BSc/MSci MidTerm Test

PHY-217: Vibrations and Waves

Time allowed: 1 hr 15 minutes

Date: 13th November 2012

Time: 9:15 – 10:30 am

Answer ALL questions in section A.

Answer ONLY ONE question from section B.

Section A carries 60 marks and Section B carries 40 marks.

An indicative marking-scheme is shown in square brackets [...] after each part of a question.

Numeric calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Important Note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from the college. Please check now to ensure that you do not have any notes in your possession. If you have any then please raise your hand and give them to an invigilator immediately. Exam papers cannot be removed from the exam room

You are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Examiner: Prof Martin Dove

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SECTION A

Attempt answers to ALL questions.

A1

An object executing one-dimensional simple harmonic motion around an equilibrium position can have its displacement x at any time t described by the function

$$x(t) = x_0 \cos(\omega t + \phi)$$

i) Define the quantities x_0, ω and ϕ. [4 marks]

ii) Sketch a graph of this function with appropriate labels on the axes. [2 marks]

A2

Consider a simple pendulum consisting of an object of mass m hanging at the end of a thin string of negligible mass. The string has length L. Denote the displacement of the object when swinging as x and y in the horizontal and vertical directions respectively.

i) The pendulum is rotated by an angle θ whilst keeping the string taught. Write equations for x and y in terms of θ, and hence obtain an equation for y in terms of x. [2 marks]

ii) Noting that the gravitational energy associated with a displacement is equal to the product of the mass, gravitational constant g = 9.8 ms^{-2}, and the vertical displacement y, write the energy of the pendulum as a function of x. [2 marks]

iii) By differentiating the potential energy, write an equation for the force experienced by the object for any rotation of the pendulum in terms of the horizontal displacement x. [2 marks]

iv) By equating the force with *mass* × *acceleration*, write the differential equation that describes the motion of the pendulum in terms of the horizontal displacement x. [2 marks]

v) Obtain an equation for the angular frequency of oscillations of the pendulum. [2 marks]

A3

Sketch a labelled graph showing *both* the potential and kinetic energy of an undamped oscillator as a function of time. [4 marks]

How will this graph change with light damping?

A4

The differential equation that describes a damped oscillator can be written as

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0$$

where x is the displacement from equilibrium.

Define the following terms using the parameters in the differential equation, and describe the motion in each case:

i) Underdamped oscillator [2 marls]

ii) Overdamped oscillator [2 marks]

iii) Critically damped oscillator [2 marks]

A5

Assume a solution for the differential equation given in question A4 of the form

$x = A_0 \exp(i\beta t)$

i) Substitute this solution into the differential equation to obtain an equation for β. [4 marks]

ii) Use the solutions for β to obtain equations for x in the underdamped and overdamped cases. [4 marks]

A6

Sketch and label plots of the amplitudes of a forced oscillator in the cases of i) no damping, and ii) light damping. Explain in words the key differences. [6 marks]

A7

i) Define the quality factor Q for a damped oscillator. [4 marks]

ii) How does the shape of a graph of absorbed energy for a forced oscillator depend on Q? [4 marks]

A8

i) Define the term "beating" for an oscillator. Give *one* example of how beating can be created. [4 marks]

ii) Describe the role of beating in the form of the transients established at the onset of an applied sinusoidal force to a lightly damped oscillator. [6 marks]

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SECTION B

Answer only ONE of the two questions in this section. No credit will be given for answers to a second question.

B1

Consider a molecule consisting of two atoms of mass m_1 and m_2, and a bond force constant k.

i) Denote the displacements of each atom as u_1 and u_2 respectively, and write an equation for the potential energy of the bond in terms of the displacements of both atoms. [6 marks]

ii) Derive equations for the force experienced by each atom. [6 marks]

iii) Assume the motions of both atoms move follow a sinusoidal vibration of angular frequency ω, but with each atom experiencing a different amplitude. Write a differential equation for the motion of each atom, and substitute in the corresponding sinusoidal function. Derive equations for the angular frequency and the relative displacements of the two atoms. [8 marks]

We have the following data for the carbon monoxide molecule:

Bond energy = –1077 kJ/mol

Bond length = 1.13×10^{-10} m

Vibration frequency = 64 THz

The bond can be modelled by a Morse potential energy function of the form

$$E(r) = E_0\left[\exp 2\alpha(r_0 - r) - 2\exp\alpha(r_0 - r)\right]$$

where r is the distance between the two atoms, and the other quantities are parameters in the model.

iv) Show by differentiation that the equilibrium separation is at $r = r_0$. [6 marks]

v) Obtain by second differentiation an equation for k in terms of α and E_0. [8 marks]

vi) From the above data, obtain values for the parameters α, r_0 and E_0. [6 marks]

B2

Four people of total mass 300 kg get into a car. The car springs are compressed and the body of the car is lowered by 3 cm. Whilst driving, the car hits a bump in the road and oscillates vertically with a period of 0.75 s.

i) Derive the value of the spring constant for the car suspension (the acceleration due to gravity can be taken as $g = 9.8$ ms^{-2}). [10 marks]

ii) Derive the total mass of the car and occupants, and hence of the car, assuming that damping is negligible. [10 marks]

iii) The damping on the suspension allows the oscillations to have decayed after 4 seconds. Assuming damping occurs through the existence of a force that is proportional to $-bv$, where b is a constant and v is the velocity, obtain a value for b and hence for the quality factor Q. [20 marks]

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