5

Errors

(2012 – 2013 Academic Year: Tutorial Questions)

Exercises

5.1 What percentage of results would one expect to obtain within $\pm 3\sigma$ of the mean value of an observable when making a set of measurements?

5.2 Compute the weighted average of the uncorrelated results 0.655 ± 0.024, 0.59 ± 0.08, and 0.789 ± 0.071.

5.3 A measurement of a complex quanity $\lambda = re^{i\phi}$ has been made, where $r = 1.0 \pm 0.1$, and $\phi = 0.0 \pm 0.2$ (in radians). What are the corresponding uncertainties on x and y, the projections of λ onto the real and imaginary axes, respectively?

5.4 What is the uncertainty σ_A on an asymmetry given by $A = (N_1 - N_2)/(N_1 + N_2)$, where $N_1 + N_2 = N$ is the total number of events obtained in the counting experiment.

5.5 The period of oscillation measured using a simple pendulum is found to be $T = (1.62 \pm 0.20)$ s, for a length of (0.646 ± 0.005) m. What is the corresponding value of g obtained from this measurement?

5.6 What is the expected improvement in the precision of g determined if instead of measuring a single oscillation, one measures ten oscillations?

5.7 Several measurements of the period of oscillation measured using a simple pendulum are made, where $T = 1.62$, 1.56, 1.59, 1.50, 1.56, 1.62, 1.59, 1.65, 1.60, 1.56 s. Given the length of the pendulum is (0.646 ± 0.005) m, what is the corresponding value of g obtained?

5.8 Compute the weighted average of the two measurements $x_1 = 1.2 \pm 0.3$ and $x_2 = 1.8 \pm 0.3$.

5.9 Compute the weighted average of the two measurements $x_1 = 1.0 \pm 0.5$ and $x_2 = 2.0 \pm 0.5$.

5.10 The ability to *tag* the value of an intrinsic quantity can be useful, however this process is often associated with a mis-assignment probability given by some parameter ω. The effective tagging efficiency is given by

$$Q = \epsilon(1 - 2\omega), \tag{5.0.1}$$

where ϵ is efficiency. Determine the general equation for the uncertainty on Q given values and uncertainties for ϵ, and ω.

5.11 Consider the measurement problem associated with determining the height of a student in a classroom. What possible sources of systematic uncertainty might be associated with that measurement?

5.12 Two experiments have measured the quantities A and B with the following precisions: $A_1 = 2.0 \pm 0.2$, $A_2 = 1.5 \pm 0.5$, $B_1 = 0.0 \pm 0.5$, and $B_2 = -1.0 \pm 0.3$. Given that the first (second) experiment reports a correlation between A and B of $\rho_{AB} = 0.5$ (0.1) compute the weighted average of these results and the covariance between the average values of A and B.

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5.14 Two experiments have measured the quantities A, B, and C with the following precisions: $A_1 = 3 \pm 1$, $A_2 = 2 \pm 1$, $B_1 = 0 \pm 1$, and $B_2 = 1 \pm 2$, and $C_1 = 0 \pm 1$, and $C_2 = 1 \pm 1$. Both experiments report that the results A, B, and C are uncorrelated with each other. Compute the weighted average for these observables.

5.15 Given the energy-mass relation $E^2 = m_0^2 c^4 + p^2 c^2$, determine the general formula for the variance on the mass m_0 given measurements of energy E and momentum p with precision of 1% each.