Vibrations and waves: exercise sheet 4

Passive amplification via an LRC circuit

Consider a circuit consisting of a series connection of a resistor, inductor and capacitor. Thus circuit is driven by an alternating voltage.

The relevant differential equation is

$$L\ddot{Q} + R\dot{Q} + Q/C = V_0 \cos\omega t$$

Show that this can be reduced to the more conventional form

$$\ddot{Q} + \gamma\dot{Q} + \omega_0^2 Q = \frac{V_0}{L}\cos\omega t$$

Just divide by L and substitute ...

Obtain the steady state solution by analogy with previous work, writing your answer in terms of the variables *L*, *C* and *R*.

Differentiate your equation with respect to time, and hence derive and equation for the current.

$$\begin{aligned} Q &= Q_0(\omega)\cos(\omega t - \delta) \\ Q(\omega) &= \frac{V_0}{\omega\sqrt{(1/\omega C - \omega L)^2 + R^2}} \\ I &= \dot{Q} = -\omega Q_0(\omega)\sin(\omega t - \delta) \\ &= \frac{V_0 \sin(\omega t - \delta)}{\sqrt{(1/\omega C - \omega L)^2 + R^2}} \end{aligned}$$

At which angular frequency will the current have maximum value?

Resonant frequency

What will be the resonant frequency and *Q* for C = 8 μF, L = 0.02 H and R = 75 Ω?