Vibrations and waves: exercise sheet 8

Long chain of coupled pendulums

Picture an infinite number of pendulums, where each pendulum is connected to its two neighbours by a spring with force constant *J*. Each pendulum has length *L* and supports a mass *m*. Neighbouring pendulums are separated by distance *a*. Consider the case where the masses move in the direction of the chain.

Write the energy for this system in terms of both the spring and gravitational energies. Compute the force acting on any selected mass when it and it's neighbours move.

Assume the travelling wave solution

$$u_j = \tilde{u}\exp\left(i(kx-\omega t)\right)$$

Show that the dispersion curve for this system is described by the equation

$$\omega^2 = \frac{4J}{m}\sin^2(ka/2)+\frac{g}{L}$$

Sketch the function for ω. Highlight points on the dispersion curve that correspond to standing waves.

Rationalise this result in the limits of zero value of *k*, and zero value of *J*.

Why do we not find the equivalent of a sound wave?