BSc/MSci MidTerm Test

PHY-217 Vibrations and Waves

Time Allowed: 40 minutes

Date: 19^{th} Nov, 2010

Time: 14:10 - 14:50

Instructions: **Answer ALL questions in section A. Answer ONLY ONE questions from section B. Section A carries 30 marks, each question in section B carries 20 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.**

Numeric calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Important Note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from the college. Please check now to ensure that you do not have any notes in your possession. If you have any then please raise your hand and give them to an invigilator immediately. Exam papers cannot be removed from the exam room

You are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Examiners: Dr L.Cerrito, Dr. A.Brandhuber

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SECTION A. Attempt answers to all questions.

A1. For a particle of mass m executing simple harmonic motion ("SHM"), its displacement from equilibrium can be written as:

$$x(t) = Asin(\omega_0 t + \phi),$$

(i) Write down the amplitude, angular frequency, period and phase at time t. **[4]**

(ii) Sketch a graph of $x(t)$. **[4]**

A2. A mass m is attached to a massless spring with constant k, and can move horizontally on a frictionless plane without air resistance.

(i) By explicit consideration of the forces involved, derive the differential equation of motion and write down the general solution. **[3]**

(ii) What is the angular frequency, expressed in terms of k and m? **[2]**

(iii) Write down the potential, kinetic, and total energy at time t. **[2]**

(iv) Write down an example of initial conditions. **[1]**

A3. For an object starting from maximum diplacement and released from rest, sketch plots of displacement versus time for: (i) underdamped SHM, (ii) critically damped SHM and (iii) overdamped SHM. **[5]**

A4. Consider a damped driven SHO in steady state. Sketch plots of (i) amplitude versus driving frequency, and (ii) average power versus driving frequency. Indicate on each plot the resonance frequency and what effect high or low oscillator's quality, Q, has on the graph. **[5]**

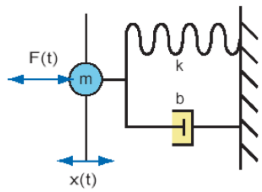
A5. Consider N equal masses coupled by springs: how many normal modes does such a system have? For a normal mode, what characterises the motion of each of the masses? **[4]**

SECTION B. Answer one of the two questions in this section.

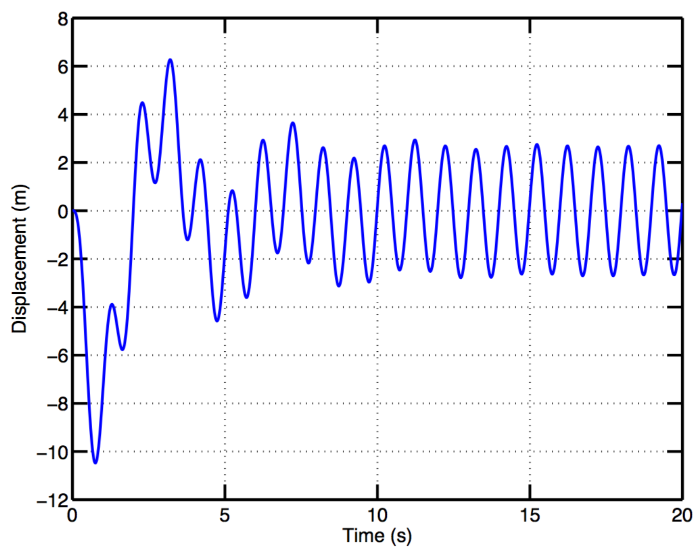
B1. Consider the simple damped spring-mass system shown in the first figure. The mass is driven by an external force given by:

$$F(t) = F_0 \sin(\omega t + \phi)$$

The mass is at rest at its equilibrium position, $x = 0$, when the force is turned on instantaneously at $t = 0$. The response of the mass to this driving force is shown in the



second figure. Assuming that the mass is $m = 1$ kg, use the graph of $x(t)$ (Displacement) to get *estimates*, giving a **thorough** explaination, for:



(i) The natural frequency of the undamped oscillator, $\omega_0/(2\pi)$ in Hz.
Hint: You may assume that γ *is small, so that* $\omega_1 \equiv \sqrt{\omega_0^2 - \gamma^2/4} \approx \omega_0$ **[10]**

(ii) The damping coefficient, b in N s/m. **[10]**

B2. The sketch below shows a mass M_1 on a frictionless plane connected to support O by a spring of stiffness k. Mass M_2 is supported by a string of length l from M_1. OA is the length of the relaxed spring. x_1 and x_2 are the positions of M_1 and M_2, respectively, relative to point A. The figure is not to scale; x_1 is much smaller than OA.

(i) By explicitly considering and *clearly identifying* all relevant forces, and assuming small angles θ, determine the differential equation of motion for M_1. **[10]**

(ii) By explicitly considering and *clearly identifying* all relevant forces, and assuming small angles θ, determine the differential equation of motion for M_2. **[10]**