The Postulates of Quantum Mechanics.

A postulate is a hypothesis, if it agrees with experiment, then it can be taken as an axiom, a truth which we cannot prove. Quantum Mechanics is based upon a number of postulates; these are stated quite differently depending on the source, vary in actual number and order etc. depending on what book one reads. In these notes, I have used the four-postulate form found in Morrison, although I have added related information from other sources.

1 Every system can be described by a wavefunction (state function), $\Psi(\mathbf{r},t)$ ($\Psi(x,t)$ in 1D), which contains <u>all</u> accessible information about the system, including its spatial and temporal evolution. $\Psi(\mathbf{r},t)$ will tell us everything!

2 The probability of finding a particle in a volume element dv, at **r**, at any one time, P dv, is given by $Pdv = |\Psi(\mathbf{r},t)|^2 dv = \Psi^*(\mathbf{r},t)\,\Psi(\mathbf{r},t)\,dv$ ($Pdx = \Psi^*(x,t)\Psi(x,t)dx$ for finding our particle in the interval dx at x and t, in 1D). This means that $\Psi^*(\mathbf{r},t)\Psi(\mathbf{r},t)$ is a probability density function for our system ($\Psi^*(x,t)\,\Psi(x,t)$). Since the particle must exist somewhere in the universe, we obtain the normalisation condition, namely:

$$\int_{-\infty}^{\infty} \Psi^*(x,t)\Psi(x,t)dx = 1 \qquad \text{for 1D}$$

3 Every observable (variable), q, is represented by an operator, $\hat{Q}$, and this operator is used to obtain information about the observable. e.g .momentum, p, operator $\hat{P}$, $\hat{P} = \frac{\hbar}{i}\frac{\partial}{\partial x}$, position, x, operator $\hat{X} = x$, and so on… Combined with the wavefunction, they give rise to eigenvalue equations of the form:

$$\hat{Q}\Psi(x,t) = q\Psi(x,t)$$

…we can also obtain the average or expectation value for any observable:

$$\langle q \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\hat{Q}\Psi(x,t)dx$$

Or, indeed, the uncertainty, Δq, $\Delta q = \sqrt{\langle q^2 \rangle - \langle q \rangle^2}$

4 The wavefunction or state function of an <u>isolated</u> system develops in time according to the Time Dependent Schrödinger Equation (TDSE), namely:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

which may appear as: $\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$ or even: $\hat{H}\Psi = \hat{E}\Psi$

Since the wavefunctions in postulate 1 have to obey the TDSE in postulate 4, there are important conditions which apply to $\Psi(x,t)$. Amongst others, both $\Psi(x,t)$ and its derivative, $\frac{\partial\Psi(x,t)}{\partial x}$ must be continuous.