7.1) $\Omega = \{0.00, 0.10, 0.15, 0.20, 0.21\}$

Compare the hypothesis that Ω is uniformly distributed (H_0) vs. the data being Gaussian with mean 0.15 and spread 0.9.

x	$P(H_0)$	$P(H_1)$	$R = \frac{P(H_0)}{P(H_1)}$
0.00	0.2	0.437	0.457
0.10	0.2	0.443	0.452
0.15	0.2	0.443	0.451
0.20	0.2	0.443	0.452
0.21	0.2	0.442	0.452

$$R = \frac{P(H_0 \mid data)}{P(H_1 \mid data)} = \frac{P(data \mid H_0)\; P(H_0)}{P(data \mid H_1)\; P(H_1)}$$

Assume $P(H_0) = P(H_1) = $ const.

$R = \prod_{i=1}^{5} R_i = 0.0191$; this indicates that H_1 is a preferred description of the data relative to H_2.

7.2) $P(\text{infected} \mid \text{+ve}) = 0.98$

$P(\text{healthy} \mid \text{+ve}) = 0.0005$

$P(\text{infected}) = 0.0001$

$P(\text{heatthy}) = 0.9999$

$$\therefore P(\text{infected} \mid \text{+ve}) = \frac{P(\text{+ve} \mid \text{infected})\, P(\text{infected})}{P(\text{+ve} \mid \text{infected})\, P(\text{infected}) + P(\text{+ve} \mid \text{healthy})\, P(\text{healthy})}$$

$$= \frac{0.98 \times 0.0001}{0.98 \times 0.0001 + 0.0005 \times 0.9999}$$

$$= 0.1639$$

$\therefore$ This is not a good test!

7.3) $P(\text{+ve}\mid\text{infected}) = 0.9999$

$P(\text{+ve}\mid\text{healthy}) = 0.0001$

$P(\text{infected}) = 0.001$

$P(\text{healthy}) = 0.999$

$$\therefore P(\text{infected}\mid\text{+ve}) = \frac{P(\text{+ve}\mid\text{infected})\,P(\text{infected})}{P(\text{+ve}\mid\text{infected})\,P(\text{infected}) + P(\text{+ve}\mid\text{healthy})\,P(\text{healthy})}$$

$$= \frac{0.9999 \times 0.001}{0.9999 \times 0.001 + 0.0001 \times 0.999}$$

$$= 0.9092$$

7.4) $P(\text{+ve}\mid\text{infected}) = 0.9999$

$P(\text{+ve}\mid\text{healthy}) = 10^{-6}$

$P(\text{infected}) = 0.10$

$P(\text{healthy}) = 0.90$

$$P(\text{infected}\mid\text{+ve}) = \frac{P(\text{+ve}\mid\text{infected})\,P(\text{infected})}{P(\text{+ve}\mid\text{infected})\,P(\text{infected}) + P(\text{+ve}\mid\text{healthy})\,P(\text{healthy})}$$

$$= \frac{0.9999 \times 0.1}{0.9999 \times 0.1 + 10^{-6} \times 0.9}$$

$$= 0.999991$$

$\Rightarrow$ A good test.

7.5) $x = \left.\begin{matrix} 1.83 \pm 0.01\,\text{cm} \\ 1.85 \pm 0.01\,\text{cm} \\ 1.87 \pm 0.01\,\text{cm} \end{matrix}\right\}$

1	1 & 2	are compatible $< 2\sigma$
2	2 & 3	" " "
3	1 & 3	" " $< 3\sigma$.

$\therefore \forall$ compatible.

Another way to look at this is $\bar{x} = 1.85 \pm 0.02$ cm

and $\forall$ central values are compatible with this result @ 1σ.

7.6) $\Omega = \{0, 1, 2, 4, 6\}$

Process is poisson

compare $H_0: \lambda = 3$

vs $H_1: \lambda = 4$

assume $P(H_i)$ = constant

$$P(\omega_i, \lambda) = \frac{\lambda^{\omega_i} e^{-\lambda}}{\omega_i!}$$

$$R_i = \frac{P(H_0 | \omega_i)}{P(H_1 | \omega_i)} = \frac{P(\omega_i | H_0)\, P(H_0)}{P(\omega_i | H_1)\, P(H_1)}$$

$$R = \prod_i R_i$$

ω_i	$P(\omega_i \vert 3)$	$P(\omega_i, 4)$	R_i
0	0.0498	0.0183	2.72
1	0.1494	0.0733	2.04
2	0.2240	0.1465	1.53
4	0.1680	0.1954	0.86
6	0.0504	0.1042	0.44

$\therefore R = 3.53$

$\Rightarrow H_0$ is favoured by the data over H_1

i.e. $\lambda = 3$ preferred.

7.7) $\Omega = \{1, 2, 3, 5, 7\}$

Process is Poisson

$H_0: \lambda = 3$

$H_1: \lambda = 4$ (as above)

ω_i	$P(\omega_i, 3)$	$P(\omega_i, 4)$	R_i
1	0.1494	0.0733	2.039
2	0.2240	0.1465	1.529
3	0.2240	0.1954	1.147
5	0.1008	0.1563	0.645
7	0.0216	0.0595	0.362

$R = \prod_i R_i = 0.8367$; so H_1 $(\lambda = 4)$ is slightly preferred over H_0 $(\lambda = 3)$.

It would be useful to gather more data in order to increase the certainty of this conclusion.

7.8) $\Omega = \{1,2,3,4,5\}$ H_0 : Binomial with $p=0.4$

H_1 : Gaussian with $\mu = 3$ and $\sigma = 1$

$R = \prod_i R_i$, where $R_i = \frac{P(H_0|\omega_i)}{P(H_1|\omega_i)} = \frac{P(\omega_i|H_0)\,P(H_0)}{P(\omega_i|H_1)\,P(H_1)}$

Assume $P(H_0) = P(H_1) = $ constant.

ω_i	$P(\omega_i\|H_0)$	$P(\omega_i\|H_1)$	R_i	
1	0.2592	0.0540	4.80	
2	0.3456	0.2420	1.42	$\therefore R = 0.238$
3	0.2304	0.3989	0.58	
4	0.0768	0.2420	0.32	H_1 is preferred by the
5	0.0102	0.0540	0.19	data (Gaussian distribution)

7.9) $\lambda = 1$ for a poisson process.

Observe 5 events → determine p value.

i.e what is $P = \sum_{i=5}^{\infty} P(i,\lambda) = 1 - \sum_{i=0}^{4} P(i,\lambda)$

$\therefore$ p-value $= 0.0037$ (4dp) ⇒ just compatible with expectations @ 3σ.

7.10) $\lambda_{sig} = 1$ As above, but with $\lambda = \lambda_{sig} + \lambda_{bg}$

$\lambda_{bg} = 2$ $= 3$

$N_{observed} = 5$

Determine the p-value $\therefore P = \sum_{i=5}^{\infty} P(i,3) = 1 - \sum_{i=0}^{4} (P_i, 3)$

$= 0.1847$

$\therefore$ p-value $= 0.1847$ (4dp) ⇒ Compatible with expectations

7.11) Expect 5 events observe 15. what is p value?

As above

$p = 1 - \sum_{i=0}^{14} P(i,5) = 0.0002$

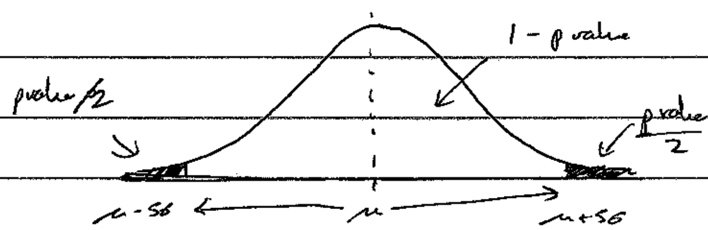
⇒ contradicts expectations

7.12) What is the p-value for the Gaussian observable

$x = 0.5 \pm 0.1$ relative to zero?

→ x is 5 sigma from zero.

→ Need to compute 2 sided interval & subtract from 1 to get the p value:



Want to know about x being non-zero; this is sign independent

$$\therefore p = 1 - \int_{-5\sigma}^{5\sigma} G(x; 0.0, 0.1)\, dx$$

assume σ from measurement & mean corresponding to hypothesis being tested.

$$\therefore p = 1 - 0.999999$$
$$= 1 \times 10^{-6}.$$

7.13) $m_W^{MARK\,2} = 91.14 \pm 0.12$ GeV

$m_W^{OPAL} = 91.1852 \pm 0.0021$ GeV

Do these results agree with each other? Assume SYST ERRORS ARE INDEPENDENT

$$\therefore \sigma_\Delta = \sqrt{0.12^2 + 0.0021^2} \text{ GeV}$$
$$= 0.12 \text{ GeV}$$

$$\Delta = m_W^{OPAL} - m_W^{MARK2}$$
$$= 0.0452$$

$\dfrac{\Delta}{\sigma_\Delta} = 0.38$; The results are in very good agreement.

7.14) Measurements of the 'Higgs-like' particle mass reported by the ATLAS and CMS experiment in 2012 are

$M_H(\text{ATLAS}) = 126.0 \pm 0.4 \pm 0.4$ GeV

$M_H(\text{CMS}) = 125.3 \pm 0.4 \pm 0.5$ GeV

$\Delta = M_H^{ATLAS} - M_H^{CMS}$

$= 0.7$ GeV

$\sigma_\Delta = \sqrt{(\sigma_m^{ATLAS})^2 + (\sigma_m^{CMS})^2}$, $(\sigma_m^{EXPT})^2 = (\sigma_{STAT}^{EXPT})^2 + (\sigma_{SYST}^{EXPT})^2$

$= 0.85$

$\therefore \frac{\Delta}{\sigma_\Delta} = 0.82$ $\Rightarrow$ results agree well.

7.15) $x \geq 0$; (A) $x_1 < 1.0 \times 10^{-6}$ @ 90% C.L.

90% C.L. upper limit is $\simeq \mu + 1.28\sigma$ for a Gaussian 1 sided interval.

(B) $x_2 = (1.5 \pm 1.0) \times 10^{-6}$

Probability (Gaussian) for such a result to be a true representation of the mean value, yet to have a subsequent experiment report a central value of ≤ 0 is $1 - 0.9332 = 6.68\%$

$\therefore$ It is reasonable to assume that both measurements are correct, given the respective datasamples studied.