QUANTUM MECHANICS B

THE FORMALISM OF QUANTUM MECHANICS

- YOU HAD A FIRST GLIMPSE AT FUNDAMENTAL POSTULATES IN QMA

- DISCUSS THEM NOW IN MORE DETAIL AND GENERALISE AND INTRODUCE NEW CONCEPTS
  - MEASUREMENT
  - EXPANSION THEOREM
  - COMMUTING OPERATORS // SIMULTANEOUS EIGENSTATES
  - OPERATORS COMMUTING WITH $\hat{H}$ AND SYMMETRIES
  - BRA-KET (DIRAC)
  - MATRIX Q.M. (APPLIED TO HARMONIC OSCILLATOR)
  - HEISENBERG EQUATIONS OF MOTION EHRENFEST THEOREM

⋮

CLASSICAL VS. QUANTUM PHYSICS

- CLASSICAL

STATE OF SYSTEM

DYNAMICAL VARIABLES (OBSERVABLES) ARE FUNCTIONS OF $\vec{x}, \vec{p}$

$\vec{x}(t), \vec{p}(t)$

$\Downarrow$

SOLUTIONS OF DYNAMICAL EQNS.

$$m\ddot{\vec{x}} = \vec{F}(\vec{x}(t)) = -\vec{\nabla}V(\vec{x}(t))$$

TRAJECTORY OF PARTICLE

ASSUMPTION: CAN MEASURE $\vec{x}$ AND $\vec{p}$ TO ARBITRARY PRECISION SIMULTANEOUSLY!

- Q.M. DESTROYS THESE INTUITIVE EXPECTATIONS

  - THERE IS A FUNDAMENTAL LIMITATION TO WHAT WE CAN KNOW ABOUT A SYSTEM

  - CONCEPTS LIKE TRAJECTORIES, SIMULT. MEASUREMENT OF $\vec{x}, \vec{p}$ DO NOT MAKE SENSE

  - AT BEST THEY ARE APPROXIMATE CONCEPTS FOR LARGE OBJECTS BUT ARE FUNDAMENTALLY WRONG AT MICROSCOPIC LEVEL.

FUNDAMENTAL OBJECT IS WAVEFUNCTION $\Psi(\vec{x},t)$ WHOSE DYNAMICS IS GOVERNED BY

TDSE $$\boxed{-\frac{\hbar^2}{2m}\vec{\nabla}^2\Psi + V(\vec{x})\Psi = i\hbar\frac{\partial}{\partial t}\Psi}$$

FUND. DYNAMICAL EQUATION

- Ψ AND $c\Psi$ ARE PHYSICALLY EQUIVALENT

POSTULATE:
IN QM, THE STATE OF THE SYSTEM IS DESCRIBED BY THE WAVEFUNCTION $\Psi(\vec{x},t)$

- MORE PRECISELY TO AN ENSEMBLE OF SYSTEMS (≡ LARGE COLLECTION OF IDENTICALLY PREPARED SYSTEMS) WE ASSOCIATE A W.F. Ψ

- Ψ CONTAINS ALL INFORMATION WE CAN EVER KNOW ABOUT THE SYSTEM (ENSEMBLE)

PROPERTIES

- $\Psi(\vec{x},t)$ HAS NO DIRECT PHYSICAL MEANING (COMPLEX)

- NORMALISABILITY: $\int d^3x\,|\Psi(\vec{x},t)|^2 = N < \infty$

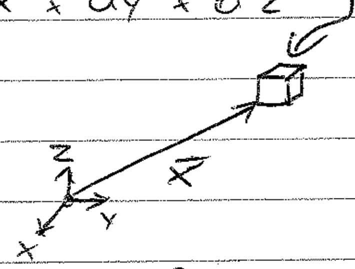
IF $N=1$ (BY CHOOSING c)

$\Rightarrow$ NORMALISED W.F.

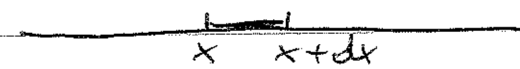
PROBABILITY INTERPRETATION (BORN)

$$P(\vec{x},t) = |\psi(\vec{x},t)|^2 \, d^3x \;\ldots$$

PROBABILITY TO FIND THE PARTICLE AT POSITION $\vec{x}$ AT TIME t IN THE VOLUME $dx \times dy \times dz$



1-DIM W.F. $|\psi(x,t)|^2 dx \equiv$ PROBABILITY



$$P_{ab} = \int_{x=a}^{b} |\psi(x,t)|^2 dx$$

PROBABILITY TO FIND PARTICLE IN A <u>SINGLE</u> MEASUREMENT

NORMALISATION CONDITION $\int_{x=-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$

JUST MEANS THAT THE PARTICLE MUST BE SOMEWHERE

- Ψ DOES NOT PREDICT OUTCOME OF A SINGLE MEASUREMENT!

- HOW CAN WE "MEASURE" $\Psi(x,t)$?

PREPARE SYSTEM ∞ MANY TIMES IN SAME STATE OR ∞ NUMBER COPIES OF IDENTICAL SYSTEMS AND MEASURE POSITION OF PARTICLE

$\Rightarrow$ ENSEMBLE INTERPRETATION OF Q.M.

- ANOTHER EXAMPLE OF AN ENSEMBLE

BEAM OF IDENTICAL PARTICLES WITH SAME MOMENTUM $\vec{p}$ AND ENERGY E

$$\Psi(x,t) = \mathcal{N} e^{i(px - Et)/\hbar}$$

de Broglie

- WHAT ELSE CAN WE PREDICT?

Q.M. EXPECTATION VALUES OF PHYSICAL OBSERVABLES

$$\langle x \rangle \equiv \int_{-\infty}^{+\infty} x \, |\Psi(x,t)|^2 dx = \int_{-\infty}^{+\infty} \Psi^* x \, \Psi \, dx$$

POSITION

MORE GENERAL OBSERVABLES ARE GIVEN BY LINEAR OPERATORS $\hat{A}$

$$\langle \hat{A} \rangle \equiv \int_{-\infty}^{+\infty} \Psi^* \hat{A} \, \Psi \, dx$$

EX: $\hat{P}_x = -i\hbar \frac{\partial}{\partial x}$ MOMENTUM OP.

OBTAINED EXPERIMENTALLY AFTER MANY REPEATED MEASUREMENTS OF x OR $\hat{A}$

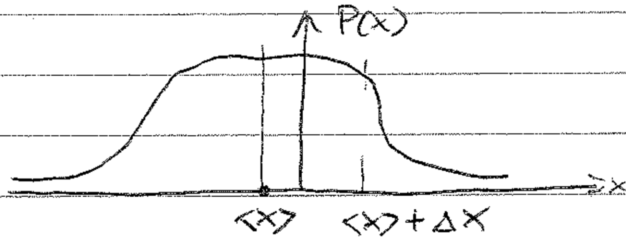
$\boxed{\text{END LECTURE 2 / WEEK 1}}$

- UNCERTAINTY Δx

$$\langle (x - \langle x \rangle)^2 \rangle = \langle x^2 - 2x\langle x \rangle + \langle x \rangle^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \qquad \langle x^2 \rangle = \int \psi^* x^2 \psi \, dx$$

"HOW MUCH IS PARTICLE SMEARED OUT"

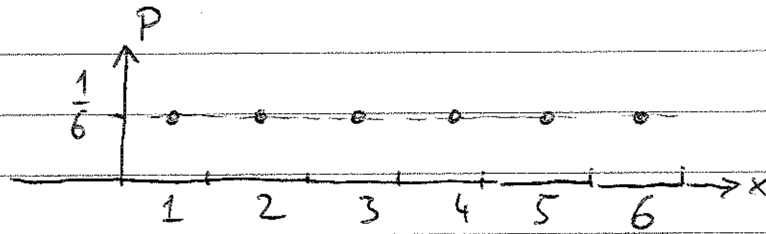


EXAMPLE 1 THROWING ~~A~~ DICE $\equiv$ PREPARING SYSTEM

POSSIBLE OUTCOMES: $x = 1, 2, 3, 4, 5, 6$

PROBABILITIES: $P_x = |\psi_x|^2 = \frac{1}{6} \qquad x = 1, \dots, 6$

$$\left.\begin{aligned} \langle x \rangle &= \tfrac{1}{6}(1 + \dots + 6) = 3.5 \\ \langle x^2 \rangle &= \tfrac{1}{6}(1 + 2^2 + \dots 6^2) = \tfrac{91}{6} \end{aligned}\right\} \Rightarrow \Delta x \cong 1.7$$



STATE OF DICE CAN BE WRITTEN AS VECTOR WITH 6 COMPONENTS

$$\vec{U} = \begin{pmatrix} U_1 \\ \vdots \\ U_6 \end{pmatrix} \quad \text{e.g. } U_x = \frac{1}{\sqrt{6}} \qquad \xrightarrow{P_x = |U_x|^2} \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \text{OUTCOME} = 3$$

$\uparrow$ STATE OF ENSEMBLE OF DICE IN THE AIR

$\uparrow$ STATE OF DICE SHOWING 3

SUPERPOSITION PRINCIPLE

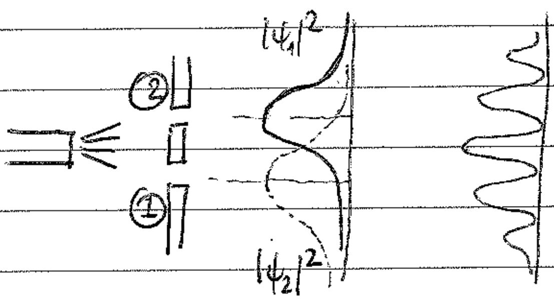
IF Ψ_1 AND Ψ_2 ARE SOLUTIONS OF TISE THEN

$$\Psi = c_1 \Psi_1 + c_2 \Psi_2 \quad \text{IS A SOLUTION}$$

REASON: TISE IS A LINEAR EQN.

$$\hat{H}\Psi = -\frac{\hbar^2}{2m}\vec{\nabla}^2\Psi + V(\vec{x})\Psi = i\hbar\partial_t\Psi$$

$\Rightarrow$ DOUBLE SLIT EXPERIMENT, ---
WAVE LIKE BEHAVIOUR
SCHRÖDINGER CAT
GHOSTLY INTERACTIONS AT DISTANCE



SEPARATELY TOGETHER

INTERFERENCE

$$\Psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$$

$$|\psi|^2 = \underbrace{\tfrac{1}{2}|\psi_1|^2 + \tfrac{1}{2}|\psi_2|^2}_{\substack{\text{classical expectation} \\ = \text{sum of probabilities} = P_1 + P_2}} + \underbrace{2\,\mathrm{Re}(\psi_1\psi_2^*)}_{\text{INTERFERENCE TERM}}$$

- COPENHAGEN/BORN INTERPRETATION AVOIDS QUESTIONS LIKE DID A SINGLE PARTICLE GO THROUGH SLIT 1 OR 2, OR BOTH?
- ~~HOWEVER MEASUREMENT OF Ψ AT [illegible] SHOWS INTERFERENCE PATTERN~~

- THE PARTICLES DO NOT FOLLOW TRAJECTORIES THROUGH 1 OR 2 AS POINTLIKE PARTICLES DO. SINCE THEN $P = P_1 + P_2$ WHICH IT IS NOT!

- BUT THE PARTICLES ARE NOT WAVES EITHER BECAUSE ON THE SCREEN WE DETECT SHARP DOTS WHERE A PARTICLE HITS!

- WE CAN TRY TO CHECK WHETHER THE PARTICLES GO THROUGH 1 OR 2 BY MEASUREMENT BUT THEN THE INTERFERENCE PATTERN IS DESTROYED!

SO PARTICLES LIKE ELECTRONS ARE POINTLIKE OBJECTS THAT DO NOT FOLLOW TRAJECTORIES!

END LECT. 3 / WEEK 1