Newton to Einstein

Mid-Term Test. 9am., Thursday 15th Nov. 2012
Duration: 50 minutes.

Use g = 10 m s^{-2} for numerical calculations.
You may use the following expressions for the dot and cross products:

$$\mathbf{u}.\mathbf{v} = u_x v_x + u_y v_y + u_z v_z$$

$$\mathbf{u} \times \mathbf{v} = \left(u_y v_z - u_z v_y, u_z v_x - u_x v_z, u_x v_y - u_y v_x\right)$$

There are **25** marks available in Part A and **25** in Part B.

Section A: Answer **ALL** Questions

A1. Given the vectors **u** = (1, 0, 0) and **v** = (1, 1, 0), find
   a) **u** + **v**
   b) **u.v**
   c) **u** × **v** **[3]**

A2. State Newton's Second Law of Motion, and explain what it says about momentum. **[3]**

A3. A ramp is at 30º to the horizontal, and a mass *m* sits on it without sliding.

   a) Give the force exerted by the ramp on the mass, and resolve it into vertical and horizontal components.

   b) Resolve the force exerted by the ramp on the mass into components parallel and perpendicular to the ramp.

   c) What is the minimum coefficient of friction? **[6]**

A4. A steel ball weighs 2kg.

   a) The ball is lifted through 20 metres vertically. Calculate its gravitational potential energy.

   b) The ball is then dropped. Calculate its kinetic energy and velocity when it has fallen 20 metres. **[3]**

A5. A dumbbell consists of two 1kg point masses, connected by a light rod one metre long. Calculate its moment of inertia about

   a) an axis perpendicular to the rod and passing through the centre,

   b) an axis perpendicular to the rod and passing through one of the masses. **[4]**

   c) State Kepler's Third Law. Derive it for a planet in a circular orbit in an inverse-square law gravitational field. **[3]**

A6. A comet is travelling at 12 km s^{-1} as it crosses Jupiter's orbit on its way to the Sun. What is its speed later in its orbit when it crosses Jupiter's orbit again? Explain your reasoning. **[3]**

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Section B. Answer **ONE** Question

B1. a) An empty cylindrical can (with no ends) weighing 100g rolls along the floor at 4 ms^{-1}. Calculate its total kinetic energy (translational plus rotational). **[5]**

b) A pendulum consists of a 10 kg point bob hanging on a light string of length 10m. It swings through an angle 2θ, i.e. θ each side of vertical.

(i) Find the tension T in the string at the end-points of the motion, i.e. when the pendulum is at the angle θ.

(ii) Give the value of the tension in the string at the end-points if the angle θ is 90°.

(iii) Give the value of the acceleration of the bob at the end-points if the angle θ is 90°. **[6]**

c) The pendulum of part (b) is swinging with the angle θ = 90°. By considering potential and kinetic energy, find

(i) the velocity of the bob at the lowest point of the motion

(ii) the tension in the string at this point. **[7]**

d) Given that $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, the mass of the Earth $M = 5.97 \times 10^{24}$ kg and the radius of the earth is R = 6380 km,

(i) Find the escape velocity for a rocket at 200km altitude.

(ii) Find the orbital speed of a satellite in a circular orbit at 200 km altitude. **[7]**

B2. a) State Kepler's Second Law and explain how it relates to angular momentum. **[5]**

b) A flywheel with a moment of inertia I = 5 kg m^2 is mounted on a light axle. It is initially stationary. A torque of 100 N m is applied to the axle for 100 s.

(i) Calculate the final angular velocity of the flywheel..

(ii) Calculate the final angular momentum of the flywheel.

(iii) Calculate the final rotational energy of the flywheel.

(iv) If a torque of 10 N m is now applied about an axis perpendicular to the axle of the flywheel, what will the motion be? Explain your answer. **[9]**

c) The potential energy of a body varies with position as $U = 2x^2$.

(i) Give an expression for the force F on the body and sketch $F(x)$.

(ii) At what position will the body be in stable equilibrium? Explain your answer. **[5]**

d) Given two vectors **u** and **v**, with magnitudes u = 1 and v = 2, and given that the magnitude of the cross product $|u \times v|$ = 1, find

(i) The angle between **u** and **v**.

(ii) The value of the dot product **u.v**. **[6]**

End of paper
Prof. D.J. Dunstan