PHY-217
Vibrations and Waves

Midterm Test
Time Allowed: 40 minutes
Date: 19 Nov. 2009
Time: 15:10
Answer ALL questions in section A. ONE question in section B.
There are 30 marks in total

NUMERIC CALCULATORS ARE PERMITTED IN THIS EXAMINATION

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO

Section A

A1: For a particle of mass m executing simple harmonic motion ("SHM"), its displacement from equilibrium can be written as:

$$x(t) = Asin(\omega_0 t + \phi),$$

(i) At time t, write down the amplitude, angular frequency, period, phase, and sketch a graph of $x(t)$. Explain your reasoning. [1 marks]
(ii) At time $t = 0$, write down the phase, the displacement, the kinetic energy, and the potential energy. Explain your reasoning. [1 marks]

A2: A mass m is attached to a massless spring with constant k. The mass can move vertically without air resistance or frictions.
(i) By explicit consideration of the forces involved, derive the differential equation of motion and write the general solution. [2 marks]
(ii) What is the angular frequency, expressed in terms of k and m? [1 marks]
(iii) Write down an example of initial conditions. [1 marks]

A3: Show that the function $y = Acos(4x) + Bsin(4x)$, where A and B are arbitrary constants, is a general solution of the differential equation: [1 marks]

$$\frac{d^2y}{dx^2} + 16y = 0$$

A4: For an object starting from maximum diplacement and zero velocity, sketch plots of amplitude versus time for (i) undamped SHM, (ii) underdamped SHM, (iii) critically damped SHM and (iv) overdamped SHM. [3 marks]

A5: Consider a damped driven SHO in steady state. Sketch plots of (i) amplitude versus driving frequency, and (ii) average power versus driving frequency. Indicate on each plot the resonance frequency and what effect does high or low Q have on the graph. [3 marks]

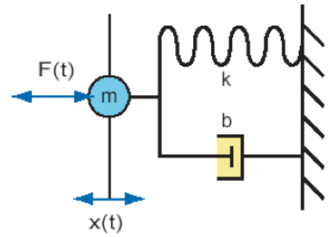
A6: Consider a vibrating system of N masses coupled by springs: explain what is meant by the term "normal mode"? How many normal modes does such system have? If the system is vabrating in one of its normal modes, what characterises the motion of each of the masses? [2 marks]

Section B

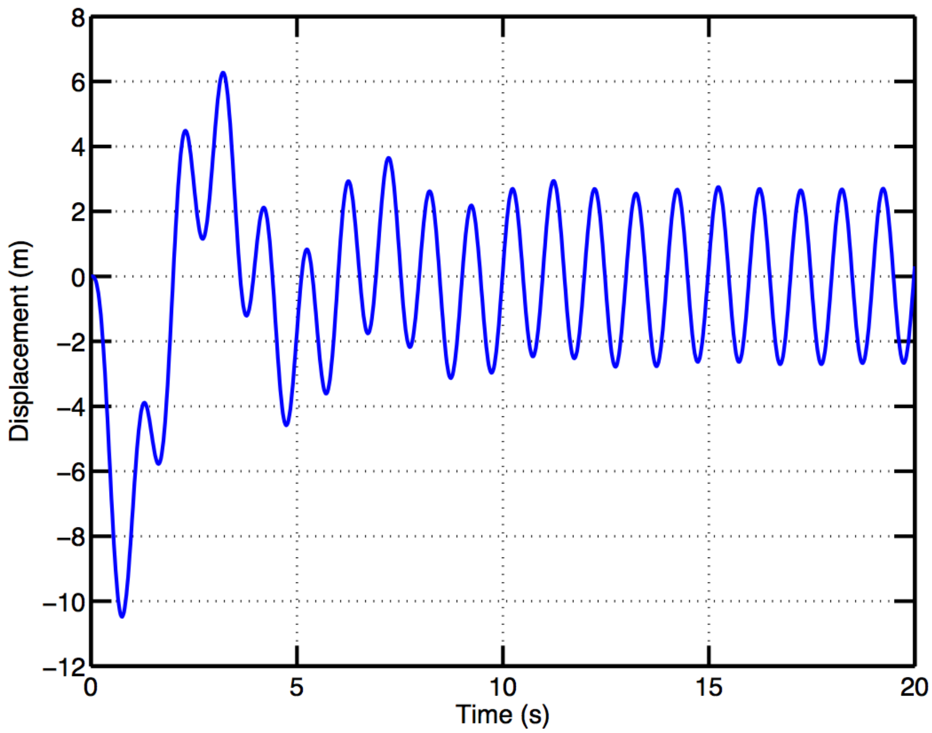
Question B1:
Consider the simple damped spring-mass system shown in the first figure. The mass is driven by an external force given by:

$$F(t) = F_0\cos(\omega t + \phi)$$

The mass is at rest at its equilibrium position, $x = 0$, when the force is turned on instantaneously



at $t = 0$. The response of the mass to this driving force is shown in the second figure. Assuming that the mass is $m = 1$ kg, use the graph of $x(t)$ (Displacement) to get *estimates* (within 20%) for:



(a) The natural frequency of the undamped oscillator, $\omega_0/(2\pi)$ in Hz.
Hint: You may assume that γ is small, so that $\omega_1 \equiv \sqrt{\omega_0^2 - \gamma^2/4} \approx \omega_0$ [3 marks]

(b) The damping coefficient, b in N s/m. [3 marks]

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(c) The frequency of the driving force, $\omega/(2\pi)$ in Hz. [3 marks]

(d) The amplitude of the driving force, F_0 in N. [3 marks]

(e) What is ϕ? [3 marks]

Question B2:
A generator of emf $V(t) = V_0 \cos\omega t$ is connected in series with a resistance R, an inductance L, and a capacitance C.

(a) Write down the differential equations for the current I in the circuit and for the charge, q, on the capacitor. [3 marks]

(b) Solve for $q(\omega, t)$. [3 marks]

(c) Solve for $I(\omega, t)$. [3 marks]

In what follows, $V_0 = 3\text{V}$, $R = 50\Omega$, $L = 100\text{mH}$, and $C = 0.01\mu\text{F}$.

(d) Plot the amplitude of the current (I_0) as a function of ω. [2 marks]

(e) At what value of ω is I_0 a maximum? [1 marks]

(f) Plot q_0 as a function of ω. [2 marks]

(g) At what frequency, ω, is the voltage across the capacitor a maximum? [1 marks]