ELECTRIC CHARGE, FORCE AND FIELD
(*Young & Freedman* Chap. 22)
(*Ohanian*, Chaps. 22, 23)

Brief review of electric charge and force

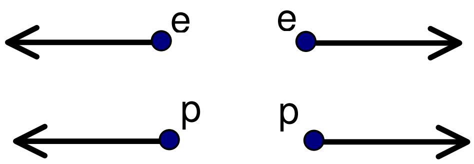
Recall: Atoms are composed of NEUTRONS, PROTONS and ELECTRONS

Electrons and protons are subject to a force which does not affect neutrons

To account for this force, electrons and protons are said to possess ELECTRIC CHARGE

Opposite charges attract:

Like charges repel:



Convention: Electrons have NEGATIVE charge
Protons have POSITIVE charge

Note:

1. The sign of electric charge is NOT the same as mathematical sign. A different convention could easily have been adopted (e.g., North and South), or the electron could have been deemed positive and the proton negative.

2. The electron and proton have equal and opposite charges of ± e (the ELECTRONIC CHARGE)

   SI system: $e = 1.602 \times 10^{-19}$ C (Coulombs - to be defined later)

3. A current of 1 Amp ≡ 1 Coulomb/second ≡ 6.3×10^{18} electrons/sec.

   This is a very large number – so, macroscopically, we can regard charge as continuous rather than discrete.

4. Macroscopically, there are equal numbers of positive and negative charges, and they are well mixed. Therefore, electric forces tend to cancel out. This allows gravity, which is much weaker, to dominate.

   Ratio of electric and gravitational forces between an electron and a proton

$$\frac{F_{\text{Elec.}}}{F_{\text{Grav.}}} = 2.3 \times 10^{39}$$

Conductors and insulators

Solid substances:

- Atoms are fixed in position.
- Nuclei and inner electrons cannot move.
- Outer electrons can be strongly or weakly bound

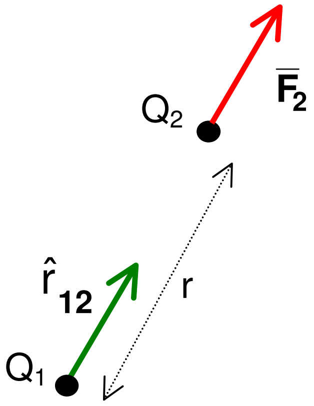
Strongly bound	INSULATOR (DIELECTRIC): even if an external electric force is applied, the electrons don't move.
Loosely bound	CONDUCTOR: electrons will move in response to an electric force, and so charge can flow.

Fluid substances:

- Neutral particles ⇒ insulator
- Charged particles ⇒ conductor: ionised atoms and free electrons can move about and carry a flow of charge
- ELECTROLYTE = liquid conductor
  PLASMA = gaseous conductor

Note: In questions on electrostatics, words like "plastic", "silk", "paper" etc. ⇒ an insulator. "Metal" ⇒ a conductor. As we shall see later, it is often important to remember whether a charged object is an insulator or a conductor.

Coulomb's law



Consider two point charges, Q_1 and Q_2, with separation r

Let $\overline{\mathbf{F}}_2$ = Force on Q_2 due to Q_1

Magnitude of $\overline{\mathbf{F}}_2$: $F_2 = K\frac{|Q_1Q_2|}{r^2}$

Direction of $\overline{\mathbf{F}}_2$: Let $\hat{r}_{12}$ be a unit vector pointing from Q_1 to Q_2

Then $\overline{\mathbf{F}}_2$ is along the direction of $\hat{r}_{12}$ if Q_1 and Q_2 have the same sign
$\overline{\mathbf{F}}_2$ is opposite to $\hat{r}_{12}$ if Q_1 and Q_2 have the opposite signs

The value of K depends on the system of units

SI units: $K = 8.99 \times 10^9 \ \mathrm{N\,m^2C^{-2}}$

or $K = \frac{1}{4\pi\varepsilon_o}$ where $\varepsilon_o = 8.85 \times 10^{-12} \ \mathrm{N^{-1}m^{-2}C^2}$

ε_o is the PERMITTIVITY CONSTANT or the PERMITTIVITY OF FREE SPACE

So $\overline{\mathbf{F}}_2 = \pm\left[\frac{|Q_1Q_2|}{4\pi\varepsilon_o r^2}\right]\hat{r}_{12}$ **Coulomb's law in vector form**

The Electric Field

Coulomb' s law describes "action at a distance". This is valid for stationary charges (electrostatics), but not for charges in motion. Applying Coulomb' s law directly in that case leads to violation of the law of conservation of momentum and the transmission of information faster than the speed of light. (See *Ohanian* p 591 for a good discussion of this point.)

The concept of the ELECTRIC FIELD gets around this problem.

Definition: The ELECTRIC FIELD, $\overline{\mathbf{E}}$, at a point in space is equal to the force, $\overline{\mathbf{F}}$, which a positive charge Q *would* experience at that point, divided by Q.

i.e., $\overline{\mathbf{E}} = \dfrac{\overline{\mathbf{F}}}{Q}$ Electric field $\equiv$ Force per unit charge

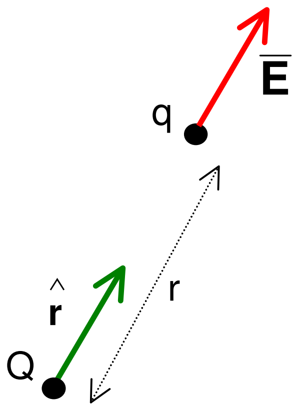
Note:
1. $\overline{\mathbf{E}}$ is a VECTOR
2. The units of $\overline{\mathbf{E}}$ are NC^{-1}
3. $\overline{\mathbf{E}}$ is not a mathematical fiction: it is a REAL PHYSICAL ENTITY
4. Note the word "would" in the definition: $\overline{\mathbf{E}}$ exists even if there is no charge present to experience it.

To find $\overline{\mathbf{E}}$: Find the force on a positive "test charge", q, and divide by q.

Magnitude : Given by Coulomb' s Law

Direction : Imagine how a small positive charge would move: that gives the direction of $\overline{\mathbf{E}}$

Example: What is the electric field at distance r from a point charge Q?



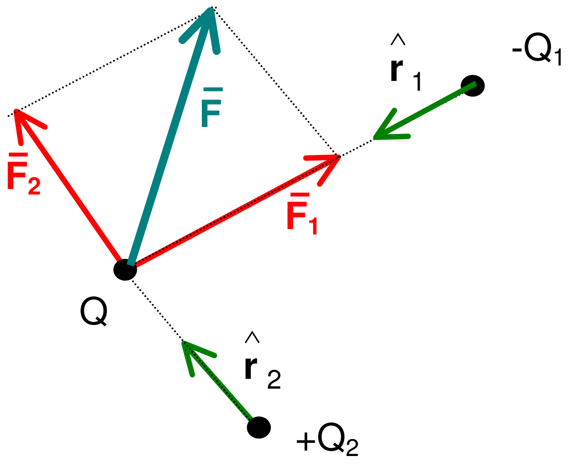
$$\overline{\mathbf{F}} = \left[\frac{Qq}{4\pi\varepsilon_o r^2}\right]\hat{\mathbf{r}} \quad \Rightarrow \quad \overline{\mathbf{E}} = \left[\frac{Q}{4\pi\varepsilon_o r^2}\right]\hat{\mathbf{r}}$$

Addition of electric forces and fields

PRINCIPLE OF SUPERPOSITION: The total force on a charge due to a number of other charges is given by the sum of all the forces on it due to the individual charges. The same applies to addition of electric fields.

i.e., Electric forces and fields are added as VECTORS

Example: A system of three charges



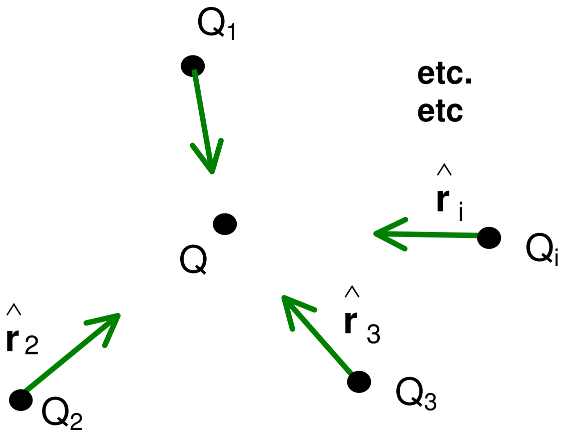
Resultant force on Q is

$$\overline{\mathbf{F}} = \overline{\mathbf{F}}_1 + \overline{\mathbf{F}}_2$$

Where $\overline{\mathbf{F}}_1$ = Force due to Q_1
$\overline{\mathbf{F}}_2$ = Force due to Q_2

In general, for a system of n other charges acting on a charge Q, the total force on Q is

$$\overline{\mathbf{F}} = \sum_{1}^{n} \left[\frac{QQ_i}{4\pi\varepsilon_o r^2}\right]\hat{\mathbf{r}}_i$$



Examples of Electrostatic Force Calculations

1. A system of three point charges in the X-Y plane
2. An electric dipole
3. Electric field on the axis of a line of charge
4. Electric field on the axis of a charged ring

See lecture notes

Electric field lines

$\overline{\mathbf{E}}$ is represented graphically by ELECTRIC FIELD LINES

- Field lines start at positive charges and end at negative charges
- The direction of a field line at any point is the direction of $\overline{\mathbf{E}}$ at that point
- The density of lines is proportional to the magnitude of $\overline{\mathbf{E}}$
- Lines do not intersect except at charges
- Field lines are NOT real - they just help us to visualise the field
- Convention: Q/ε_o field lines originate from a charge Q

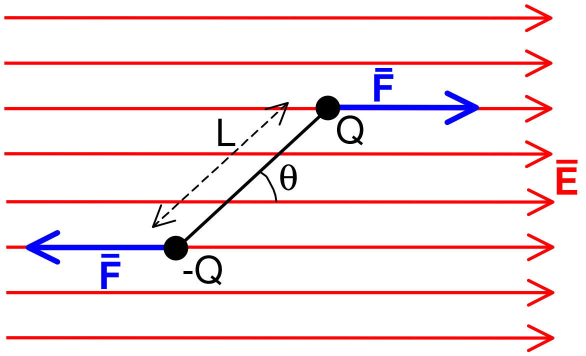
Examples: using the field lines concept to find the electric field

1. $\overline{\mathbf{E}}$ due to an infinite plane sheet of uniform charge density
2. $\overline{\mathbf{E}}$ due to an infinite line of uniform charge density

See lecture notes

An electric dipole in an electric field

Recall: an ELECTRIC DIPOLE two equal and opposite charges separated by a fixed distance. Although it is electrically neutral, it will be influenced by an electric field because the charges are separated.



Consider a dipole (±Q, separation L) in a uniform electric field, and let the dipole axis be at an angle θ to the field direction, as shown.

Charge Q experiences a force $\overline{\mathbf{F}}$ as shown, with magnitude $F = QE$
Charge -Q experiences a force $\overline{\mathbf{F}}$ as shown, with magnitude $F = QE$

The two equal and opposite forces have different lines of action, and so generate a TORQUE, τ. From the diagram, the torque tends to rotate the dipole so as to reduce θ - i.e., to align the dipole axis with the field.

Convention: Positive torque tends to increase θ (anticlockwise rotation)
Negative torque tends to decrease θ (clockwise rotation)

By definition,

Torque = (Force)(Perpendicular distance between lines of action)

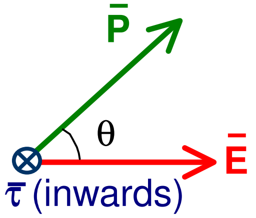
$\Rightarrow$ in this case, $\tau = -QEL\sin\theta$

Recall: QL = Dipole Moment, P $\Rightarrow$ $\tau = -PE\sin\theta$

This looks like the magnitude of a VECTOR CROSS PRODUCT, and it is.

The torque is a vector defined as $\overline{\tau} = \overline{\mathbf{P}} \times \overline{\mathbf{E}}$

$\overline{\mathbf{P}}$ is the ELECTRIC DIPOLE MOMENT VECTOR



Direction of $\overline{\mathbf{P}}$: From the negative charge towards the positive charge
Direction of $\overline{\tau}$: By the right hand rule, $\overline{\tau}$ is perpendicular to both $\overline{\mathbf{P}}$ and $\overline{\mathbf{E}}$ (into the page as drawn here).