Strings in the Quantum World.

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INTRODUCTION

Quantum Physics

Our understanding of the sub-atomic world relies crucially on a novel way of looking at the world, which differs fundamentally from the pre-twentieth century *classical* physics.

Probabilities and a new Uncertainty Principle play a previously unexpected role

Einstein, Bohr, Heisenberg, Schrodinger ..

Quantum Physics $\rightarrow$ Quantum Fields.

The quantum description of matter and forces at the microscopic level culminates in “Quantum Field Theory. ” (QFT)

Dirac, Feynman, Yang , Mills , ’t Hooft ...

What are Quantum Fields ?

Quantum description of strings : Results

Calculation of quantum interactions between gravitons and other particles, of the standard model kind.

Four dimensional Quantum Field Theory (4D QFT) achieves very limited success with this unification.
String Theory $\rightarrow$ 4D QFT.

Extra dimensions required by consistency of string theory.

String Theory $\rightarrow$ 10 D QFT.

String Theory : The Secret

Successes are its relations to 4D QFT and 10D QFT.

Secret Weapon : its relation to 2D QFT

2D QFT

Simpler

We can do more with it.

Cleverness of String Theory

Relate all the things we can do with 2D QFT
To things we would like to do with 4D QFT,
With a detour via 10D QFT.

OUTLINE

- Classical Physics
  - Motion of Apples and Planets
  - Waves in the Sea and classical fields
  - Waves in the Vacuum
- Quantum Physics
  - The Heisenberg uncertainty priciple
  - Schrodinger waves and Quantum states
  - The unexpected role of Probability

OUTLINE

- Quantum Fields
  - Fields, Waves and Quantum States

- Quantum Strings
  - Worldlines and Worldsheets
  - Quantum Fields on Worldsheets
  - Spacetime states from worldsheet states

OUTLINE

- Results from String Theory
  - Gravitons, particles and their interactions.
  - Extra dimensions : WHY TEN ?
- More on the 2D secret
  - Interactions from QFT on footballs, donuts, pretzels ...

CLASSICAL PHYSICS

- Mechanics : Motion of particles.
- Particle is a point like-object. It has a position in space described by (x, y, z). It moves under the influence of forces.

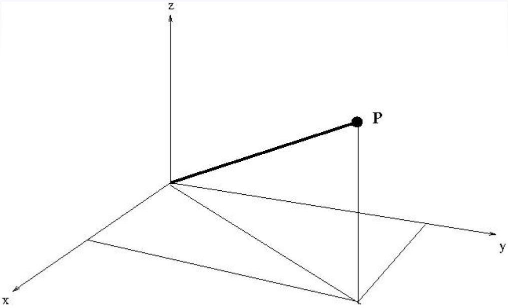

Figure: Coordinates of particle

Mechanics

- The motion is described by thinking of the coordinates (x, y, z) as functions of time $(x(t), y(t), z(t))$
- The equations of mechanics describe how the the functions $(x(t), y(t), z(t))$ evolve with time.
- For example, given position when we start out stopwatch,

$$(x(0), y(0), z(0))$$

  we can calculate, position at time $t = 10s$.

$$(x(t = 10s), y(t = 10s), z(t = 10s))$$

Mechanics

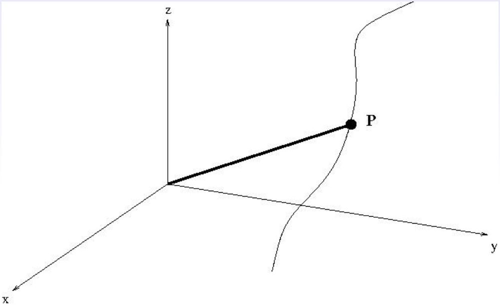

Figure: Motion = Time-dependent coordinate functions

Mechanics

- Motion is influenced by forces.
- Gravitational forces exist between any two massive bodies.
- The Law of gravitation tells us how the force depends on the masses.
- $$F = \frac{GM_1 M_2}{R^2}$$

Mechanics

- Once we know the initial positions $(x(0), y(0), z(0))$ and the initial velocities of particles $(v_x(0), v_y(0), v_z(0))$, we can work out the gravitational forces, and the equations of motion determine the future trajectories.
- This framework explains diverse things such as :
  - Motion of apples
  - Moons around the planets
  - Planets in the solar system
- It has applications such as
  - Sending satellites into outer space
  - And the Moon and Planets.

- The realm of classical mechanics also covers the oscillatory motion of guitar strings and water waves.

- Water wave : The height of the water surface varies with position x, y on a horizontal plane, and the time t. We express this by saying h is a function of x, y, t, written as

$$h(x, y, t)$$

Electromagnetic Waves

- Waves can also exist in the vacuum.
- Light, radio waves , x-rays etc. are all examples of electromagnetic waves.
- These are fluctuating patterns of electric and magnetic fields in space, which can transmit energy.
- The variables of interest are now

Electric	$E(x, y, z, t)$
Magnetic	$B(x, y, z, t)$

Electromagnetic Waves : Fields

- The electric and magnetic fields E, B are force fields, related to electric and magnetic forces that are exerted on charged particles such as ions, electrons etc.
- Compare to mechanics $(x(t), y(t), z(t))$, where x, y, z are functions of time.
- Quantities such as the height of of water waves $h(x, y, t)$ and the electric and magnetic fields $E(x, y, z, t), B(x, y, z, t)$, which are functions of space-coordinates and time are called classical fields.

Limitations of Classical Physics

- The classical mechanics of the pre-twentieth century lead to spectacular failures when applied to the microscopic world.
- The smallest atom, hydrogen was known to be made of proton and electron, which are positive and negative particles.
- Applying the laws of classical electromagnetism to an orbiting electron around a proton shows that the electron continuously loses energy and its orbit collapses to zero size.

Quantum Physics

- Quantum physics was formulated to provide the foundations of microscopic physics.

- In classical physics, we can always safely ask someone, without any second thoughts
  Where are you ? and Where are you going ?
- Not so in Quantum Physics !!

- Small objects in nature, such as electrons and protons, do not have well-defined positions and velocities at a given time !!

Quantum Physics : Uncertainty Principle

- This, in essence, is the Heisenberg Uncertainty Principle.

- Cannot specify (x, y, z) and (v_x, v_y, v_z) simultaneously as initial conditions. Equivalently momenta $(p_x, p_y, p_z) = (mv_x, mv_y, mv_z)$ cannot be specified simultenaously with position.
- There is a new fundamental constant of nature h

$$\Delta x \Delta p_x \geq \frac{h}{2\pi}$$

Quantum Physics

- The quantum physics of a particle is described in terms of its wavefunction $\Psi(x, y, z, t)$
- The wavefunctions give probabilities of finding the particle in different positions or with different velocities.
- When trajectories involve distances large compared to the wavelength, classical physics is a good approximation.

Quantum Physics of fields

- A parallel development in the Early twentieth century was the theory of relativity, which gave the correct description of motion when speeds approach the speed of light.
- Dirac's attempt to combine quantum mechanics with relativity showed that the electron has a positively charged partner, with the same mass. This was called the positron and was subsequently discovered in the lab.
- Electrons and positrons can annihilate to form a photon.

- In relativity, it becomes extremely useful to describe the motion of particles with space-time diagram.

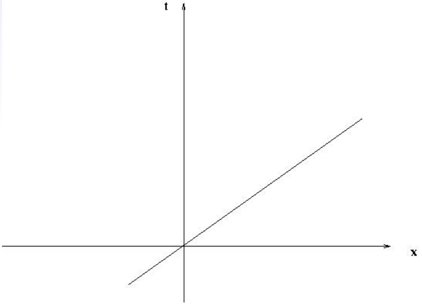

Figure: Depicting motion in spacetime diagram

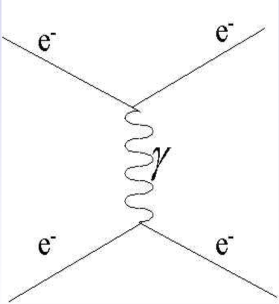

Figure: Space-time Diagram of Creation annihilation

- The description of such processes requires a new more abstract mathematical language, going beyond $(x(t), y(t), z(t))$.
- It requires a language which accomodates creation and annihilation operators for electrons, positrons and photons.
- At the same time this new language captures thewave properties of electrons, photons.

Quantum Field Theory

- This is the mathematical language of Quantum Field theory.
- Like the classical fields which describe waves, they depend on space time, but they are not numbers.

$$\Phi(x, y, z, t)$$

- They need to have the flexibility to describe the creation and annihilation of particles.

Quantum Field Theory

- The Language must contain symbols corresponding to notions such as
  - A Vacuum state with no particles.
  - A creation operator for electron (or positron or photon) in each wave-like configuration.
  - An annihilation operator for particle in each wave-like configuration.

Quantum Field Theory

- The vacuum state is denoted by $|0\rangle$.
  The creation operator $a^{\dagger}_{k,s}$ creates a particle of type s in a wave-like configuration k.
  An annihilation operator $a_{k,s}$ which destroys a particle of type s in wave-like configuration k.
- The state with one particle of type s, in configuration k

$$a^{\dagger}_{k,s}|0>$$

- Requiring that $a_{k,s}$ annihilates leads to rules for manipulating products

$$a_{k,s}a^{\dagger}_{k,s}$$

  which allow simplifications

$$a_{k,s}a^{\dagger}_{k,s}|0> \sim |0>$$

- $a^{\dagger}_{k,s}$ not numbers, but more general mathematical objects.

String Theory

- In String Theory, particles in space-time are replaced by strings or one-dimensional objects, in spacetime.
- When a particle propagates in time it describes a worldline. When a string propagates in time, it describes worldsheet.

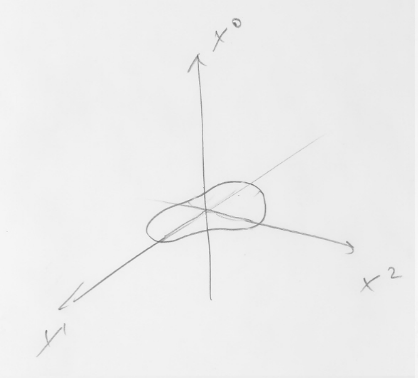

Figure: String in Space-time

String Theory

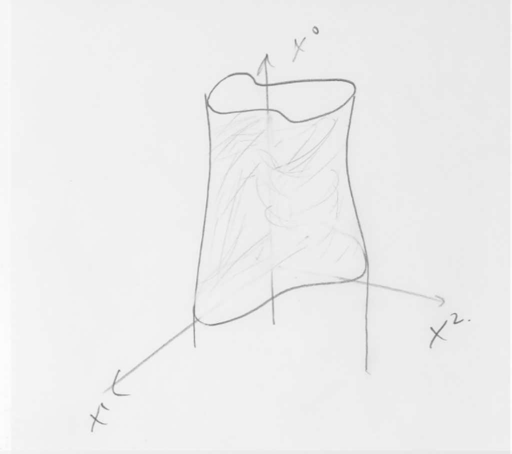

Figure: String worldsheet in Space-time

String Theory

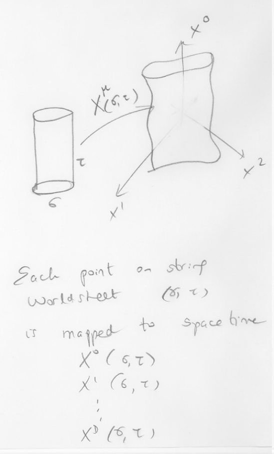

Figure: String worldsheet in Space-time

String Theory

- In string theory, the space-time co-ordinates $X^0(\sigma,\tau), X^1(\sigma,\tau), X^2(\sigma,\tau), X^3(\sigma,\tau)$ are quantum fields in two dimensions.
- For strings propagating in M-dimensions, we have M-spacetime fields.

String Theory

- From composites of these quantum fields, we can construct the states of spacetime
- Schematically :

$$\partial X^{\mu} \partial X^{\nu} |0>$$

  is a state in 2D QFT which corresponds to graviton in space-time QFT.

-

$$\partial X^{\mu} |0>$$

  is a state in 2D QFT which corresponds to photon in space-time QFT.

String Theory

- Quantum consistency of the worldsheet theory coming from reparametrization invariance leads to $M = 10$.
- Hence ten dimensions.

String Theory

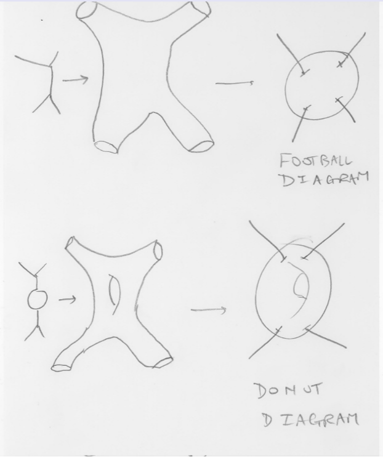

Figure: Interaction diagrams

String Theory

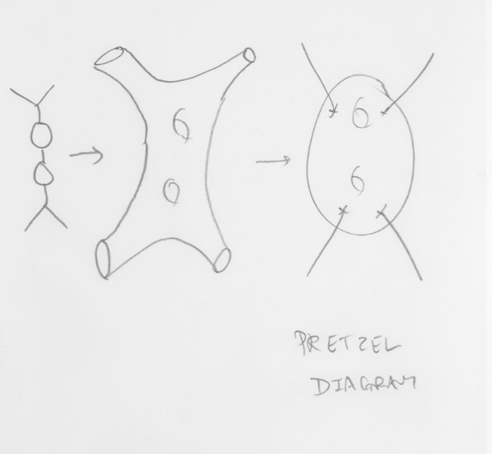

Figure: Interaction diagrams