PHY4116 From Newton to Einstein
Coursework Sheet 6: Gravity & Coriolis force

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You will need the following constants:

G=6.67 ×10^{-11} Nm^{2}kg^{-2}
R_{Earth} = 6.37×10^6 m M_{Earth} = 5.97×10^{24} kg
R_{Moon} = 1.74×10^6 m M_{Moon} = 7.35×10^{22} kg

A1) State Kepler's Third Law. Derive it for a planet in a circular orbit in an inverse-square law gravitational field. [5]

A2) a) Calculate the escape velocity for an object on the surface of the earth.
b) Repeat the above calculation for the surface of the moon.
c) Given that the average. speed for a typical air molecule is 100s of ms^{-1} comment on the stability of a hypothetical lunar atmosphere. [3]

A3) Prove that an object of mass m in circular orbit, radius r, around an object of mass M has total energy: $-\frac{GmM}{2r}$. [2]

A4) A body travels in a highly elliptical orbit around the earth. Given that its speed at a radius of 10^{8}m is 2.2×10^{3}ms^{-1} (the apogee),
a) Calculate its speed at the perigee, where the radius is 2×10^{7}m.
b) What is the angle between its direction of travel (its velocity) and its radius (its position vector) when it is at a distance of 8×10^{7}m from the earth? [3]

A5) A planet of mass m is in a circular orbit of radius R about a star of mass $M >> m$. Calculate its kinetic energy and its gravitational potential energy (the zero of potential energy is at infinity). Hence or otherwise show that the escape velocity is $\sqrt{2}$ times the orbital velocity. [5]

B6) A child is on a playground roundabout rotating clockwise (viewed from above) and crawls radially towards the centre of the roundabout.
a) Explain what the Coriolis force is and why the child experiences it.
b) On a suitable sketch, indicate the direction of the Coriolis force.
c) What can you say about the force (magnitude and direction) as the child crosses the centre of the roundabout and then moves radially outwards?
iv) Give a derivation for the magnitude of the Coriolis force and calculate it for a rotation speed of 1 radian per second, mass of child 20 kg, and the radial speed of child is 1 ms^{-1}. [9]