Vibrations and waves: exercise sheet 2

Some physics of our cuckoo clock

The purpose of this exercise is to think about a practical example of a damped harmonic oscillator, namely the pendulum that drives our cuckoo clock. The pendulum consists of an object with some mass, that is attached to a very light rod such that it can be slid up or down in order to adjust the period of swing. Because the rod is light and doesn't extend all the way up to the pivot point, the pendulum can be treated as a simple pendulum with mass at a single point.

I have measured the total mass of the pendulum to be 12 g, and the distance from the pivot to the centre of the mass is 20 cm.

1. Assume that the weight can be treated as a point mass m, located at a distance h from the pivot point. From the equation for the simple pendulum, deduce the period of oscillation of the pendulum.

The equation from the lectures is

$$\omega^2 = \frac{g}{h} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{h}{g}} = 2\pi \times \sqrt{\frac{0.2}{9.8}} = 0.9 \text{ s}$$

2. The timing of the clock can be adjusted by sliding the weight up or down. If the clock loses 10 minutes over 24 hours, in which direction should the weight be moved. Show that the relationship between a change in height h is related to a change in period T by

$$\frac{\Delta h}{h} = 2\frac{\Delta T}{T}$$

   and thereby calculate the required movement of the weight.

To make the clock mechanism go faster, the frequency of the pendulum needs to increase, in which case the weight needs to be moved upwards. For this particular case, we have

$$\Delta h = 2h\frac{\Delta T}{T} = 2 \times 0.2 \times \frac{10}{24 \times 60} \text{ m} = 2.8 \text{ mm}$$

3. The weight on the pendulum has an amplitude of 2 cm. Calculate the potential energy of the pendulum when it has moved its maximum amount.

In the lectures we developed these equations:

$$E = mgy$$

$$L - y = L\cos\theta \simeq L\left(1 - \frac{\theta^2}{2}\right)$$

$$y = \frac{1}{2}L\theta^2$$

$$x = L\sin\theta \simeq L\theta$$

$$y = \frac{1}{2L}x^2$$

$$E = \frac{mg}{2L}x^2$$

On this basis, we calculate the potential energy as

$$E = \frac{mg}{2h}x^2 = \frac{12\times10^{-3}\times9.8\times0.02^2}{2\times0.2} = 1.176\times10^{-4}\ \text{J}$$

4. The pendulum is an underdamped oscillator and will eventually come to a halt. The energy loss is replenished from the gravitational energy released by the lowering of a larger weight of 275 g. This larger weight falls 1.8 m in 24 hours. Calculate the energy loss per period of oscillation. Equate the rate of change of gravitational energy with the energy loss of the pendulum. Use this to calculate a value of the figure or merit Q.

The change in potential energy of the large weight over one period is the change in potential energy over 24 hours divided by the number of periods in the same time; the length of one period was derived in question 1:

$$\Delta E = mg\times1.8\times\frac{0.9}{24\times3600} = 0.275\times9.8\times1.8\times\frac{0.9}{24\times3600} = 5.053\times10^{-5}\ \text{J}$$

From the lectures we showed that over one period of oscillation

$$\frac{\Delta E}{E} = \frac{2\pi}{Q} \quad\Rightarrow\quad Q = 2\pi\times\frac{1.176\times10^{-4}}{5.053\times10^{-5}} = 14.6$$