QMA Mid-term Test 13:00 on Tuesday the 1st of March 2011

Write your answers in the space provided. **NAME**:

Useful Stuff: $\int_{-\infty}^{\infty} e^{-Cx^2}\,dx = \sqrt{\frac{\pi}{C}}$ $\int_{-\infty}^{\infty} x\ e^{-Cx^2}\,dx = 0$ $\int_{-\infty}^{\infty} x^2 e^{-Cx^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{C^3}}$

Q1.

a) Write down the one-dimensional, time-dependent Schrödinger equation for a particle in a potential $V(x,t)$.

b) Which condition allows us to derive the time-independent Schrödinger equation?

c) Write down the time-independent Schrödinger equation.

d) Write down a resulting general form for $\Psi(x,t)$ in the time independent case.

Q2. A particle exists in an eigenstate $\Psi(x,t)$; an observable, q , is represented by an operator $\hat{Q}$.

a) Write an expression for the expectation value of the observable, $\langle q \rangle$.

b) Write an expression for the uncertainty in q , namely Δq .

c) What is the Born interpretation of $\Psi(x,t)$?

Q3. Write down expressions for the operators relating to position and momentum, namely: $\hat{X}$, $\hat{P}$, $\hat{X}^2$ and $\hat{P}^2$.

Q4. Including as much relevant information as reasonably possible, sketch the potential, the wavefunction and the probability density for the ground state <u>and</u> first excited state of:

a) A particle confined to an infinite 1D square well.

b) A particle confined to a finite 1D square well.

Q5. At $t = 0$ a wave packet moving in one dimension is prepared in a state corresponding to the wave function shown in fig.1.

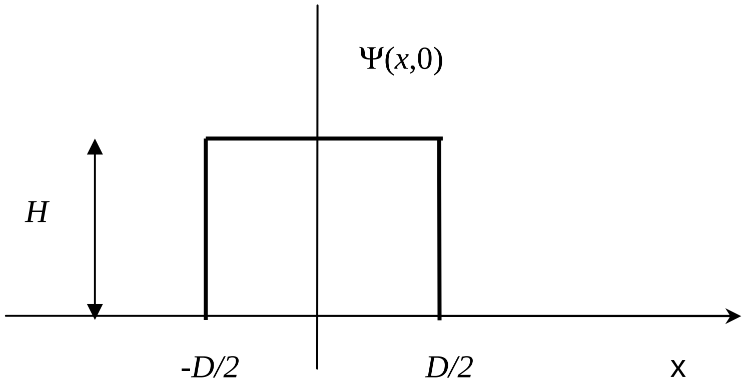

Fig.1: A wave packet at t = 0

This can be written as: $\Psi(x,0) = H$ for $-\frac{D}{2} \leq x \leq \frac{D}{2}$ and $\Psi(x,0) = 0$ elsewhere

a) By normalising the wave function, $\Psi(x,0)$, prove that $H = \frac{1}{\sqrt{D}}$.

Q5. b) Prove that the uncertainty in position, Δx , is given by: $\Delta x = \dfrac{D}{2\sqrt{3}}$.

Q6. Given the normalised ground state wavefunction of a quantum mechanical harmonic oscillator:

$$\Psi(x,t) = \left(\frac{a}{\pi}\right)^{\frac{1}{4}} e^{-\frac{ax^2}{2}} e^{-\frac{1}{2}i\omega_0 t} \text{ where } a = \frac{m\omega_0}{\hbar}.$$

Prove that the momentum uncertainty, Δp, is given by: $\Delta p = \sqrt{\frac{\hbar^2 a}{2}}$.