5.1) 99.73 %

5.2) $x_1 = 0.655 \pm 0.024$

$x_2 = 0.59 \pm 0.08$

$x_3 = 0.789 \pm 0.071$

$$\bar{x} = \frac{\sum_{i=1}^{3} x_i / \sigma_i^2}{\sum_{i=1}^{3} 1/\sigma_i^2}$$

$$\sigma^2 = \sum_{i=1}^{3} 1/\sigma_i^2$$

as the results are ∀ uncorrelated.

x_j	0.655	0.59	0.789	
σ_j	0.024	0.08	0.071	
σ_j^2	0.000576	0.0064	0.005041	
$1/\sigma_j^2$	1736.11	156.25	198.373	$\sum 1/\sigma_j^2 = 2090.73$
x_j/σ_j^2	1137.15	92.1875	156.516	$\sum x_i/\sigma_i^2 = 1385.86$

$\therefore \bar{x} = 0.663 \pm 0.022$

5.3) $\lambda = re^{i\phi}$ $\quad r = 1.0 \pm 0.1$, $\phi = 0.0 \pm 0.2$ rad.

$x = r\cos\phi$ $\qquad y = r\sin\phi$

$$\sigma_x^2 = \left(\frac{\partial x}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial x}{\partial \phi}\right)^2 \sigma_\phi^2 \qquad \sigma_y^2 = \left(\frac{\partial y}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial y}{\partial \phi}\right)^2 \sigma_\phi^2$$

$$= \cos^2\phi\, \sigma_r^2 + r^2 \sin^2\phi\, \sigma_\phi^2 \qquad = \sin^2\phi\, \sigma_r^2 + r^2\cos^2\phi\, \sigma_\phi^2$$

$$= 0.01 \qquad = 0.04$$

$\therefore \sigma_x = 0.1$ & $\sigma_y = 0.2$

5.4) $A = \dfrac{N_1 - N_2}{N_1 + N_2}$; $N = N_1 + N_2$

$= \dfrac{N_1}{N} - \dfrac{N - N_1}{N}$

$= 2\epsilon - 1$ where $\epsilon = N_1 / N$

$\sigma_a^2 = \left(\dfrac{\partial A}{\partial \epsilon}\right)^2 \sigma_\epsilon^2 = 4\sigma_\epsilon^2$

σ_ϵ^2 is given by the binomial uncertainty $\epsilon(1-\epsilon)/N$; thus

$\sigma_a^2 = 4\dfrac{\epsilon(1-\epsilon)}{N} \Rightarrow \sigma_a = 2\sqrt{\dfrac{\epsilon(1-\epsilon)}{N}}$

5.5) $T = 1.62 \pm 0.20$ s

$L = 0.646 \pm 0.005$

what is g? $g = \dfrac{4\pi^2 L}{T^2}$ $\therefore \sigma_g^2 = \left(\dfrac{8\pi^2 L}{T^3}\right)^2 \sigma_T^2 + \left(\dfrac{4\pi^2}{T^2}\right)^2 \sigma_L^2$

$\therefore g = 9.72$ m/s² ± 2.40 m/s²

$\therefore g = (9.7 \pm 2.4)$ m/s²

5.6) One would expect a factor of 10 improvement (neglecting σ_L), as $\sigma_g \propto \sigma_T$ and the uncertainty on one oscillation, when ten are measured, is 1/10th of the uncertainty of measuring just a single oscillation.

5.7) $L = (0.646 \pm 0.005)$ m

$\sigma_T = 0.2$ s

T	1.62	1.56	1.59	1.50	1.56	1.62	1.59	1.65	1.60	1.56
g	9.718	10.480	10.088	11.335	10.480	9.718	10.088	9.368	9.962	10.479

$\langle T \rangle = 1.585$ s $\sigma_T^{meas} = 0.042$ s

$\langle g \rangle = 10.171$ m/s² $\sigma_g = 0.553$ m/s²

5.8)

$x_1 = 1.2 \pm 0.3$

$x_2 = 1.8 \pm 0.3$

$$\bar{x} = \frac{\sum_{i=1}^{2} x_i/\sigma_i^2}{\sum_{i=1}^{2} 1/\sigma_i^2} \pm \left(\sum_{i=1}^{2} 1/\sigma_i^2 \right)^{-1/2}$$

$$\therefore \sum \frac{1}{\sigma_i^2} = \frac{1}{0.09} + \frac{1}{0.09} = 22.222$$

$$\Rightarrow \sigma = 0.212$$

$$\sum_i x_i/\sigma_i^2 = 13.\dot{3} + 20 = 33.\dot{3}$$

$$\therefore \bar{x} = 1.5 \pm 0.2$$

5.9)

$x_1 = 1.0 \pm 0.5$

$x_2 = 2.0 \pm 0.5$

$$\sum_i 1/\sigma_i^2 = \frac{1}{0.25} + \frac{1}{0.25} = 8 \quad \Rightarrow \sigma = 1/\sqrt{8} = 0.354 \text{ (3 dp)}$$

$$\sum_i x_i/\sigma_i^2 = 4 + 8 = 12$$

$$\therefore \bar{x} = 1.5 \pm 0.354 \quad \text{or } \bar{x} = 1.5 \pm 0.4 \text{ (1 dp)}$$

5.10)

$$Q = \epsilon(1 - 2\omega)$$

$$\sigma_Q^2 = \left(\frac{\partial Q}{\partial \epsilon}\right) \sigma_\epsilon^2 + \left(\frac{\partial Q}{\partial \omega}\right)^2 \sigma_\omega^2$$

$$= (1-2\omega)^2 \sigma_\epsilon^2 + (-2\epsilon)^2 \sigma_\omega^2$$

$$\therefore \sigma_Q = \sqrt{(1-2\omega)^2 \sigma_\epsilon^2 + 4\epsilon^2 \sigma_\omega^2}$$

5.11)

Posture of student (slouching / upright)

offset from footwear

accuracy of measuring device

use of measuring device (along axis of height of student; or rotated relative to that?)

Time of day of the measurement (relative to the time the person wakes up, as height changes depending on how long we have been lying down / asleep).

etc...

5.12) Expt 1:

$A_1 = 2.0 \pm 0.2$ $B_1 = 0.0 \pm 0.5$ $e_{AB} = 0.5$

Expt 2:

$A_2 = 1.5 \pm 0.5$ $B_2 = -1.0 \pm 0.3$ $e_{AB} = 0.1$

$$\bar{x} = \left[\sum_i V_i^{-1}\right]^{-1} \left[\sum_i V_i^{-1} \cdot x_i\right]$$

$$V = \left[\sum_i V_i^{-1}\right]^{-1}$$

$$\therefore x_1 = \begin{pmatrix} 2.0 \\ 0.0 \end{pmatrix} \quad e_1 = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix} \Rightarrow V_1 = \begin{pmatrix} 0.04 & 0.05 \\ 0.05 & 0.25 \end{pmatrix}$$

$$x_2 = \begin{pmatrix} 1.5 \\ -1.0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 1 & 0.1 \\ 0.1 & 1 \end{pmatrix} \Rightarrow V_2 = \begin{pmatrix} 0.25 & 0.015 \\ 0.015 & 0.09 \end{pmatrix}$$

$$\therefore V = \begin{pmatrix} 0.0293 & 0.01299 \\ 0.1299 & 0.0662 \end{pmatrix} \quad \bar{x} = \begin{pmatrix} 1.819 \\ -0.738 \end{pmatrix}$$

i.e. $A = 1.82 \pm 0.17$

$B = -0.74 \pm 0.26$

$\epsilon_{AB} = 0.013$

5.13) $A_1 = 2.0 \pm 0.5$ $B_1 = 0.0 \pm 0.1$ $e_{AB} = 0.0$

$A_2 = 1.5 \pm 0.5$ $B_2 = 0.5 \pm 0.2$ $e_{AB} = 0.3$

~~V observables are correlated~~

$$x_1 = \begin{pmatrix} 2.0 \\ 0.0 \end{pmatrix} \quad e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow V_1 = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.01 \end{pmatrix}$$

$$x_2 = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix} \quad e_2 = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix} \Rightarrow V_2 \begin{pmatrix} 0.25 & 0.03 \\ 0.03 & 0.04 \end{pmatrix}$$

$$\therefore V = \begin{pmatrix} 0.120 & 0.003 \\ 0.003 & 0.008 \end{pmatrix} \quad \bar{x} = \begin{pmatrix} 1.59 \\ 0.11 \end{pmatrix} \quad \therefore \epsilon_{AB} = 0.003$$

$\therefore A = 1.59 \pm 0.35$

$B = 0.11 \pm 0.09$

5.14 $A_1 = 3 \pm 1$ $B_1 = 0 \pm 1$ $C_1 = 0 \pm 1$

$A_2 = 2 \pm 1$ $B_2 = 1 \pm 2$ $C_2 = 1 \pm 1$

$\sigma_{ij} = 0 \quad \forall\ i,j \quad i \neq j.$

$$\therefore x_1 = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \qquad V_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$x_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \qquad V_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

One can follow the same procedure as the previous 2 questions or revert to the uncorrelated variant of these to compute the average values; actually as there are no correlations one can treat A, B & C as indept. problems.

eg. A: $\sum 1/\sigma_i^2 = 1/1^2 + 1/1^2 = 2 \implies \sigma_A = 1/\sqrt{2}$

B: $= 1/1^2 + 1/2^2 = 1\frac{1}{4} \implies \sigma_B = 2/\sqrt{5}$

etc.

$$\therefore \bar{x} = \begin{pmatrix} 2.5 \\ 0.2 \\ 0.5 \end{pmatrix} \quad \& \quad V = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 4/5 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

$\implies A = 2.5 \pm 1/\sqrt{2} = 2.5 \pm 0.7$ (1dp)

$B = 0.2 \pm 2/\sqrt{5} = 0.2 \pm 0.9$ (1dp)

$C = 0.5 \pm 1/\sqrt{2} = 0.5 \pm 0.7$ (1dp)

5.15) $E^2 = m_o^2 c^4 + p^2 c^2$ $\qquad \frac{\sigma_E}{E} = 0.01$

$\therefore m_o = \sqrt{\frac{E^2 - p^2 c^2}{c^4}}$ $\qquad \frac{\sigma_p}{p} = 0.01$

$$\sigma_{m_o}^2 = \left(\frac{\partial m_o}{\partial E}\right)^2 \sigma_E^2 + \left(\frac{\partial m_o}{\partial p}\right)^2 \sigma_p^2$$

$$\frac{\partial m_o}{\partial E} = \frac{1}{c^2} \cdot \frac{\partial}{\partial E}\left(E^2 - p^2 c^2\right)^{1/2}$$

$$= \frac{1}{2c^2} \cdot \frac{2E}{\sqrt{E^2 - p^2 c^2}}$$

$$\therefore \left(\frac{\partial m_o}{\partial E}\right)^2 = \frac{E^2}{c^4 (E^2 - p^2 c^2)}$$

$$\frac{\partial m_o}{\partial p} = \frac{1}{c^2} \frac{\partial}{\partial p}\left(E^2 - p^2 c^2\right)^{1/2}$$

$$= \frac{1}{2c^2} \cdot \frac{-2 p^2 c^2}{\sqrt{E^2 - p^2 c^2}}$$

$$\therefore \left(\frac{\partial m_o}{\partial p}\right)^2 = \frac{p^2}{E^2 - p^2 c^2}$$

$$\therefore \sigma_{m_o}^2 = \frac{E^2 \cdot \sigma_E^2}{c^4 (E^2 - p^2 c^2)} + \frac{p^2 \sigma_p^2}{E^2 - p^2 c^4}$$; where $\sigma_E = 0.01\,E$, $\sigma_p = 0.01\,p$

$$\therefore \sigma_{m_o}^2 = \frac{10^{-4}}{E^2 - p^2 c^2}\left[\frac{E^4}{c^4} + p^4\right]$$

$$\therefore \sigma_{m_o} = \frac{10^{-2}}{\sqrt{E^2 - p^2 c^2}}\left[\frac{E^2}{c^4} + p^4\right]^{1/2}$$