4.1) $P_H = P_T = 0.5$

$n = 5$, $r = 0$ or 5 $\qquad P(r;p,n) = p^r(1-p)^{n-r} \cdot \dfrac{n!}{r!(n-r)!}$

$P(0;0.5,5) = 0.5^5$

$P(5;0.5,5) = 0.5^5$

$\therefore P(\emptyset \text{ or } 5\ H) = 2 \cdot (0.5^5) = 0.0625$

4.2) $p = 0.4$

$P(0;0.4,5) = 0.07776$

$P(5;0.4,5) = 0.01024$

$\therefore P(\emptyset \text{ or } 5\ H) = 0.088$

4.3) $p = 0.4 \qquad n = 5 \qquad r = 3$

$$P(3;0.4,5) = 0.4^3(1-0.4)^2 \cdot \frac{5!}{3!\,2!} = 0.2304$$

4.4) Given $\lambda = 3$; what is $P(0,3) = \dfrac{3^0 e^{-3}}{0!}$

$$= e^{-3} = 0.0498$$

4.5) Given $\lambda = 3$ Compute

$$P = \sum_{i=3}^{\infty} P(i,3) = \sum_{i=3}^{\infty} \frac{3^i e^{-3}}{i!}$$

$$= 1 - \sum_{i=0}^{2} \frac{3^i e^{-3}}{i!} = 1 - 0.4232$$

$$= 0.5768$$

(compute sum by hand or use reference table).

4.6) $R = \dfrac{P(1,4)}{P(5,4)}$ for a poisson distribution

$$= \frac{0.07326}{0.15630} = 0.4688$$

SDA 4 – DISTRIBUTIONS

4.7) $P(x) = G = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma}$

$n\sigma$	Prob
1	0.6827
2	0.9545
3	0.9973

use integral tables.

4.8) Use one sided table & read off value: $z = 1.28$ has an integral for $z \in [-\infty, 1.28]$ of 0.8997. $z = 1.29$ has an integral of 0.9015; $\therefore$ 90% bound is close to $z = 1.28$.

4.9) $P(+1\sigma; 0, 1) = 0.24197$

$P(-1\sigma; 0, 1) = P(+1\sigma; 0, 1)$

$P(0; 0, 1) = 0.39894$

$\therefore L(z = \pm 1\sigma) = 0.6065$

4.10) $\chi^2 = 5$, $\nu = 4$ $\Rightarrow$ $P(\chi^2, \nu) = 0.2873$ (this is reasonable)

4.11) $\chi^2 = 1$, $\nu = 10$ $\Rightarrow$ $P(\chi^2, \nu) = 0.9998$ (this is reasonable)

4.12) $\chi^2 = 8$, $\nu = 2$ $\Rightarrow$ $P(\chi^2, \nu) = 0.0183$ (this is not reasonable)

4.13) $\lambda = \lambda_1 + \lambda_2$; the sum of two Poisson distributions is a poisson distribution.

This can be seen as:

$$P_1 = \frac{\lambda_1^{r_1} e^{-\lambda_1}}{r_1!} \qquad P_2 = \frac{\lambda_2^{r_2} e^{-\lambda_2}}{r_2!}$$

$P_{12} = P_1 + P_2$

Given $\langle r \rangle = \sum_{r=0}^{\infty} r P(r, \lambda) = \lambda$ & $V(r) = \lambda$

then $\langle r_{12} \rangle = \langle r_1 \rangle + \langle r_2 \rangle = \lambda_1 + \lambda_2$

and $V_{12} = V_1 + V_2 = \lambda_1 + \lambda_2$

4.14) Compute the expectation value of x for $P(x) = a + e^{-x}$ over $x \in [0, 1]$.

$$\langle x \rangle = \int_0^1 x \; P(x) \, dx$$

$$= \int_0^1 ax + xe^{-x} \, dx$$

let $u = x$ $\quad v = e^{-x}$

$u' = 1$ $\quad v' = -e^{-x}$

$$= \left[\frac{ax^2}{2}\right]_0^1 + \left[xe^{-x}\right]_0^1 - \int_0^1 e^{-x} \, dx$$

$$= \left[\frac{ax^2}{2} + xe^{-x}\right]_0^1 + \left[e^{-x}\right]_0^1$$

$$= \left[\frac{ax^2}{2} + xe^{-x} + e^{-x}\right]_0^1$$

$$= \left(\frac{a}{2} + e^{-1} + e^{-1}\right) + 1$$

$$= \frac{a}{2} + 1 + 2e^{-1}$$

4.15) $$\langle V \rangle = \int_0^1 (x - \bar{x})^2 \; P(x) \, dx$$

; $P(x) = a + e^{-x}$

$\bar{x} = \frac{a}{2} + 1 + \frac{2}{e}$

$$= \int_0^1 ax^2 + x^2 e^{-x} \, dx - \left(\frac{a}{2} + 1 + 2e^{-1}\right)^2$$

as $V(x) = \langle x^2 \rangle - \langle x \rangle^2$

$$\langle x^2 \rangle = \left[\frac{ax^3}{3} - x^2 e^{-x}\right]_0^1 + 2\int_0^1 x e^{-x} \, dx$$

$$= \left[\frac{ax^3}{3} - x^2 e^{-x} - 2xe^{-x}\right]_0^1 + 2\int_0^1 e^{-x} \, dx$$

$$= \left[\frac{ax^3}{3} - x^2 e^{-x} - 2xe^{-x} - 2e^{-x}\right]_0^1$$

$$= \frac{a}{3} - e^{-x} - 2e^{-x} - 2e^{-x} + 2$$

$$= \left(2 + \frac{a}{3} - \frac{5}{e}\right)$$

$$\therefore \langle V \rangle = \left(2 + \frac{a}{3} - \frac{5}{e}\right) - \left(\frac{a}{2} - \frac{2}{e} + 1\right)^2$$

$$= 1 - \frac{2a}{3} - \frac{a^2}{4} - \frac{4}{e^2} - \frac{1}{e} + \frac{2a}{3}$$