Duality and the nature of quantum space-time

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Strings provide a working theory of quantum gravity existing in harmony with particle physics

Successes include computations of quantum graviton scattering, new models of beyond-standard-model particle physics based on branes, computation of black hole entropy
A powerful new discovery: Duality

- An unexpected equivalence between two systems, which a priori look completely unrelated, if not manifest opposites.

- Large and Small (T-duality)

- Gravitational and non-Gravitational. (Gauge-String duality)
Unification and Duality

- They are both powerful results of string theory. They allow us to calculate things we could not calculate before.

- Unification: We asked for it and found it. We kind of know why it had to be there.

- Duality: We didn’t ask for it. We use it. We don’t know what it really means.
Unification v/s Duality

- Unification has an illustrious history dating back to the days of Maxwell.

- Before Maxwell, we thought magnets attracting iron on the one hand and lightning on the other had nothing to do with each other.

- After Maxwell: Magnets produce $B$-field. Electric discharge in lightning is caused by $E$-fields. The coupled equations of both allow fluctuating $E, B$-fields which transport energy travelling at the speed of light. In fact light is electromagnetic waves.
Unification v/s Duality

- Einstein tried to unify gravity with quantum physics.

- String Theory goes a long way.

- Computes graviton interaction probabilities.
We think that asking the question of whether something is large or small has a unique answer.

T-duality of string theory says: Not always!!
Unification v/s Duality

- We think, as physicists, we know whether we are dealing with gravity or not.

- In applications to the real world, gravity dominates at large distances. Non-gravitational forces dominate at small distances.

- Gauge-String duality says a theory of quantum gravity in 10 dimensions is equivalent to a generalized theory of photons in 4 dimensions!!
OUTLINE

- Large-small Duality (T-duality of strings)
- Gauge-String Duality
- What is this telling us?
  Something deep about space-time... We think.
Consider string theory in 10 dimensions.

Let us say $X^0, X^1 \cdots , X^8$ are infinite.

But $X^9$ is finite.

Further let us say : $X^9 = X^9 + 2\pi R$. 
T-duality

- Equivalently, $X^9$ is a coordinate along a circle.

- The physical states include particles with different amounts of momentum along the circle.

- Any state in the spectrum has a definite momentum along $X^9$. 
T-duality

- Momentum is related to the waveform on the circle.

- Because the waveform is periodic, with periodicity $2\pi R$, the wavelength is quantized.

- There is a momentum quantum number $n$, and $p_n = \frac{n}{R}$
T-duality

- Because this is a string theory, the string can wind around the circle.

- It can wind multiple times.

- Any state has a winding number.
T-duality

Figure: String Winding Number
T-duality

- Any state $\psi(n, m; q)$ has a momentum quantum number $n$, a winding quantum number $m$ and other quantum numbers $q$.

- The Energy of the state is $E(R, n, m; q)$.

- T-duality says that

$$E(R, n, m; q) = E\left(\frac{1}{R}, m, n; q\right)$$

- Physics on circle of radius $R$ and $1/R$ are identical, as long as momentum and winding modes are exchanged!!
Gauge-String Duality

- The two-dimensional sphere $S^2$ is described by the equation

$$x_1^2 + x_2^2 + x_3^2 = 1$$

- The five-dimensional sphere $S^5$ is described by the equation

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = 1$$

- Change some signs in the equation for $S^5$ and you get a space called anti-de-Sitter space $AdS_5$. 
Gauge-String Duality

- Consider string theory on a ten dimensional space, five of which are form $S^5$ and the remaining five of which form $AdS_5$.

- This theory manifestly contains gravity and is a quantum theory.

- Gauge String Duality says: It is in fact equivalent to a theory in 4 dimensions.
Gauge-String Duality

- QCD is a theory of quarks and gluons, where there are 3 colours of quarks and $3 \times 3 = 9$ types of gluons.

- Colour is a generalization of charge. The gluons are generalizations of photons, i.e. light

- A further generalization is one where you have $N$ colours.
Gauge-String Duality

- A theory of N-colour gluons is actually equivalent to String Theory on $\text{AdS}_5 \times S^5$, in the limit, of $N$ going to infinity.

- A consequence of D-Brane physics, called the AdS/CFT correspondence or the Maldacena correspondence, and an example of Gauge-String Duality.

How can this be?
Gauge-String Duality

- The $S^5$ part of the 10 D space is compact.

- We can Fourier transform the on the $S^5$ to write the theory in terms of the remaining non-compact space $AdS_5$.

- Euclidean $AdS_5$ is a 5-ball, whose boundary is a 4-sphere.
Gauge-String Duality

- This AdS/CFT correspondence is a bulk-boundary correspondence.

- Gravity, Black Holes, gravitons live in the bulk.

- Gluons live on the boundary.

- All the physics of the bulk can be reproduced by the physics of the gluons.
Gauge-String Duality

Figure: Bulk Boundary correspondence
What does it mean? T-duality

- In ordinary QFT in 10 D spacetime which includes a circle, it is impossible to have an equivalence of large and small

- While string theory connects to 10D QFT, the QFT is a low-energy approximation. Going to higher and higher energy involves making the QFT more and more complicated.

- Large-Small duality means that the QFT way of thinking about spacetime physics using $\Phi(x, y, z, t)$ misses some crucial equivalences.
What does it mean? T-duality

- If we find the right string theory which looks like the standard model below a TeV, SUSY theory around a TeV, more complicated SUSY theory at higher energy scale ...

- Then the full string theory is more than the sequence of QFTs..

- It has qualitatively new features such as large-small equivalence in the extra dimensions which is not manifest in the sequence of QFTs
What does it mean? Gauge-String duality

- The theory looks like a 5D theory of quantum gravity.

- Yet its physical degrees of freedom are 4D.

- So perhaps there is a way of thinking about the gravity where it is manifest that the fifth dimension is not real; so that we would not even begin to describe the bulk theory by $\Phi(x_1, x_2, x_3, x_4, t)$. 
What does it mean? Gauge-String duality

- Perhaps there is a formulation of gravity where it is manifest that the theory is gravitational and four dimensional?

- We have no idea what such a formulation might look like.

- Whatever the gravity field is doing in 4-directions determines what it is doing in the fifth? A new type of space-time uncertainty? Analogous to the Heisenberg uncertainty which taught us that $x, p$ are redundant. Only one could be specified.
A new kind of space-time uncertainty?

- There are hints from black hole physics that something new is needed.

- Hawking Radiation poses a paradox for the standard understanding of when quantum gravity effects can be ignored and QFT can be used as a low energy approximation.

- String Theory and duality tell us that a full theory of quantum gravity such as string theory can have non-localities very unlike our naive expectations based on ordinary quantum fields.
Some questions

- Implications of duality and associated non-localities for:
  - Black Holes
  - Early universe
  - Vacuum selection problem.