## SECTION A

**A1**

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(\underline{r},t)\right]\Psi(\underline{r},t) = i\hbar\frac{\partial\Psi(\underline{r},t)}{\partial t}$$

$$= \left[\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(\underline{r},t)\right]\Psi(\underline{r},t) = i\hbar\frac{\partial\Psi(\underline{r},t)}{\partial t}$$

If $V(\underline{r},t)$ is static in time i.e. $V = V(\underline{r})$, then the TDSE above reduces to the TISE. Where

$$\Psi(\underline{r},t) \rightarrow \psi(\underline{r})\,\phi(t)$$

and

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(\underline{r})\right]\psi(\underline{r}) = E\,\psi(\underline{r})$$

$$\frac{d\,\phi(t)}{dt} = \frac{-i\,E}{\hbar}\,\phi(t) \quad \Rightarrow \quad \phi(t) = Ae^{-iEt/\hbar}$$

where $E$ is the energy eigenvalue of the state and $A$ is a normalisation constant.

[5 MARKS]

A2

$\hat{P}$ IS AN OPERATOR FOR SPATIAL INVERSION:

$$\hat{P}\,\underline{r} = -\underline{r}$$
$$\hat{P}\,\psi(\underline{r}) = \psi(-\underline{r})$$

$\hat{P}\Psi = \eta_p \Psi$ where $\Psi$ is an eigen function of the operator $\hat{P}$ and $\eta_p$ is the corresponding eigen value

$$(\hat{P})^2\,\Psi = (\eta_p)^2\,\Psi$$

$$\Rightarrow \eta_p = \pm 1$$

If $\eta_p = +1$ the wavefunction $\Psi$ is even under ~~interchange~~ inversion of spatial coordinates (parity)

If $\eta_p = -1$ the wavefunction $\Psi$ is odd under parity transformations

[5 marks]

A3

a) $|\Psi(\underline{r},t)|^2 \, d\underline{r}^3$ corresponds to the probability of finding the particle described by $\Psi$ in the volume element given by $d\underline{r}^3$ at time $t$, on making a measurement of the state $\Psi$.

b) $\hat{x} = x$

$\hat{p} = -i\hbar \dfrac{\partial}{\partial x}$

c) $[\hat{x}, \hat{p}] = (\hat{x}\hat{p} - \hat{p}\hat{x})\,\psi(x)$

$= \left(-i\hbar x \dfrac{\partial}{\partial x} + i\hbar \dfrac{\partial}{\partial x}(x)\right)\psi(x)$

$= -i\hbar x \dfrac{\partial \psi}{\partial x} + i\hbar\left[x\dfrac{\partial \psi}{\partial x} + \psi\right]$

$= i\hbar\,\psi(x)$

d) As $\hat{x}$ and $\hat{p}$ do not commute they are not compatible observables, hence it is not possible to simultaneously determine position and momentum of a particle exactly; one is bounded by the limit given by the Heisenberg uncertainty relationship.

[5 marks]

OMB3

A4

a) $\Psi = c_1 \psi_1 + c_2 \psi_2$ [2 marks]

b) $|c_1|^2$ [2 marks]

c) $\Psi \to \psi_n(x)$ on measurement of the state yielding the energy eigenvalue $E_n$. [1 mark]

[5 marks]

A5

$$\Psi = \sum_{i=1}^{N} c_i \psi_i(x,t)$$ is the wave function in general

where

$$\langle \hat{H} \rangle = \int_{-\infty}^{\infty} \psi_n^*(x,t)\, \hat{H}\, \psi_n(x,t)\, dx$$

$$= E_n$$

and $$\int_{-\infty}^{\infty} \psi_i^* \hat{H} \psi_j \, dx = \int_{-\infty}^{\infty} (\hat{H} \psi_i)^* \psi_j \, dx$$

$$\Rightarrow (E_j - E_i)^* \int_{-\infty}^{\infty} \psi_i^* \psi_j \, dx = 0$$

$$\Rightarrow \int_{-\infty}^{\infty} \psi_i^* \psi_j \, dx = \delta_{ij}$$

i.e states are orthonormal

[5 marks]

## A6

a) $\hat{\underline{L}} = \hat{r} \times \hat{p}$

$$= -i\hbar \begin{vmatrix} i & j & k \\ x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$$

$$\therefore \hat{L}_x = -i\hbar\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right) = y\hat{p}_z - z\hat{p}_y$$

$$\hat{L}_y = -i\hbar\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) = z\hat{p}_x - x\hat{p}_z$$

$$\hat{L}_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) = x\hat{p}_y - y\hat{p}_x$$

$$\hat{L} = \hat{L}_x\, i + \hat{L}_y\, j + \hat{L}_z\, k$$

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

$$= -\hbar^2\left\{\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)^2 + \left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right)^2 + \left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)^2\right\}$$

Eigenvalues of $\hat{L}_z$ are $m\hbar$, $m \in \{l, l-1, \ldots -l\}$

Eigenvalues of $\hat{L}^2$ are $l(l+1)\hbar$

[5 marks]

$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$$

$$[\hat{L}_y, \hat{L}_z] = i\hbar \hat{L}_x$$

$$[\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y$$

$[\hat{L}_z, \hat{L}^2] = 0$ ; $[\hat{L}_x, \hat{L}^2] = 0$ & $[\hat{L}_y, \hat{L}^2] = 0$

[5 marks]

TOTAL A6 = 10 MARKS

A7

$$\Delta A \, \Delta B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle \right|$$

~~IF TWO OPERATORS COMMUTE THE OBSERVABLES ASSOCIATED WITH THEM CAN BE DETERMINED PRECISELY.~~

[2 marks]

A8

a) $$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

PAULI SPIN MATRICES

$$S_i = \frac{\hbar}{2} \sigma_i$$

RELATIONSHIP BETWEEN SPIN OPERATORS & MATRIX

[5 marks]

EXAMPLES (ONE OF THE FOLLOWING)

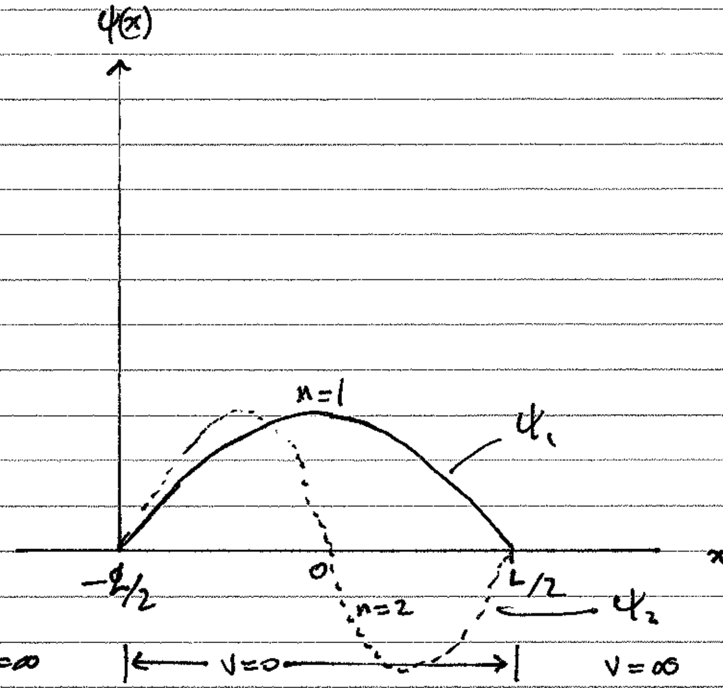
b) • The results of the Stern Gerlach experiments of Ag atoms for example (Alkali metals have one exposed valence $e^-$); [1 mark]

• Hyperfine splitting → non-degeneracy of emission lines IN SPECTROSCOPIC ANALYSIS.

TOTAL FOR A8
6 MARKS

A9

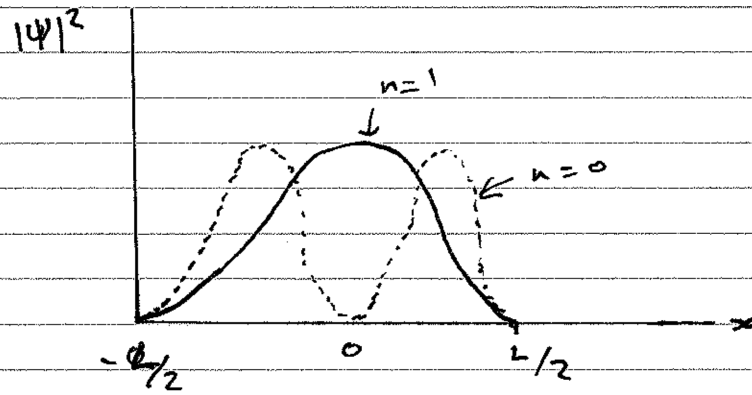
a)




$n=1$ = ground state
$n=2$ = 1st excited state.

[2]

b)




[2]

c) $P\psi_1 = \psi_1$ ; WAVE FUNCTION IS EVEN UNDER PARITY

$P\psi_2 = -\psi_2$ ; WAVE FUNCTION IS ODD UNDER PARITY

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

so $$E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$$

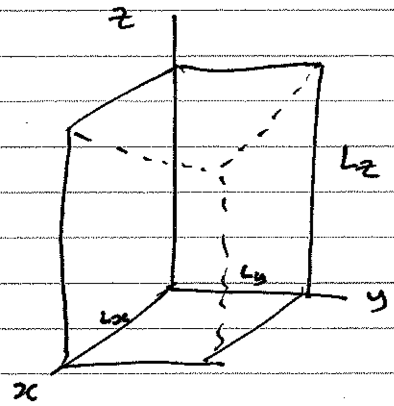
$$E_2 = \frac{4\pi^2 \hbar^2}{2mL^2} = 4E_1$$ [3]

A9 TOTAL 7 MARKS

QMB7

B1

a)




$\psi(x,y,z) = 0$ for $x \leq 0$, $x \geq L_x$
$y \leq 0$, $y \geq L_y$
$z \leq 0$, $z \geq L_z$

$V(x,y,z,t) = 0$ ; hence TDSE → TISE

TISE:

$$\frac{-\hbar^2}{2m} \nabla^2 \psi = E\psi \qquad \text{within the box.}$$

[5]

b) So $x, y$ and $z$ are orthogonal hence:

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi = E\psi \longrightarrow (1)$$

reduces to three coupled equations; one in each of $x, y, z$ where

$$\psi(x,y,z) = \psi(x)\,\psi(y)\,\psi(z)$$
$$= X(x)\,Y(y)\,Z(z)$$

∴ (1) becomes

$$-\frac{1}{X}\frac{\partial^2 X}{\partial x^2} - \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} - \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = \frac{2mE}{\hbar^2} = k^2$$

This can be broken down into 3 equations:

$$\frac{\partial^2 X}{\partial x^2} = -k_x^2 X$$
$$\frac{\partial^2 Y}{\partial y^2} = -k_y^2 Y$$
$$\frac{\partial^2 Z}{\partial z^2} = -k_z^2 Z$$

where $k^2 = k_x^2 + k_y^2 + k_z^2$

CONTD.

B1 : b)

where symmetry of the problem yields:

$$X(x) = N_x \sin\left(\frac{n_x \pi}{L_x} x\right) \qquad n_x = 1, 2, 3, \ldots$$

$$k_x = \frac{n_x \pi}{L_x}$$

$$Y(y) = N_y \sin\left(\frac{n_y \pi}{L_y} y\right) \qquad n_y = 1, 2, 3, \ldots$$

$$k_y = \frac{n_y \pi}{L_y}$$

$$Z(z) = N_z \sin\left(\frac{n_z \pi}{L_z} z\right) \qquad n_z = 1, 2, 3, \ldots$$

$$k_z = \frac{n_z \pi}{L_z}$$

where the $N_i$ are normalisation constants.

$$\int_0^{L_x} N_x^2 \sin^2\left(\frac{n_x \pi}{L_x} x\right) dx = 1 \qquad \text{similarly for } y \,\&\, z$$

so $\cancel{N_x \left[ - \frac{L_x}{n_x \pi} \cos \frac{n_x \pi}{L_x} x \right]_0^{L_x}}$

$$\frac{N_x^2}{2} \int_0^{L_x} 1 - \cos\left(\frac{2 n_x \pi}{L_x} x\right) dx = 1$$

$$= \frac{N_x^2}{2} \left[ x - \sin\left(\frac{2 n_x \pi x}{L_x}\right) \frac{L_x}{2 n_x \pi} \right]_0^{L_x}$$

$$= \frac{N_x^2}{2} L_x$$

$$\therefore \quad N_x = \sqrt{\frac{2}{L_x}}$$

Similarly $N_y = \sqrt{\frac{2}{L_y}}$ ; $N_z = \sqrt{\frac{2}{L_z}}$

So

$$\psi(x, y, z) = \sqrt{\frac{8}{L_x L_y L_z}} \sin\left(\frac{n_x \pi}{L_x} x\right) \sin\left(\frac{n_y \pi}{L_y} y\right) \sin\left(\frac{n_z \pi}{L_z} z\right)$$

CONTD

B1 : 6)

and $$E = \frac{k^2 \hbar^2}{2m}$$
$$= \frac{\hbar^2}{2m} \cdot \pi^2 \cdot \left( \frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right)$$

eigenvalues → energy. [12]

c) $\underline{(n_x, n_y, n_z) = (2, 1, 1)}$ unseen

$$\psi_{211} = \sqrt{\frac{8}{L_x L_y L_z}} \sin\left(\frac{2\pi x}{L_x}\right) \sin\left(\frac{\pi y}{L_y}\right) \sin\left(\frac{\pi z}{L_z}\right)$$

$$E_{211} = \frac{\pi^2 \hbar^2}{2m} \left( \frac{4}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right)$$

$\underline{(1, 2, 1)}$ :

$$\psi_{121} = \sqrt{\frac{8}{L_x L_y L_z}} \sin\left(\frac{\pi x}{L_x}\right) \sin\left(\frac{2\pi y}{L_y}\right) \sin\left(\frac{\pi z}{L_z}\right)$$

$$E_{121} = \frac{\pi^2 \hbar^2}{2m} \left( \frac{1}{L_x^2} + \frac{4}{L_y^2} + \frac{1}{L_z^2} \right)$$

$\underline{(1, 1, 2)}$

$$\psi_{112} = \sqrt{\frac{8}{L_x L_y L_z}} \sin\left(\frac{\pi x}{L_x}\right) \sin\left(\frac{\pi y}{L_y}\right) \sin\left(\frac{2\pi z}{L_z}\right)$$

$$E_{112} = \frac{\pi^2 \hbar^2}{2m} \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{4}{L_z^2} \right)$$

If $L_x = L_y = L_z = L$; then $E_{112} = E_{121} = E_{211}$ & the three states become degenerate where

$$E_{degen} = \frac{6\pi^2 \hbar^2}{2m L^2} = \frac{3\pi^2 \hbar^2}{m L^2}$$ [8]

TOTAL B1 = 25 MARKS

B2

a) • The rigid rotator model assumes the atomic nuclei are fixed at some distance $r$ from each other at equillibrium. This describes free rotational motion of the molecule.

• There may be small amounts of vibrational motion about the Equillibrium point; so a SHO potential can be added.

• The rotational & vibrational modes are assumed to be independent of each other

$$\Rightarrow \Psi_{rot} \cdot \Psi_{VIB} = \Psi_{TOT}$$ for properly normalized wavefunctions

$$\Rightarrow E_{ROT} + E_{VIB} = E_{TOT}$$

$$\hat{H} = \frac{-\hbar^2}{2\mu}\nabla^2 + V(r) + \frac{1}{2}m\omega_0^2 r^2$$ ; $\mu$ = reduced mass of system : $\frac{m_1 m_2}{m_1 + m_2}$

central potential

[5 marks]

b)

$$E_{ROT} = \frac{\hbar^2}{2I} l(l+1) \qquad l = 0, 1, 2, \ldots$$

$$E_{VIB} = (n+\tfrac{1}{2})\hbar\omega \qquad n = 0, 1, 2, \ldots$$ (SHO contribution)

$$\therefore E_{TOT} = \frac{\hbar^2}{2I} l(l+1) + (n+\tfrac{1}{2})\hbar\omega$$ where $I = \mu a^2$ & $a$ is equillibrium separation distance

CONSIDER ROTATIONAL MOTION vs VIB.

$$E_{ROT} = \frac{\hbar^2}{2I} l(l+1)$$

$\sim 10^{-5} - 10^{-4}$ eV small compared to thermal energy.

Typically $E = kT \gg E_{ROT}$

$E_{VIB} \sim 0.5$ eV for diatomic molecules (IR)

$E_{ROT} \sim 10^{-4}$ eV microwave.

$\Rightarrow$ rotational states are easily excited by thermal motion.

$E_{ROT}(l) \rightarrow E_{ROT}(l \pm 1)$ requires a change in total orbital angular momentum of the molecule

i.e $l \Rightarrow l \pm 1$

in these excitations $\Rightarrow$ the allowed radiative transitions are those where the total angular momentum is conserved.

CONTD...

2B b contd) this makes sense when one considers also the spin of an emitted photon $= 1$.

i.e.

$$\Delta l = \pm 1$$ for emission $(-1)$ or absorption $(+1)$ of a single photon with $m = 0, \pm 1$.

**EIGENSTATES:**

$$\Psi = \psi(r)\,\psi(\theta)\,\psi(\phi)$$

where the separation of $r$ from the angular coordinate is the result of assuming ROT & VIB modes are independent.

$\psi(\theta)\,\psi(\phi)$ are the $Y_{lm}(\theta,\phi)$ solutions found for the Hydrogen atom i.e.

$$Y_{lm}(\theta,\phi) = \psi(\theta)\,\psi(\phi)$$
$$= N\, P_l^{|m|}(\cos\theta)\, e^{im\phi}$$

where $N$ is a normalisation constant

$P_l^{|m|}(\cos\theta)$ is a legendre polynomial

$m$ is the magnetic QN : $m \in \{-l, -l+1 \dots +l\}$

$$\psi_n(r) = \frac{1}{\sqrt{n!}}\left(\hat{a}^{\dagger}\right)^n \psi_0(r) \; ; \quad \psi_0(r) = \left(\frac{\beta^2}{\pi}\right)^{1/4} e^{-\beta^2 x^2/2}$$

and $\hat{a}^{\dagger} = \frac{1}{\sqrt{2}}\left(\beta\hat{r} - i\alpha\hat{p}\right)$ is the creation operator for SHO.

[10 marks]

CONTD ...

2B c) $\Psi(r,\theta,\phi) = \psi(r)\, Y_{1m}(\theta,\phi)$

$l = 1 \quad ; \quad m = 0, \pm 1$

$l = +1 \quad ; \quad m = 0$

$$Y_{1m} = \sqrt{\frac{3}{4\pi}} \cos\theta \qquad |Y_{1m}|^2 = \frac{3}{4\pi} \cos^2\theta$$

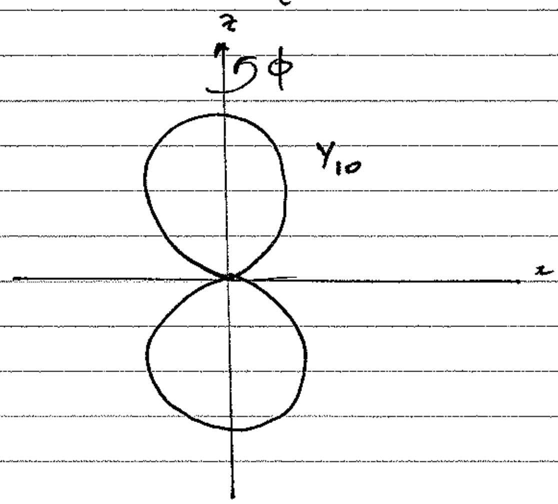
$l = +1 \quad ; \quad m = +1$

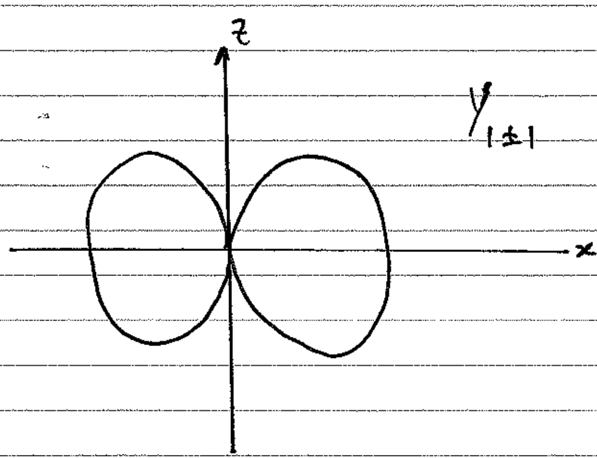
$$Y_{1m} = -\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{i\phi} \qquad |Y_{1m}|^2 = \frac{3}{8\pi} \sin^2\theta$$

$l = +1 \quad ; \quad m = -1$

$$Y_{1m} = \sqrt{\frac{3}{8\pi}} \sin\theta\, e^{-i\phi} \qquad |Y_{1m}|^2 = \frac{3}{8\pi} \sin^2\theta$$

$\theta = 0$ direction (coincident with the $L_{\hat{z}}$ direction)



B3

a) $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$

$\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$

$\chi_+^\dagger \chi_- = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = 0$

$\chi_-^\dagger \chi_+ = \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = 0$

$\chi_+^\dagger \chi_+ = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = 1$

$\chi_-^\dagger \chi_- = \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = 1$

$\therefore$ the $\chi_\pm$ are orthonormal [5 marks]

b) $S_x\chi' = \frac{\hbar}{2}\lambda\chi'$

let $\chi' = \begin{pmatrix}a\\b\end{pmatrix}$ s.t. $|a|^2 + |b|^2 = 1$

$\therefore S_x\begin{pmatrix}a\\b\end{pmatrix} = \frac{\hbar}{2}\lambda\begin{pmatrix}a\\b\end{pmatrix}$ $S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

$\therefore \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}b\\a\end{pmatrix} = \frac{\hbar}{2}\lambda\begin{pmatrix}a\\b\end{pmatrix}$

$\therefore b = \lambda a$ or $a = \lambda b$

$\Rightarrow \lambda^2 = 1$ ; $\lambda = \pm 1$

As $|a|^2 + |b|^2 = 1$ ; and $b = \pm a$

$2|a|^2 = 1$

$\Rightarrow a = \frac{1}{\sqrt{2}}$ & $\therefore b = \pm\frac{1}{\sqrt{2}}$ for $\lambda = \pm 1$

$\Rightarrow \chi'_+ = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ ; $\chi'_- = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

i.e $\chi'_\pm = \frac{1}{\sqrt{2}}\left(\chi_+ \pm \chi_-\right)$

contd.

B3 b) contd

$$S_y \chi'' = \frac{\hbar}{2} \lambda \chi'' \qquad \chi'' = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$S_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\therefore \text{LHS: } \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{\hbar i}{2} \begin{pmatrix} -b \\ a \end{pmatrix}$$

$$\text{RHS: } \frac{\hbar}{2} \lambda \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\Rightarrow \quad i \begin{pmatrix} -b \\ a \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix}$$

so $b = \lambda i a$
$\quad ia = \lambda b$

$\therefore \lambda^2 = 1 \quad \Rightarrow \quad \lambda = \pm 1$ as usual

if $\lambda = +1$ then $b = i a$
if $\lambda = -1$ then $b = -i a$

using $|a|^2 + |b|^2 = 1$ we find that

$$|a|^2 + (ia)(ia)^* = 1$$
$$\therefore 2|a|^2 = 1 \quad \Rightarrow \quad a = \frac{1}{\sqrt{2}}$$
$$\Rightarrow \quad b = \pm \frac{i}{\sqrt{2}} \qquad \text{for } \lambda = \pm 1$$

$$\therefore \chi''_{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \left( \chi_+ \pm i \chi_- \right)$$

[10 marks]

contd.

B3 CONTD

c) $\Psi = \frac{1}{\sqrt{2}}\left(\psi_1 \phi_2 - \phi_1 \psi_2\right)$ $\psi = |K^0\rangle$

$\phi = |\bar{K}^0\rangle$

i) • IF THE 1ST PARTICLE IS MEASURED AS A $K^0$; THEN THE SECOND ONE MUST BE A $\bar{K}^0$ AT THAT SAME INSTANT.

• THE PROBABILITY OF THIS OUTCOME CAN BE SEEN FROM $|c_i|^2 = \frac{1}{2}$.

[2 MARKS]

ii) Before $\Psi = \frac{1}{\sqrt{2}}\left(\psi_1 \phi_2 - \phi_1 \psi_2\right) = \Psi_i$

AFTER $\Psi = \psi_1 \phi_2 = \Psi_f$

[1 mark]

iii) $K_s = p\psi - q\phi$

$K_L = p\psi + q\phi$

$\Psi_i = K_L$

$\Psi_f = \frac{2p\psi}{\sqrt{2}} = A K_s + B K_L$

$A(p\psi - q\phi) + B(p\psi + q\phi)$

$= (A+B)p\psi + (B-A)q\phi$

$\Rightarrow B = A$ ; hence 2nd term vanishes

$\Rightarrow (A+B)\,p\psi = \frac{2p\psi}{\sqrt{2}}$

$\Rightarrow A+B = \frac{1}{\sqrt{2}}$

$\Rightarrow A = B = 1$

$\Rightarrow \Psi_f = K_s + K_L$

$\Rightarrow K_s$ beam has been regenerated. 7 marks

[10 for part c]

25 marks B3