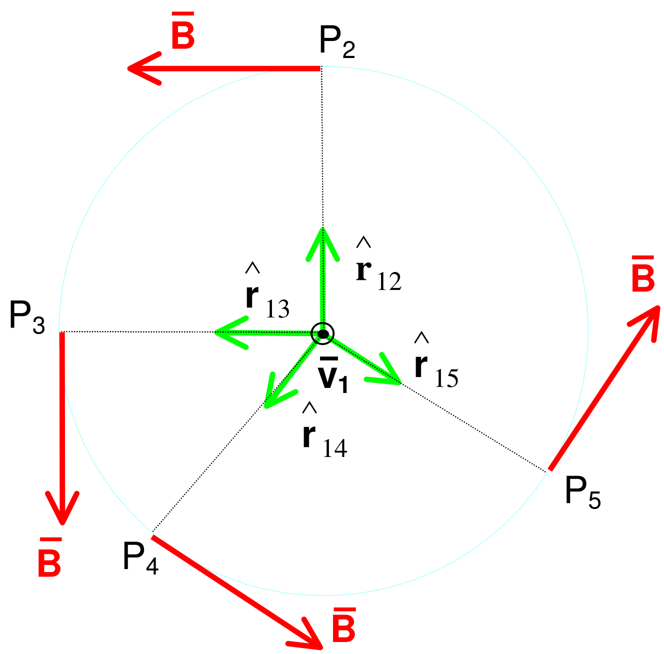
Imagine we view ALONG THE DIRECTION OF MOTION, so that Q_1 appears to be coming straight at us:

$\overline{\mathbf{v}}_1$ is OUT OF PAGE

If we consider points on a circle (e.g., P_2 - P_5) then the direction of the unit vector $\hat{\mathbf{r}}_{12}$ etc. depends on which point we consider.

$\overline{\mathbf{v}}_1$ is always out of the page.

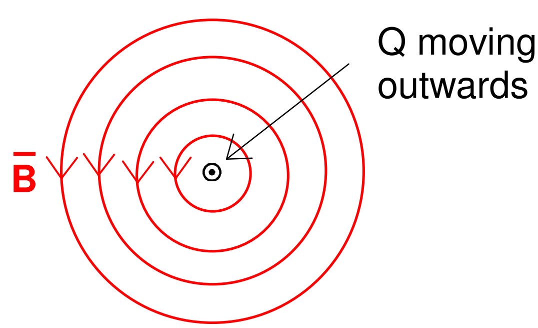
Apply the right hand rule to each point



→ $\overline{\mathbf{B}}$ is always tangential

⇒ Lines of $\overline{\mathbf{B}}$ form CLOSED LOOPS

Convention: As before, line spacing is used to indicate the magnitude of $\overline{\mathbf{B}}$ (closely spaced lines ⇒ strong field).



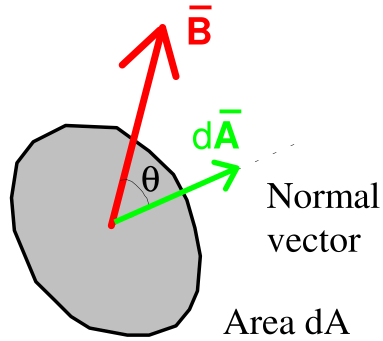
Magnetic flux, ψ

This is defined in exactly the same way as electric flux.

Consider a small flat area dA.

Let $\overline{\mathbf{B}}$ be the magnetic field at its centre.

Assume that dA is so small that $\overline{\mathbf{B}}$ can be regarded as uniform over the whole of dA.



DEFINITION: The MAGNETIC FLUX, dψ through the area dA is the product of dA and the normal component of $\overline{\mathbf{B}}$.

or $d\psi = (B\cos\theta)dS$ so $d\psi = \overline{\mathbf{B}} \cdot d\overline{\mathbf{A}}$

where $d\overline{\mathbf{A}}$ is the NORMAL VECTOR of the area dA:

Magnitude of $d\overline{\mathbf{A}}$ is: dA
Direction of $d\overline{\mathbf{A}}$ is: perpendicular to dA

Note: 1. Magnetic flux is a SCALAR.

2. *Young & Freedman* uses the symbol Φ_B for magnetic flux

3. The SI unit of magnetic flux is the Weber (Wb):

$1 \text{ Wb} \equiv (1 \text{ Tesla})(1 \text{ m}^2)$ or $1 \text{ T} \equiv 1 \text{ Wb m}^{-2}$.

Magnetic flux for the case of a non-uniform field passing through an arbitrary surface

Proceeding exactly as we did for the electric flux (see handout on electric flux and Gauss's Law), we can show that the total magnetic flux crossing a surface S is

$$\Psi = \int_S \overline{\mathbf{B}} \cdot d\overline{\mathbf{A}}$$

What is Ψ for a closed surface?

Recall: Gauss's Law for $\overline{\mathbf{E}}$:

$$\Phi = \oint \overline{\mathbf{E}} \cdot d\overline{\mathbf{A}} = \frac{Q_{enclosed}}{\varepsilon_o}$$

Lines of $\overline{\mathbf{E}}$ begin and end on electric charges. But lines of $\overline{\mathbf{B}}$ form closed loops (there is no equivalent of charge - i.e., no "magnetic monopoles").

⇒ as many magnetic field lines will leave a given volume as enter it (no enclosed "magnetic charge").

⇒ THE TOTAL MAGNETIC FLUX THROUGH A CLOSED SURFACE IS ZERO

or

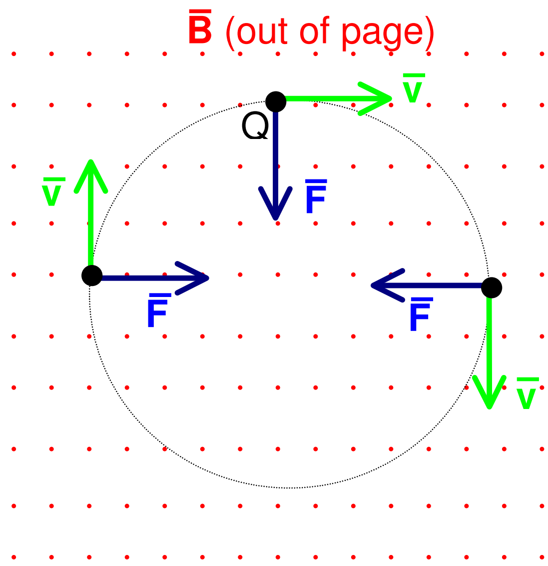
$$\boxed{\Psi = \oint \overline{\mathbf{B}} \cdot d\overline{\mathbf{A}} = 0}$$

THIS IS **GAUSS'S LAW** FOR THE MAGNETIC FIELD
MAXWELL'S 2nd EQUATION

This is sometimes written as $\operatorname{div}\overline{\mathbf{B}} = 0$ or $\nabla \cdot \overline{\mathbf{B}} = 0$.

Force on a charged particle moving in a uniform magnetic field

Consider a uniform $\overline{\mathbf{B}}$ coming out of the page (usually represented by dots):



Let charge Q have velocity $\overline{\mathbf{v}}$ perpendicular to $\overline{\mathbf{B}}$

$$\overline{\mathbf{F}} = Q(\overline{\mathbf{v}} \times \overline{\mathbf{B}})$$

$\overline{\mathbf{F}}$ is always perpendicular to $\overline{\mathbf{v}}$ and $\overline{\mathbf{B}}$

$\Rightarrow$ no component of force along the direction of $\overline{\mathbf{v}}$

$\Rightarrow$ THE SPEED NEVER CHANGES

$F = QvB = \text{constant}$

Constant force $\perp$ to velocity $\Rightarrow$ MOTION IN A CIRCLE

Force required for circular motion is $F = \dfrac{mv^2}{r}$ m = mass

r = radius

$$\Rightarrow \quad QvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{QB}$$

Speed = v, Circumference = $2\pi r$

$$\Rightarrow \text{ Frequency is } \quad f = \frac{v}{2\pi r} = \frac{QB}{2\pi m} = \text{CYCLOTRON FREQUENCY}$$

Note: 1. f is independent of v
2. f depends only on B and fundamental constants
$\Rightarrow$ it can be used to find B.

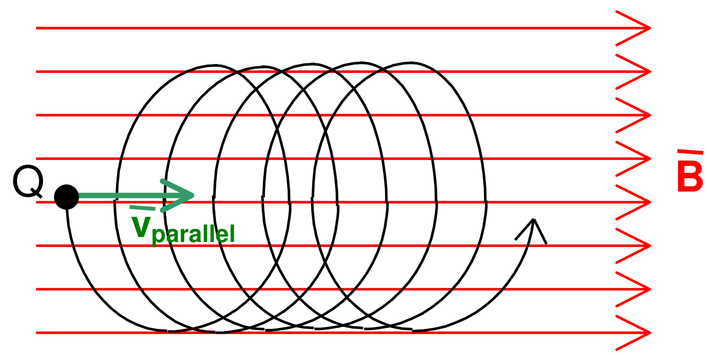
What if $\overline{\mathbf{v}}$ also has a component of motion along the direction of $\overline{\mathbf{B}}$?

Call this component $\overline{\mathbf{v}}_{\mathbf{parallel}}$

$\overline{\mathbf{B}}$ and $\overline{\mathbf{v}}_{\mathbf{parallel}}$ are parallel,

so $\overline{\mathbf{v}}_{\mathbf{parallel}} \times \overline{\mathbf{B}} = 0$

$\Rightarrow$ no force in this direction.



$\Rightarrow$ motion along $\overline{\mathbf{B}}$ is not affected $\Rightarrow$ MOTION IN A SPIRAL

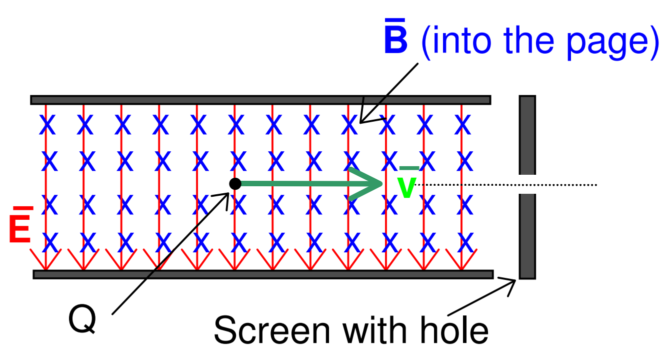
Motion of a charged particle in combined electric and magnetic fields: the Lorentz force

If a point charge Q moves with velocity v in an electric field $\overline{\mathbf{E}}$ and a magnetic field $\overline{\mathbf{B}}$, the resultant force on it is

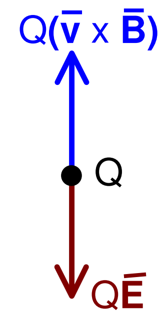
$$\overline{\mathbf{F}} = Q\left[\overline{\mathbf{E}} + (\overline{\mathbf{v}} \times \overline{\mathbf{B}})\right] \qquad \text{THE LORENTZ FORCE}$$

The velocity selector

Consider a charged particle moving with velocity $\overline{\mathbf{v}}$ perpendicular to both an electric field, $\overline{\mathbf{E}}$, and a magnetic field, $\overline{\mathbf{B}}$.



Q experiences both electric and magnetic forces:

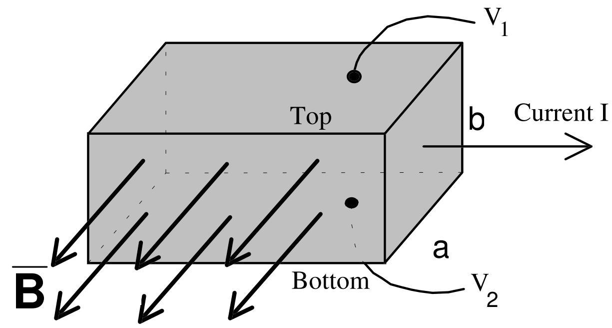


Now $\overline{\mathbf{v}} \perp \overline{\mathbf{B}}$ so $Q\left|\overline{\mathbf{v}} \times \overline{\mathbf{B}}\right| = QvB$

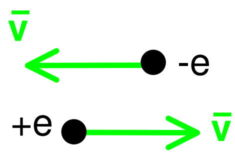
$\Rightarrow$ the two forces balance exactly if $QvB = QE$
$\Rightarrow$ the particle is not deflected if $v = E/B$
$\Rightarrow$ only particles for which $v = E/B$ pass through the hole
$\Rightarrow$ the output beam is of uniform velocity (mono-energetic).

The Hall effect

- Rectangular sample of conductor or semiconductor carrying a current I
- Magnetic field $\overline{\mathbf{B}}$ applied perpendicular to the direction of the current

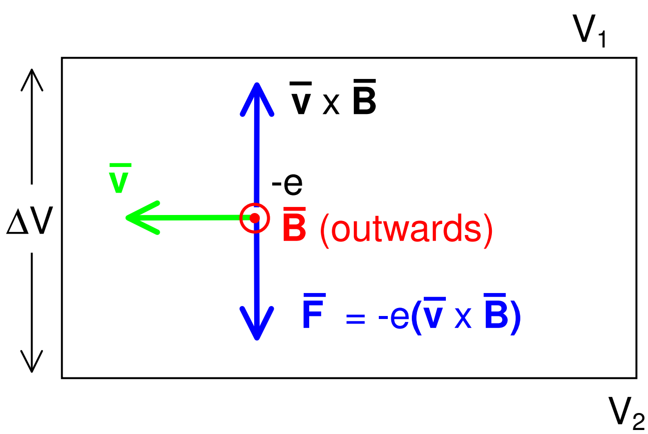


- Current could be carried by either electrons or holes



- Voltmeter between top and bottom sides measures $\Delta V = V_1 - V_2$

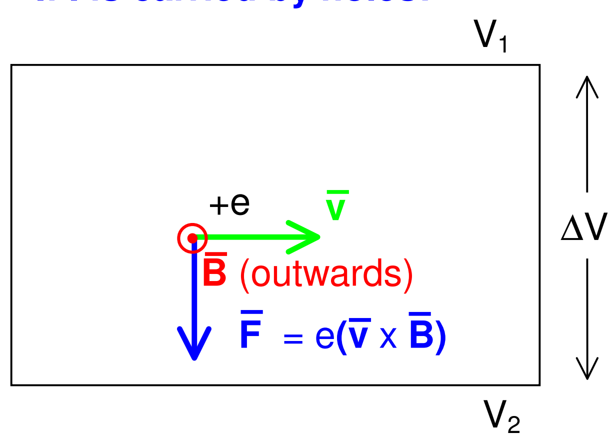
If I is carried by electrons:



Build-up of negative charge on the bottom side

$V_1 > V_2$ ΔV is positive

If I is carried by holes:



Build-up of positive charge on the bottom side

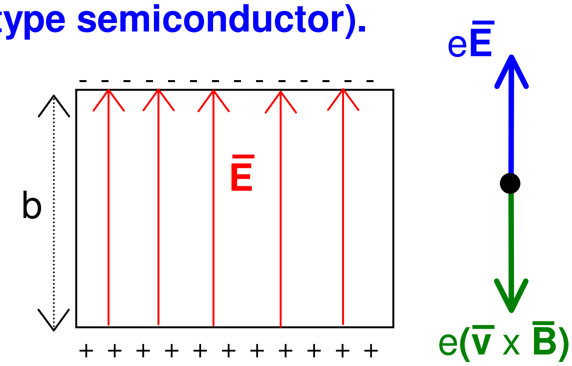
$V_2 > V_1$ ΔV is negative

$\Rightarrow$ **we can find out whether the current is carried by electrons (n-type semiconductor) or holes (p-type semiconductor).**

Charge build-up $\rightarrow$ electric field, $\overline{\mathbf{E}}$, between top and bottom sides

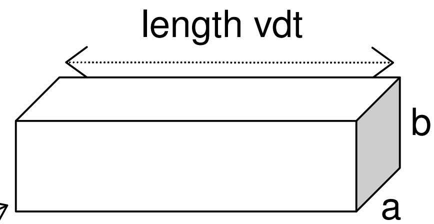
$\overline{\mathbf{E}}$ opposes further charge build-up

$\Rightarrow$ Equilibrium established when $eE = evB$



$\Rightarrow e\,(\Delta V/b) = evB \quad \Rightarrow \quad \Delta V = vBb$ (b = distance between top and bottom sides)

We can relate the speed, v, to the current, I:



Let n = no. of carriers per unit volume (each with charge e)

Consider the amount of charge, dQ, crossing area ab in time dt:

dQ = amount of charge in <u>this volume</u>

So $dQ = ne(ab)(vdt) \Rightarrow I = dQ/dt = neabv$

So $v = \dfrac{I}{neab} \Rightarrow \Delta V = \dfrac{IB}{nea}$ ΔV is the HALL POTENTIAL

By measuring the sign and magnitude of ΔV we can :

- Find n if B is known - i.e., investigate the sign and number density of the charge carriers in a sample of material

- Or find B if n is known - i.e., measure an unknown magnetic field with a HALL PROBE