From Newton to Einstein

Mid-Term Test, 12a.m. Thur. 13th Nov. 2008
Duration: 50 minutes. There are 20 marks in Section A and 30 in Section B.

Use g = 10 ms^{-2} in numerical calculations.
You may use the following expressions for vector dot and cross products:

$$\mathbf{u.v} = u_x v_x + u_y v_y + u_z v_z$$
$$\mathbf{u} \times \mathbf{v} = \left(u_y v_z - u_z v_y, u_z v_x - u_x v_z, u_x v_y - u_y v_x\right)$$

Section A: Answer ALL Questions

A1. State, giving reasons, whether the following quantities are vectors or scalars:
   a) (1, 2, 3)
   b) $\left(\frac{\partial}{dx} + \frac{\partial}{dy} + \frac{\partial}{dz}\right)$
   c) $0\hat{\mathbf{i}} + 1\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ **[3]**

A2. State Newton's Third Law of Motion, and explain what it says about momentum. **[3]**

A3. A ramp is at 45º to the horizontal, and a mass m slides down at constant speed.
   a) Indicate on a sketch the force exerted by gravity on the mass, and resolve it into vertical and horizontal components. Give their values.
   b) On a new sketch, indicate the components parallel and perpendicular to the ramp of the force exerted by gravity on the mass. Give their values.
   c) On a new sketch, indicate the force exerted by the ramp on the mass. Give its value.
   d) Calculate the coefficient of friction between the mass and the ramp. **[7]**

A4. A light stick of length 2m has 1 kg point masses attached at the centre and at one end (end A). Find the moments of inertia about
   a) An axis perpendicular to the stick and passing through the centre.
   b) An axis perpendicular to the stick and passing through the other end (end B) **[3]**

A5. A planet mass M in a circular orbit around a star has an orbital speed of v. Give expressions for its kinetic energy and gravitational potential energy. **[4]**

Paper continues

Section B: Answer ONE Question

B1. a) With the technology available to the Ancient Greeks, how could they have found

(i) The diameter of the Earth,

(ii) The distance from the Earth to the Moon,

(iii) The distance from the Earth to the Sun? **[9]**

b) A flywheel with a moment of inertia I = 10 kg m^2 is mounted on a light axle. It is initially stationary. A torque of 10 Nm is applied to the axle for 100 s.

(i) Calculate the final angular velocity of the flywheel.

(ii) Calculate the final angular momentum of the flywheel.

(iii) Calculate the final rotational energy of the flywheel.

(iv) If a torque of 5 N m is now applied about an axis perpendicular to the axle of the flywheel, what will the motion be? Explain your answer.**[12]**

c) The potential energy of a nitrogen atom in an ammonia (NH_3) molecule varies with position as $U = x^4 - 4x^2$.

(i) Sketch $U(x)$. Give an expression for the force F on the nitrogen atom and sketch $F(x)$.

(ii) At what positions will the nitrogen atom be in stable equilibrium? Explain your answer. **[9]**

B2. a) A pendulum consists of a 2 kg bob attached to a high ceiling by a 10 m light rope. It is pulled to one side and released to swing back and forth. As the rope swings through the vertical, the speed of the bob is 14.14 $m.s^{-1}$.

(i) What is the acceleration of the bob, in magnitude and direction, at this instant?

(ii) What is the tension in the rope at this instant?

(iii) How close to the ceiling will the ball be at the end-points of the motion? **[9]**

b) A steel hoop weighing 5kg rolls without slipping down a ramp inclined at 30° to the horizontal. Calculate its acceleration. **[9]**

c) Given the planet of Question A5, if its speed v = 10 kms^{-1}, to what speed must it be accelerated if it is to leave its circular orbit and escape to infinity? Does it matter in what direction it is accelerated? Give reasons. **[12]**

End of Paper

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