Newton to Einstein

Section 0: Basic Definitions and Concepts

The Principle of Relativity. Galileo and Newton took this as fundamental. You cannot determine, from measurements carried out in a closed laboratory, if you are stationary or travelling at some speed or another. So any statement of position or of velocity always begs the question, position or velocity relative to what?

Frames of Reference. These are what positions and velocities (and accelerations) can be measured with respect to (relative to). A frame of reference may be thought of as points laid out from an origin with metre rules and with clocks, so that any point has coordinates (x, y, z) and any event has coordinates (x, y, z, t). A Cartesian frame has rectilinear coordinates, but in general a frame of reference may be accelerating and rotating, and its geometry may be curvilinear.

Inertial and Non-Inertial Frames of Reference.
Cartesian and Non-Cartesian Frames of Reference.
Euclidean and Non-Euclidean Frames of Reference.
Most of our every-day frames of reference are non-inertial, non-Cartesian or non-Euclidean. So, e.g., a sailor has to handle spherical geometry to navigate, and a meteorologist needs to account for the Earth's rotation to make good weather forecasts.

Scalars and Vectors. Some physical quantities are simply magnitudes at a point (e.g. temperature). Some are magnitudes between two points (e.g. distance, voltage). These are scalar quantities. Some physical quantities have magnitude *and* direction at a point (e.g. wind velocity, electric field) or between two points (e.g. displacement). These are vector quantities. A vector is something having both magnitude and direction.

Scalar and Vector Notation. A scalar will always be italic regular font (x, T). A vector may be embellished italic or simply bold Roman ($\underline{w}$, $\vec{u}, \vec{v}, \mathbf{x}$). The magnitude of the vector $\vec{v}$ may be expressed as its *modulus*, $|\vec{v}|$, or simply by the same symbol as a scalar, $u \equiv |\mathbf{u}|$.

Vector Components. A vector is a physical thing, with a real magnitude and a real direction (e.g. an arrow drawn on the blackboard). Given a frame of reference, or coordinate system, the vector acquires *components*, which are the *projections* of the vector onto the axes of the coordinate system.

Vector Component Equations. The following equations are easily derived from trigonometry and Pythagoras and are essential for vector manipulation:

$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$\theta = \tan^{-1}\frac{y}{x}$$

Vector Relationships. Vectors can be

- Added. Parallelogram rule, or add the components.
- Multiplied, by the dot product, yielding a scalar:
  $$\vec{u}.\vec{v} = uv\cos\theta$$
- Multiplied, by the cross product, yielding a vector normal to both:
  - Magnitude: $|\vec{u}\times\vec{v}| = uv\sin\theta$
  - Direction: $\vec{w} = \vec{u}\times\vec{v}$ is at right angles to both $\vec{u}$ and $\vec{v}$
  - And $\vec{v}\times\vec{u} = -\vec{w}$
  - Right-Hand Rule: Rotate from $\vec{u}$ to $\vec{v}$, then $\vec{u}\times\vec{v}$ points along the direction a screw would move in under that rotation.