PHY217: Vibrations and Waves

Assessed Problem set 1

Issued: 5 November 2012

Deadline for submission: 5 pm Thursday 15th November, to the V&W pigeon hole in the Physics reception on the 1st floor of the GO Jones building.

Question 1

The central assumption behind the course so far is that the function describing the energy can be approximated as quadratic in the variable of interest. For two examples discuss the validity of this approximation. [10 marks]

Question 2

1. Define the terms "under-damped", "over-damped" and "critically-damped". Give examples of when two of these properties can be exploited in real-world applications. [4 marks]
2. Explain the meaning of the quantities γ and ω_0 in the differential equation that describes a damped oscillator. [3 marks]
3. Explain why consideration of forced oscillations requires the inclusion of damping in the model. [3 marks]
4. Describe the difference between steady-state motions of a forced oscillator and the transient state. Explain why the transient state is a valid part of the solution of the differential equation that describes forces and accelerations. [4 marks]

Question 3

Explain why the angular frequency of an oscillator is a property of the system when it is allowed to move freely when starting from an initial state, but is imposed when there is a continuous sinusoidal applied force. In which case is the amplitude of motion established by the initial conditions? [10 marks]

Question 4

1. Define the figure of merit Q for a damped oscillator (unforced or forced). [5 marks]
2. Explain why Q is a useful quantity in the study of forced oscillations. [5 marks]

Question 5

Consider a hydrogen fluoride molecule (atomic mass of H is 1 g/mole, and of F is 19 g/mole).

1. Write the energy of the system in terms of the displacements of both atoms. [3 marks]
2. Write the equation for force = mass × acceleration as a differential equation for both atoms, obtaining the force from the differential of the energy. [3 marks]
3. Show that the following equations for the displacements of the atoms are solutions of the differential equation

$$x_{\mathrm{H}} = \frac{x_0}{m_{\mathrm{H}}}\cos\omega t$$

$$x_{\mathrm{F}} = -\frac{x_0}{m_{\mathrm{F}}}\cos\omega t$$

and hence obtain an equation for the angular frequency ω. [4 marks]

4. The vibration frequency is measured as 124 THz; use this result to obtain the effective spring constant of the bond between the H and F atoms. [3 marks]
5. Predict the frequency of the molecule if the hydrogen atom is replaced by a deuterium atom (atom mass 2 g/mole). [3 marks]

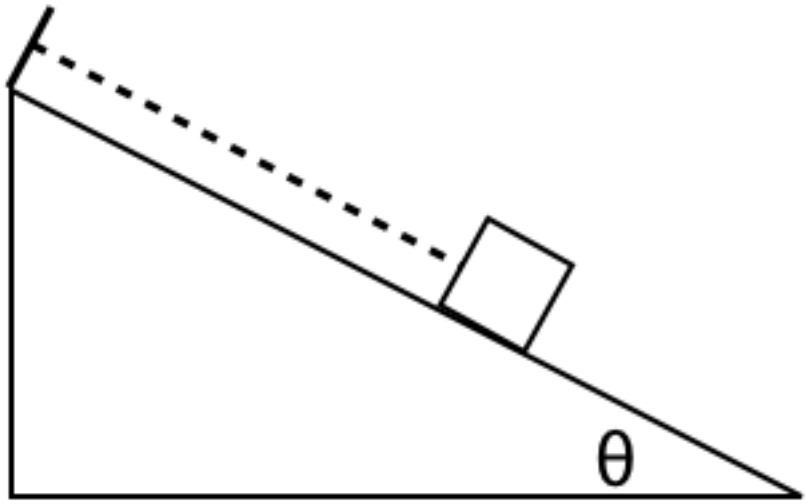
Question 6

Find solutions of the following differential equations:

1. $\ddot{y} + 16y = 0$ [3 marks]
2. $\ddot{y} + 4\dot{y} + 16y = 0$ [3 marks]
3. $\ddot{y} + 8\dot{y} + 16y = 0$ [3 marks]
4. $\ddot{y} + 16\dot{y} + 16y = 0$ [3 marks]

Question 7

A mass m is attached to a massless spring with a spring constant k. The mass can slide along a frictionless plane inclined as an angle θ to the ground (as illustrated below).



1. Write an equation for the energy as the mass is displaced from its equilibrium position, assuming that the only force acting is gravitational. [4 marks]
2. Derive the differential equation describing the force and acceleration. [4 marks]
3. Show the the period of motion is independent of the angle θ. [4 marks]

Question 8

An object of mass m = 0.2 kg is hung from a spring whose spring constant is 80 $\mathrm{Nm^{-1}}$. The body is subject to a resistive force given by $-bv$, where v is its velocity and b = 4 $\mathrm{Nm^{-1}s}$.

1. Set up the differential equation of motion for free oscillations of the system, and find the period of such oscillations. [4 marks]
2. The object is subjected to a sinusoidal force given by $F(t) = F_0 \sin(\omega t)$, where $F_0 = 2$ N and $\omega = 30$ rad/s. In steady state, what is the amplitude of the forced oscillation? [4 marks]
3. Instead of a driving force, we now oscillate the end of the spring at the top end vertically with a harmonic displacement $X = X_0 \sin(\omega t)$. Set up the differential equation of motion for this driven oscillator. [4 marks]
4. What is the amplitude of the mass in steady state for $\omega = 0$, 30 and 300 rad/s, if $X_0 = 0.5$ cm in each case? [4 marks]