2

Probability

(2012 – 2013 Academic Year: Tutorial Questions)

Exercises

2.1 Compute the probability of being dealt an ace from a new deck of cards.

2.2 Two people are playing blackjack. What is the probability that the first two cards being dealt are both aces?

2.3 Compute the probability of being dealt either a picture card or 10 from a new deck of cards.

2.4 In a game of blackjack with two participants, compute the probability of being dealt an ace from a deck of cards, followed by either a picture card or a 10, assuming that the first card is dealt to you.

2.5 The weather forecast tomorrow is for rain. 70% of the time when it rains, the rain has been correctly forecast. When there is no rain forecast, it rains 5% of the time, and on average it rains 30 days of the year. Given this, what is the probability that it will rain tomorrow?

2.6 A storm is forecast for tomorrow. When storms occur 95% of the time, the forecast was correct, and when there was no storm was forecast, only 2% of the time would one happen. Storms occur on average 20 days ofthe year in England. Given this, what is the probability that a storm will occur tomorrow?

2.7 The weather forecast tomorrow is for rain. 70% of the time when it rains, the rain has been correctly forecast. When there is no rain forecast, it rains 5% of the time, and on average it rains 100 days of the year. Given this, what is the probability that it will rain tomorrow? Compare the result obtained with that assuming that on average it is equally likely to rain or not.

2.8 The weather forecast tomorrow is for rain. 90% of the time when it rains, the rain has been correctly forecast. When there is no rain forecast, it rains 5% of the time, and on average it rains 30 days of the year. Given this, what is the probability that it will rain tomorrow? Compare the result obtained with that assuming that on average it is equally likely to rain or not.

2.9 If there are two doors leading from a corridor, where nothing but an empty room lies behind one, and a staircase lies behind the other, use Bayes theorem to compute the probability of opening the left hand door and finding an empty room.

2.10 You appear on a gameshow where the host presents you with three doors. Behind one of the doors there is a car and if you correctly select this door you will win it. There is nothing behind the other two. You select one of the doors. After this, the host opens one of the two remaining doors to reveal nothing behind it. What should you do to maximise your chance of winning the car, and what is the corresponding probability?1

2.11 The parabola $y = N(3-x)^2$ corresponds to PDF. Compute the normalisation constant N, and re-express the PDF as a likelihood function with a most probable value of one.

2.12 Compute the PDF normalisation coefficient N for the function given by $y = N(1+x+x^2)$ in the range $x \in [0,2]$, and write down the corresponding likelihood function with a most probable value of one.

2.13 Compute the PDF normalisation coefficient for an exponential distribution approximating the exponential decay of a muon with lifetime $\tau_\mu = 2.2\,\mu$s over the range $t \in [0,20]$ μs, and write the corresponding PDF and likelihood functions, where $\max[\mathcal{L}] = 1$.

2.14 What is the likelihood ratio for $x = 1$ compared to $x = 2$ for the function $\mathcal{L} = e^{-x}$, valid for $x > 0$?

2.15 What is the likelihood ratio for $x = -1$ compared to $x = +1$ for the function $\mathcal{L} = e^{-x^2}$?

1 Note this is the famous Monty Hall problem, named after the gameshow host.