Queen Mary
University of London

BSc/MSci RESIT EXAMINATION

PHY-304 Physical Dynamics

Time Allowed: 2 hours 15 minutes

Date: May 2005

Time:

Instructions: **You should attempt THREE questions, answering the ONE question from section A and TWO questions from section B. The compulsory question in section A is worth 40 %; the questions in section B are worth 30 % each. The mark** *provisionally allocated* **for each sub-section of a question is indicated in square brackets. Calculators are permitted in this paper.**

Mathematical formulae: A formula sheet containing mathematical results that may be of help in various questions appears at the end of the examination paper.

DO NOT TURN TO THE FIRST PAGE OF THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR

 Version Final

SECTION A: Answer all parts of the **ONE** question in this Section.

1 (a) A system of N point particles with masses m_i, $i = 1, \ldots, N$, moves under the influence of external and internal forces where $\boldsymbol{F}_i^{(e)}$ is the external force on particle i and $\boldsymbol{F}_{ij}$ is the force on particle i due to particle j.

i) Making appropriate assumptions about forces and starting from Newton's Second Law applied to each particle in an inertial frame of reference, show that the total linear momentum of the system satisfies the equation **[6]**

$$\frac{d\boldsymbol{P}_{tot}}{dt} = \boldsymbol{F}_{tot}^{(e)} \quad .$$

ii) What additional assumptions about forces are necessary in order that the angular momentum of the system as a whole is conserved? **[3]**

iii) Define what is meant by the centre of mass $\boldsymbol{R}$ of this system and by the centre of mass frame of reference. Show that the total kinetic energy of the system in an inertial frame T can be expressed as

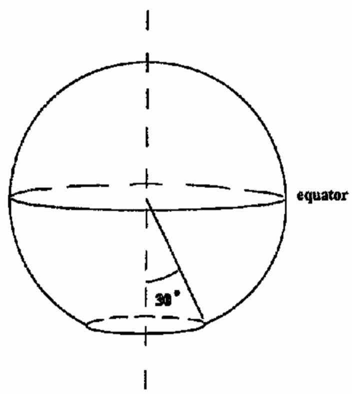
$$T = \frac{1}{2}M\dot{\boldsymbol{R}}^2 + T^* \quad ,$$

where M is the total mass of the system and T^* is the kinetic energy in the centre of mass frame. **[9]**

(b) Explain what is meant by a set of generalised coordinates q_j for a conservative mechanical system. Define the Lagrangian for the system and state Lagrange's equations in terms of the generalised coordinates. **[4]**

i) A particle of mass m is constrained to slide under the influence of gravity and without friction on the inner surface of a sphere of radius a. Using an appropriate set of spherical polar coordinates, write down the Lagrangian function L for this system and obtain Lagrange's equations of motion for the point mass. Obtain expressions for all the conserved quantities for this system. **[10]**

ii) At the bottom of the spherical surface a small circular hole is cut out whose radius subtends an angle of 30 ° at the centre of the sphere. At time $t = 0$ the particle is located at a point on the equator of the sphere and is moving with a velocity $\sqrt{ga}\boldsymbol{e}_\theta + 2\sqrt{ga}\boldsymbol{e}_\phi$ where $\boldsymbol{e}_\theta$ and $\boldsymbol{e}_\phi$ are the usual unit vectors in spherical polar coordinates. Find the values of the conserved quantities for this set of initial conditions. Show that in the subsequent motion the particle will not fall through the hole in the bottom of the sphere. **[8]**



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SECTION B:

Answer TWO questions from this Section.

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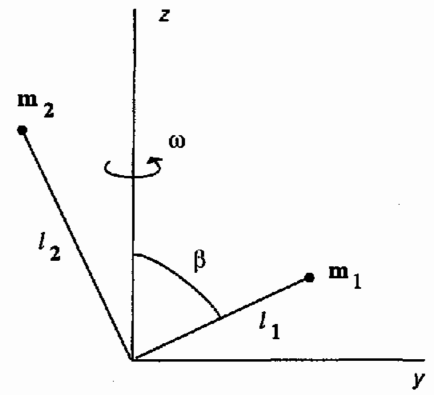
(a) In an inertial frame of reference a rigid body rotates at angular velocity $\boldsymbol{\omega}$ about a fixed axis. Show that the angular momentum of the rigid body, $\boldsymbol{L}$, is related to its angular velocity $\boldsymbol{\omega}$ by a linear equation of the form $\boldsymbol{L} = \boldsymbol{I} \cdot \boldsymbol{\omega}$, where the moment of inertia tensor $\boldsymbol{I}$ is represented as a three-by-three matrix. Obtain explicit expressions for the elements of this matrix. Explain what is meant by a *principal axis* system for a rigid body. In a certain Cartesian coordinate system a rigid body is reflection-symmetric with respect to the $x-y$ and $x-z$ coordinate planes. Prove that this is a principal axis system. **[12]**

(b) Two point masses m_1 and m_2 are attached to the ends of light rigid rods of lengths l_1 and l_2 respectively. As shown in the figure below, the other ends of the rods are fixed rigidly at right angles to each other and are also fixed rigidly to a light bearing mounted on a vertical shaft about which the whole assemblage rotates at constant angular velocity $\boldsymbol{\omega}$. The figure shows the system at an instant when the masses and rods lie in the $y-z$ coordinate plane with the bearing located at the origin and the angle β denoting the fixed angle between the vertical and the rod ℓ_1.

i) Calculate the elements of the moment of inertia tensor for the system of two masses in the coordinate system shown. What condition must hold for the quantities m_1, m_2, l_1, l_2 in order that this be a principal axis system? **[10]**

(ii) Write down the components of the angular momentum vector $\boldsymbol{L}$ for the rotating assembly. State how the net torque acting on the system is related to $\boldsymbol{L}$. Determine the components of the net torque vector at the instant shown below. Determine the value of the angle β at which this torque is greatest in magnitude. **[8]**



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(a) Briefly describe (no detailed mathematical derivations) examples of *precession* in each of the following physical systems:

i) A moving bicycle wheel

ii) The rotating Earth **[10]**

(b) The Euler equations of motion for a torque-free rotating rigid body have the form

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3 \quad ,$$
$$I_2\dot{\omega}_2 = (I_3 - I_1)\omega_3\omega_1 \quad ,$$
$$I_3\dot{\omega}_3 = (I_1 - I_2)\omega_1\omega_2 \quad .$$

In what frame of reference are these equations valid? Why is this frame of reference useful? What are the quantities I_1, I_2, I_3? **[4]**

For a particular rigid body we have $I_1 > I_2 > I_3$. The angular velocity vector for the body has the form $\boldsymbol{\omega} = (\epsilon_1, \omega_2, \epsilon_3)$ where $|\epsilon_1| << \omega_2$ and $|\epsilon_3| << \omega_2$. Working to first order in the small quantities ϵ_1 and ϵ_3 use the Euler equations to show that the motion is unstable in the sense that ϵ_1 and ϵ_3 can grow exponentially quickly. Obtain an expression for the rate of this exponential growth in terms of ω_2, I_1, I_2, I_3.

[16]

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(a) Briefly compare and contrast the Lagrangian and Hamiltonian formulations of classical mechanics. Explain how the Hamiltonian function is defined and write down the Hamilton equations of motion for a system of n degrees of freedom. [10]

(b) Define what is meant by the *action* for a mechanical system and state Hamilton's principle. [5]

(i) In special relativity the only time and space dependent quantity, invariant under Lorentz transformations, that we can form for a free point particle of rest mass m_0 is the invariant interval associated with its worldline. Use the invariant interval to formulate the action for the particle. Use Hamilton's principle to deduce the Lagrangian L for the free relativistic particle. [9]

(ii) Starting from the Lagrangian for the free relativistic particle obtain its Hamiltonian. [6]

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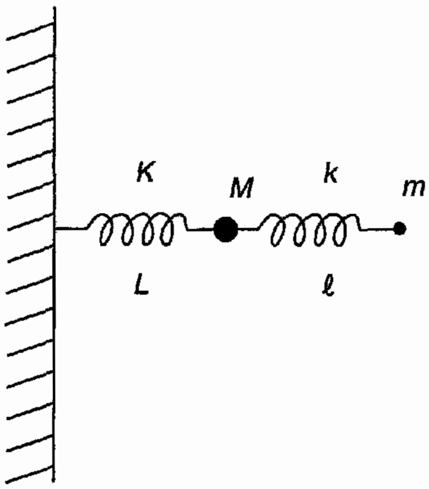
(a) A conservative mechanical system of n degrees of freedom is disturbed slightly from a stable equilibrium configuration. Explain briefly what is meant by the *small oscillation approximation* and by a *normal mode* of such a system. How many normal modes will such a system possess? Briefly describe one experimental technique used to explore the normal mode frequencies and amplitudes for a molecule undergoing small oscillations. **[10]**

(b) A diatomic molecule consisting of two atoms of mass M and m respectively is adsorbed on a solid surface. We can model the forces of attraction in this system (see below) by a spring of force constant K and natural length L holding the atom M to the wall and a second spring of force constant k and natural length ℓ holding atom m to atom M. Assuming that the atoms move only in the direction normal to the crystal surfaces, write down the Lagrangian for the molecule using appropriate coordinates. Obtain the small oscillation equations of motion for the system. **[8]**

(c) Determine the frequencies and amplitudes of the normal modes of this system if $M = 4m$ and $K = 3k$. **[12]**



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(a) The Euler equation of motion for an ideal incompressible fluid can be written as

$$\rho\frac{D\boldsymbol{v}}{Dt} = \rho\frac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \boldsymbol{F} - \nabla p \quad .$$

Explain briefly the meaning of the quantities $\frac{D\boldsymbol{v}}{Dt}$ and $\boldsymbol{F}$. [6]

(b) Starting from Euler's equation, stating clearly any assumptions made, derive Bernoulli's theorem. Explain qualitatively why a light ball supported in a vertical jet of air is stable. [12]

(c) A canal of rectangular cross-section and horizontal bed carries water of depth d in a channel of width w at a steady speed v_0. At some distance downstream the width remains the same but the canal bed rises through a height L, where $L < d$, to form another horizontal portion of bed. Use Bernoulli's theorem to discuss the flow in this raised region. Show that the water surface is lowered relative to its upstream level as it flows over this elevated portion of bed. [12]

END OF EXAM - R. B. JONES

FORMULA SHEET

PLANE POLAR COORDINATES:

$$\boldsymbol{r} = r\,\boldsymbol{e}_r \quad ,$$

$$\boldsymbol{v} = \dot{r}\,\boldsymbol{e}_r + r\dot{\theta}\,\boldsymbol{e}_\theta \quad ,$$

$$T = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \quad .$$

SPHERICAL POLAR COORDINATES:

$$\boldsymbol{r} = r\,\boldsymbol{e}_r \quad ,$$

$$\boldsymbol{v} = \dot{r}\,\boldsymbol{e}_r + r\dot{\theta}\,\boldsymbol{e}_\theta + r\sin\theta\dot{\phi}\,\boldsymbol{e}_\phi \quad ,$$

$$T = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\right) \quad .$$

CYLINDRICAL POLAR COORDINATES:

$$\boldsymbol{r} = \rho\,\boldsymbol{e}_\rho + z\,\boldsymbol{e}_z \quad ,$$

$$\boldsymbol{v} = \dot{\rho}\,\boldsymbol{e}_\rho + \rho\dot{\theta}\,\boldsymbol{e}_\theta + \dot{z}\,\boldsymbol{e}_z \quad ,$$

$$T = \frac{m}{2}\left(\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2\right) \quad .$$

VECTOR IDENTITIES

$$\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c}) = \boldsymbol{b}\cdot(\boldsymbol{c}\times\boldsymbol{a}) = \boldsymbol{c}\cdot(\boldsymbol{a}\times\boldsymbol{b})$$

$$\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}$$

$$(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = \frac{1}{2}\boldsymbol{\nabla}v^2 + (\boldsymbol{\nabla}\times\boldsymbol{v})\times\boldsymbol{v} \quad .$$