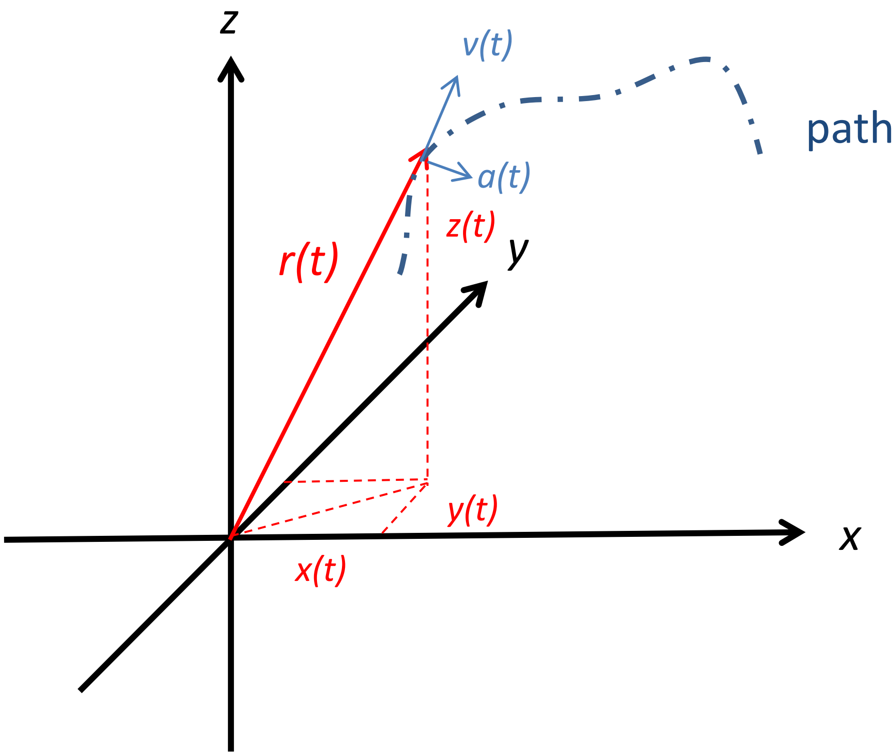
REVISION

Kinetics (kinematics)

Motion of bodies without considering forces

Consider an object moving in space in a generalised 3D path



$$\overrightarrow{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$$

Its instantaneous velocity, **v**, is simply the rate of change of **r,** so

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} \quad \text{or} \quad \underline{\dot{r}}$$

or

$$\vec{v}(t) = \frac{dx(t)}{dt}\hat{\imath} + \frac{dy(t)}{dt}\hat{\jmath} + \frac{dz(t)}{dt}\hat{k}$$

where: $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, $v_z = \frac{dz}{dt}$

$$\vec{v}(t) = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

so it can be written in terms of its own components

We can define the acceleration **a** at time *t* as

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \dot{\vec{v}} = \ddot{\vec{r}} = \underline{\ddot{r}}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d\{v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}\}}{dt}$$

$$\vec{a}(t) = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k}$$

$$\vec{a}(t) = \frac{d\frac{dx}{dt}}{dt}\hat{\imath} + \frac{d\frac{dy}{dt}}{dt}\hat{\jmath} + \frac{d\frac{dz}{dt}}{dt}\hat{k}$$

$$\vec{a}(t) = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath} + \frac{d^2z}{dt^2}\hat{k} = \frac{d^2\vec{r}(t)}{dt^2}$$

or simply in terms of its own components

$$\vec{a}(t) = a_x\hat{\imath} + a_y\hat{\jmath} + \ a_z\hat{k}$$

where,

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}\ , \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}\ , \quad a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2}$$

Running the maths backwards now…

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt}$$

$$\int \vec{a}dt = \int d\vec{v}(t)$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \int_{t_i}^{t_f} \vec{a}(t)dt$$

Where

$$\vec{v}_i = initial\ velocity$$

$$\vec{v}_f = final\ velocity$$

$$t_i = initial\ velocity$$

$$t_f = final\ velocity$$

Of course we can also do this for velocity

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$$

$$\int \vec{v}(t)dt = \int d\vec{r}(t)$$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \int_{t_i}^{t_f} \vec{v}(t)dt$$

Where,

$$\vec{r}_i = initial\ position$$

$$\vec{r}_f = final\ position$$

Usually t_i=0 and we also have boundary (initial) conditions to define the constants of integration.

Since we can write everything in terms of independent (orthogonal) components,

$$\overrightarrow{r(t)} = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k},$$

$$\vec{v}(t) = v_x\hat{\imath} + v_y\hat{\jmath} + \ v_z\hat{k},$$

etc.,

$$\Delta x = \int_{t_i}^{t_f} v_x dt, \qquad \Delta y = \int_{t_i}^{t_f} v_y dt, \qquad \Delta z = \int_{t_i}^{t_f} v_{yz} dt$$

and

$$\Delta v_x = \int_{t_i}^{t_f} a_x dt, \qquad \Delta v_y = \int_{t_i}^{t_f} a_y dt, \qquad \Delta v_z = \int_{t_i}^{t_f} a_z dt$$

Remember that <u>average</u> velocity over interval Δt,

$$\vec{v}_{Average} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} \neq \frac{dx(t)}{dt}$$

Consider the special case of constant acceleration, i.e. not *a*(t) but simply *a*

$$a = \frac{dv}{dt}$$

$$\int a dt = \int dv$$

$$\Delta v = [at]_0^t$$

$$\Delta v = at$$

$$v_f - v_i = at$$

Let $v_f = v$ and $v_i = u$,

$$\boxed{v = u + at}$$

We also have over interval t,

$$\Delta x = S = \int_0^t (u + at)dt$$

$$S = ut + \frac{1}{2}at^2$$

$$\boxed{S = ut + \frac{1}{2}at^2}$$

Use $t = \frac{v-u}{a}$ and substitute into the above,

$$S = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$aS = u(v-u) + \frac{1}{2}(v-u)^2$$

$$aS = uv - u^2 + \frac{1}{2}(v^2 - 2uv + u^2)$$

$$2aS = 2uv - 2u^2 + \frac{1}{2}(v^2 - 2uv + u^2)$$

$$2aS = v^2 - u^2$$

$$\boxed{v^2 = u^2 + 2aS}$$

$$V_{Average} = \frac{S}{t} = u + \frac{1}{2}at = u + \frac{1}{2}\left(\frac{v-u}{a}\right)a = \frac{v+u}{2}$$

this is true <u>only</u> for constant a.