Symmetries and Surprises in String Theory

Sanjaye Ramgoolam

December 09, 2013
Psi-Star

OUTLINE

Symmetries - geometrical figures, Newtonian Physics

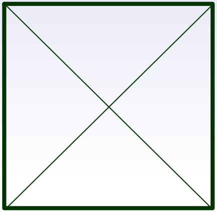
Beyond Newton : quantum field theory (QFT) , string theory, extra dimensions

The big surprise : AdS/CFT correspondence

10D strings and gravity hidden in 4D quantum fields.

Symmetries - in the origins and the workings of AdS/CFT.

Symmetry in Geometry

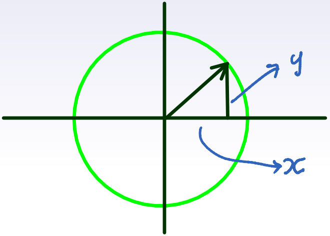


Symmetries : $R_{90}, R_{180}, R_{270}, R_{360} = R_0$.

Composing symmetries : $R_{90} \circ R_{90} = R_{180}$

SYMMETRY GROUP - here $\mathbb{Z}_4$.

Symmetry in Geometry

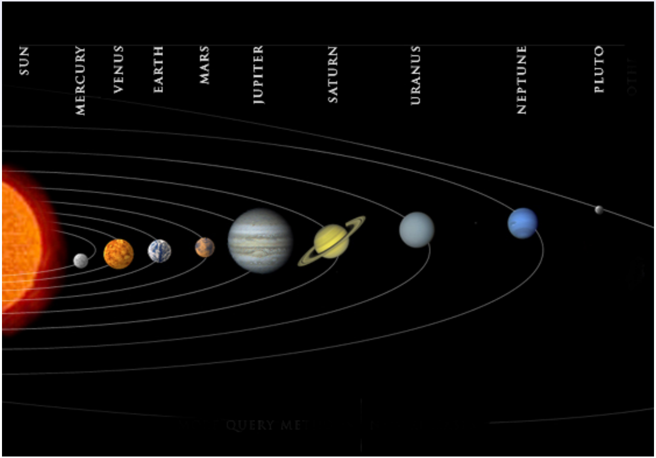


For circle of unit radius $\sqrt{x^2+y^2}=1 \implies x^2+y^2=1$.

Continuous symmetries of rotations by any angle θ. Each rotation R_θ is a transformation $R_\theta : (x,y) \to (x',y') : x^2+y^2=(x')^2+(y')^2$.

This group of symmetries has a name. It is called $SO(2)$.

Symmetry in Physics : Gravitation



Newton's Law of Universal Gravitation

$$F = \frac{GM_1 M_2}{r^2}$$

Newton's Law of Universal Gravitation

$$F = \frac{GM_1 M_2}{r^2}$$

If one particle is at the origin of a Cartesian coordinate system, and the second particle is at a position (x_1, x_2, x_3) then the force is given by

$$F = \frac{GM_1 M_2}{(x_1^2 + x_2^2 + x_3^2)}$$

Symmetry : $SO(3)$ – group of symmetries that moves x_1, x_2, x_3, while keeping fixed the combination $x_1^2 + x_2^2 + x_3^2$.

From Newton to String Theory

From Newton to String Theory

Maxwell's Equations : Unified electricity and magnetism ; light as fluctuations of $\vec{E}(\vec{x}, t), \vec{B}(\vec{x}, t),$ which can propagate even in a vacuum.

Special Relativity (SR) : Particles cannot travel faster than light. Symmetry $SO(3) \rightarrow SO(3, 1)$.

Quantum Physics (QP) : Atomic spectra, Heisenberg Uncertainty.

General Relativity (GR) : Gravity as Curved space-time, predicts back holes, describes expanding universe.

From Newton to String Theory

Maxwell's Equations : Unified electricity and magnetism ; light as fluctuations of $\vec{E}(\vec{x}, t), \vec{B}(\vec{x}, t),$ which can propagate even in a vacuum.

Special Relativity (SR) : Particles cannot travel faster than light. Symmetry $SO(3) \to SO(3,1)$.

Quantum Physics (QP) : Atomic spectra, Heisenberg Uncertainty.

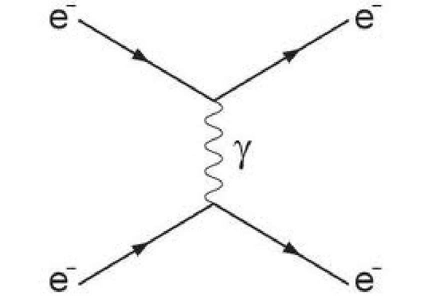
General Relativity (GR) : Gravity as Curved space-time, predicts back holes, describes expanding universe.

SR + QP $\to$ Quantum Field Theory (QFT)

QFT $\to$ Standard Model: electrons, Quarks, Gluons, Higgs.

Quantum Field Theory

Quantum Field Theory : Electrons repel because they are exchanging photons (particles of light)



Force is exchange of particles.

Standard Model

Quantum Field Theory (QFT) is a beautiful theoretical framework for describing matter and forces in a way compatible withquantum physics and relativity.

There are infinitely many possible QFTs – One of them is called the standard model of particle physics. The STD model works incredibly well, its latest success the discovery of the Higgs particle – but it is a bizarre zoo.

3 types of forces : electromagnetic force, weak force, strong force. Quarks (found inside protons) experience the strong force. There are 3 colours of quarks, and $8 = 3 \times 3 - 1$ colours of gluons (strong force carriers). Why 3 ?

3 generations of electrons : electron, muon , tau. Why 3 ?

QFT to String Theory

Standard Model works amazingly well for particle physics experiments.

General Relativity works very well for motion of planets, stars and galaxies

Straightforward incorporation of gravity into QFT fails, because of a problem called non-renormalizability. No known systematic way to control infinities.

String theory solves this theoretical problem by starting from strings rather than particles as fundamental. It can incorporate quantum physics, particle physics, general relativity .. BUT

String Theory : What is it ?

So what is string theory ?

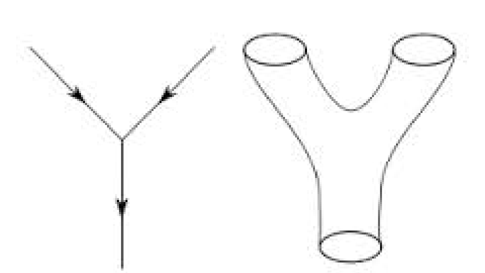
Particles are bundles of energy.

$E = mc^2$

Everything is made from fundamental strings. Different modes of vibration – different types of energy - are different particles.

In the model universes of string theory, some of these string excitations are a lot like the electrons, quarks and gluons we see, others are like gravitons - particles that mediate the force of gravity.

String Theory : Unifies particles, unifies interactions



Interactions of electrons with electrons, electrons with gravitons, gravitons with gravitons all come from the same string Feynman diagram.

String Theory : work in progress

String theory - 10-dimensional world.

Many possible four-dimensional worlds, consistent with unified QFT + gravity.

Is it the theory that describes the real world ?

String Theory : work in progress

String theory - 10-dimensional world.

Many possible four-dimensional worlds, consistent with unified QFT + gravity.

Is it the theory that describes the real world ?

We don't know. Most string theorists believe the theory should be understood better before we can answer that question.

More restrictive than just QFT , but not restrictive enough - yet.

String Theory : It can surprise us

The big surprise delivered by string theory : the AdS/CFT correspondence

Consider six-dimensional space - described by six coordinates - instead of the usual three, $(x_1, x_2, x_3, x_4, x_5, x_6)$ and then restrict to

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = R^2$$

This is a 5-dimensional sphere, denoted S^5, described by an equation in six dimensions.

It has symmetry group $SO(6)$ – just like 2-dimensional sphere in 3-dimensional space has symmetry $SO(3)$.

Anti-de-Sitter space

Now consider the space

$$y_1^2 + y_2^2 + y_3^2 + y_4^2 - y_5^2 - y_6^2 = -R^2$$

This is 5-dimensional Anti-de-Sitter space, denoted AdS_5. Its symmetry is called $SO(4,2)$.

The 10-dimensional space $(\vec{x}, \vec{y})$ is one that allows strings to propagate.

The full string theory has a big symmetry – let's call it

$$SUPER(6|4,2)$$

A distinguished solution of string theory

This is a maximally symmetric solution of string theory, containing gravity, strings and also extended objects called branes.

compare : black holes as symmetric solutions of General relativity

This is one half of the AdS/CFT correspondence.

The other half is CFT. This is an ordinary QFT in four dimensions - no gravity, no strings. The kind that we could have chosen to describe a possible "standard model"

Except we consider the number of gluon types N^2 to be a large number , and we treat N as an arbitrary parameter.

A distinguished QFT – CFT4-max

What is remarkable about this particular CFT4 (conformal field theory) is that it has symmetry $SO(3,1)$ like all relativistic theories, but a much bigger symmetry.

In addition to rotations of space-time, it has scaling and inversion – roughly

$$x \to \lambda x$$
$$x \to x^{-1}$$

These combine into the symmetry group $SO(4,2)$.

A distinguished CFT

Alongside these generalized space-time symmetries, it has symmetries which mix different quantum fields.

In fact there are 6 Higgs-like fiields, hence an $SO(6)$ symmetry of Higgs rotations. Call them $\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6$.

Starting with one quantum field, you can change it to a quantum superposition of quantum fields

$$\Phi_1 \to \Phi_3 + \Phi_4$$

while leaving the equations of the theory unchanged.

Distinguished CFT - a big symmetry we saw before

Altogether – with the enhanced space-time symmetries of 4 dimensions and the Higgs rotating symmetries, we have the symetry group

$$SUPER(6|4,2)$$

the same one that the theory of strings propagating on $AdS_5 \times S^5$ has.

Same Symmetry - Secretly same system ?

In 1996, Maldacena argued that this cannot be a coincidence. Building on this coincidence, with other pobservations about branes in string theory, he suggested that

String theory on $AdS_5 \times S^5$ with $Super(6|4,2)$ symmetry

is equivalent to

CFT4-max with $Super(6|4,2)$ symmetry

He proposed concrete equations relating the parameters of the two theories.

$$(\frac{R}{l_s})^4 = g_{YM}^2 N$$
$$g_s = g_{YM}^2$$

AdS/CFT - Well tested

$SUPER(6|4,2)$ was the inspirational symmetry which lead to the AdS/CFT conjecture. Has passed many tests.

Example : Graviton scattering in 10 dimensions can be computed using a purely 4-dimensional calculation.

This is the big surprise : CFT4 calculations have to do with 10 dimensional quantum fields, without four-dimensional gravity fields.

But by a complicated transformation, which is only partially understood, manages to reproduce the physics of 10 dimensional strings, branes and gravitons.

AdS/CFT - How does it work ?

How does this transformation work ?

Very little is known about this. But what is becoming clear is that there is a hidden symmetry in this AdS/CFT physics which is central in organizing the dictionary between 4D and 10 D physics.

This hidden symmetry is actually a tower of symmetries.

Permutation symmetries

For each integer n, there is a group called S_n, which is the permutation group of n distinct objects. The operation of re-arranging the n objects. An example for $n = 3$.

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

A tower of permutation symmetry groups

Consider all n : for $n = 1, 2, 3....$

Putting together all the S_n together, using a branch of mathematical physics known as topological field theory, forms a mathematical structure which knows a lot about AdS/CFT.

Can reduce the graviton scattering processes to simple counting problems related to permutations.

Permutations have to do with different ways of constructing gauge invariants from $\Phi_1, \cdots, \Phi_6$.

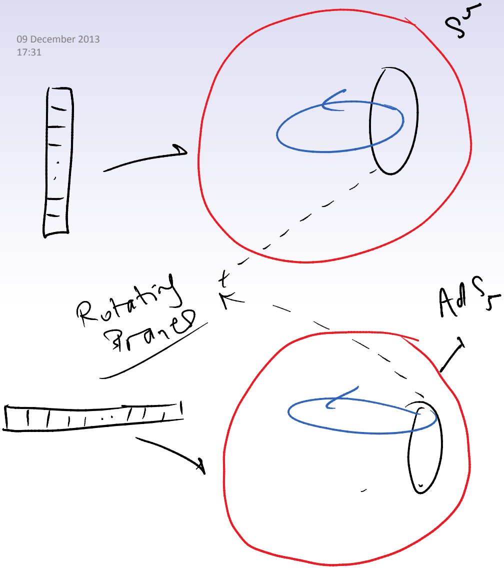
$$
\begin{array}{l}
(\Phi\Phi\Phi) \\
(\Phi\Phi)(\Phi) \\
(\Phi)(\Phi)(\Phi)
\end{array}
$$

Representations of symmetry groups and Branes

Can organize, the dictionary of relations between quantum states in CFT to the branes of string theory

using the representation theory of permutation groups, which is described using the combinatorics of Young diagrams.

09 December 2013
17:31



Some open problems

Improvements in our understanding of the AdS/CFT dictionary would shed light on problems such as :

- How 4D CFT calculations describe black holes in the 10 dimensional string theory ? and their evaporation by emission of Hawking radiation ?
- How does the ten dimensional theory of quantum gravity manage to package all its information into a four dimensional theory without gravity ? A weakening of our usual notions of locality ? And a reduction in the amount of information you can pack into a volume of quantum gravitational space-time ?