

# BSc/MSci MidTerm Test

PHY-217

Vibrations and Waves

Time Allowed: 40 minutes

Date: 18<sup>th</sup> Nov, 2011

Time: 9:10 - 9:50

Instructions: **Answer ALL questions in section A. Answer ONLY ONE questions from section B. Section A carries 25 marks, each question in section B carries 25 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.**

Numeric calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work which is not to be assessed.

**Important Note:** The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from the college. Please check now to ensure that you do not have any notes in your possession. If you have any then please raise your hand and give them to an invigilator immediately. Exam papers cannot be removed from the exam room

**You are not permitted to read the contents of this question paper until instructed to do so by an invigilator.**

*Examiner: Dr L.Cerrito*

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## SECTION A. Attempt answers to all questions.

- A1.** For a particle of mass  $m$  executing simple harmonic motion (“SHM”), its displacement from equilibrium can be written as:

$$x(t) = A\sin(\omega_0 t + \phi).$$

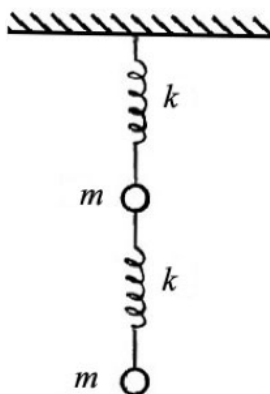
- (i) Write down the amplitude, angular frequency, period and phase at time  $t$ . [3]
  - (ii) Sketch a graph of  $x(t)$ . [2]
- A2.** A mass  $m$  is attached to a massless spring with constant  $k$ , and can move horizontally on a frictionless plane without air resistance.
- (i) By explicit consideration of the forces involved, derive the differential equation of motion and write down the general solution. [2]
  - (ii) What is the angular frequency, expressed in terms of  $k$  and  $m$ ? [2]
  - (iii) Write down the potential, kinetic, and total energy at time  $t$ . [2]
  - (iv) Write down an example of initial conditions. [1]
- A3.** Show that the function  $y = A\cos(4x) + B\sin(4x)$ , where  $A$  and  $B$  are arbitrary constants, is a general solution of the differential equation: [4]

$$\frac{d^2 y}{dx^2} + 16y = 0.$$

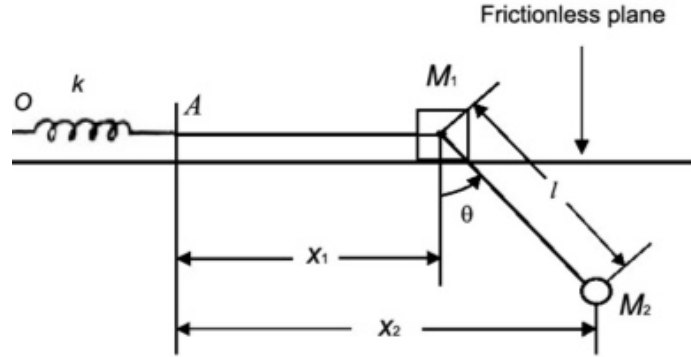
- A4.** For an object starting from maximum displacement and released from rest, sketch plots of displacement versus time for: (i) underdamped SHM, (ii) critically damped SHM and (iii) overdamped SHM. (iv) How is the oscillator’s Quality Factor,  $Q$ , defined? [4]
- A5.** Consider a damped driven simple harmonic oscillator in steady state. Sketch plots of (i) amplitude versus driving frequency, and (ii) average power versus driving frequency. Indicate on each plot the resonance frequency and what effect high or low oscillator’s quality factor,  $Q$ , has on the graph. [3]
- A6.** Consider  $N$  equal masses coupled by springs: how many normal modes does such a system have? For a normal mode, what characterises the motion of each of the masses? [2]

**SECTION B. Answer one of the two questions in this section.**

- B1.** Two equal masses are connected as shown with two identical massless springs of spring constant  $k$ .
- (i) By explicit consideration of the forces and considering only motion in the vertical direction, derive the equations of motion of the two masses. Do you need to consider gravitational forces? [10]
- (ii) Show that the angular frequencies of the two normal modes are given by  $\omega^2 = (3 \pm \sqrt{5})k/2m$  and hence that the ratio of the normal mode frequencies is  $(\sqrt{5}+1)/(\sqrt{5}-1)$ . [8]
- (iii) Find the ratio of amplitudes of the two masses in each separate mode. [7]



- B2.** The sketch shows a mass  $M_1$  on a frictionless plane connected to support  $O$  by a spring of stiffness  $k$ . The mass  $M_2$  is supported by a string of length  $l$  from  $M_1$ .  $OA$  is the length of the relaxed spring.  $x_1$  and  $x_2$  are the positions of  $M_1$  and  $M_2$ , respectively, relative to point  $A$ .



- (i) Assuming small angles  $\theta$ , write down the differential equation of motion for  $M_1$ . [8]
- (ii) Assuming small angles  $\theta$ , write down the differential equation of motion for  $M_2$ . [7]
- (iii) Now let  $O$  oscillate harmonically in the horizontal direction, driven by an external force, and its position  $X(t)$  is given by  $X_0 \cos(\omega t)$ . Let both masses be equal to  $M$ . Write down the differential equation of motion for each mass. [5]
- (iv) What are the amplitudes (steady state) of the two masses as a function of  $k$ ,  $M$ ,  $X_0$ ,  $l$  and  $\omega$ ? (*Hint: use Cramer's rule*) [5]