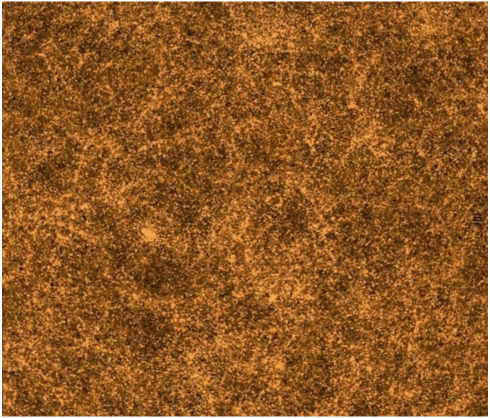
Conserved Quantities in Lemaître-Tolman-Bondi Cosmology



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Following work in arXiv:1403.7661 by AL and Karim A. Malik

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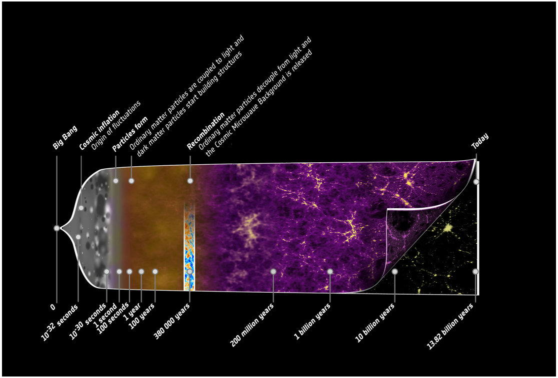
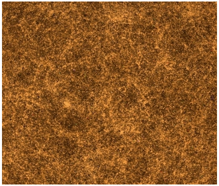
Figure: Universe timeline - ESA



Overview



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Why Perturb LTB Cosmology?

Why Perturb LTB Cosmology?

- Locally Rotationally Symmetric Cosmologies appealing, explaining Dark Energy interpreted observations (SN1a observations) with inhomogeneous expansion - no DE
- Of the many possible LRS cosmologies, LTB type models still lie within the constraints provided by observations
- Ongoing work interpreting observations (galaxy surveys, large scale structure surveys, CMB) and other redshift dependent observations in context of LTB background
- Ongoing numerical studies perturbing LTB: analytically derived results useful for comparison
- Structure formation within a region or scale over which LTB assumed valid requires understanding of how perturbations within region evolve

The Standard Model of Cosmology - Flat FLRW

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- Perturbed metric:

$$ds^2 = -(1 + 2\Phi)dt^2 + 2aB_{,i}dx^i dt + a^2(\delta_{ij} + 2C_{ij})dx^i dx^j, \quad (1)$$

- Further decomposition, revealing curvature perturbation ψ:

$$C_{ij} = E_{,ij} - \psi\delta_{ij}, \quad (2)$$

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- Gauge Transformations:

$$\widetilde{\psi_{\rm FRW}} = \psi_{\rm FRW} + \frac{\dot{a}}{a}\delta t\,, \quad (3)$$

$$\widetilde{\delta\rho_{\rm FRW}} = \delta\rho_{\rm FRW} + \dot{\bar{\rho}}\delta t\,, \quad (4)$$

The Standard Model of Cosmology - Flat FLRW

The Standard Model of Cosmology - Flat FLRW

- Gauge choice, $\widetilde{\delta\rho_{\rm FRW}} = 0$,

$$\delta t = -\frac{\delta\rho_{\rm FRW}}{\dot{\bar{\rho}}}\,, \tag{5}$$

- Gauge invariant curvature perturbation, ζ,

$$-\zeta \equiv \psi_{\rm FRW} + \frac{\dot{a}/a}{\dot{\rho}}\delta\rho\,, \tag{6}$$

- Evolution equations for ζ by taking d.w.t.t, for $\delta\rho$ from energy conservation $\nabla_\mu T^{\mu\nu} = 0$...

Conserved Quantities in Perturbed LTB

Conserved Quantities in Perturbed LTB

- The background metric:

$$ds^2 = -dt^2 + X^2(r,t)dr^2 + Y^2(r,t)\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{7}$$

- Metric Perturbations:

$$\delta g_{\mu\nu} = \begin{pmatrix} -2\Phi & XB_r & YB_\theta & Y\sin\theta B_\phi \\ XB_r & 2X^2C_{rr} & XYC_{r\theta} & XY\sin\theta C_{r\phi} \\ YB_\theta & XYC_{r\theta} & 2Y^2C_{\theta\theta} & Y^2\sin\theta C_{\theta\phi} \\ Y\sin\theta B_\phi & XY\sin\theta C_{r\phi} & Y^2\sin\theta C_{\theta\phi} & 2Y^2\sin^2\theta C_{\phi\phi} \end{pmatrix}. \tag{8}$$

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- Inhomogeneous spacetime, with 1+3 decomposition does not allow further decomposition (S,V,T), but...
- gives simpler expressions, easing constructing conserved quantities.

Conserved Quantities in Perturbed LTB

Conserved Quantities in Perturbed LTB

- Background Energy Conservation:

$$\dot{\rho} + \rho(H_X + 2H_Y) = 0\,. \tag{9}$$

- Perturbed Energy Conservation:

$$\begin{aligned} \delta\dot{\rho} \quad &+ \quad (\delta\rho + \delta P)\,(H_X + 2H_Y) + \bar{\rho}' v^r \\ &+ \quad \bar{\rho}\Big(\dot{C}_{rr} + \dot{C}_{\theta\theta} + \dot{C}_{\phi\phi} + \partial_r v^r + \partial_\theta v^\theta + \partial_\phi v^\phi \\ &+ \quad \left[\frac{X'}{X} + 2\frac{Y'}{Y}\right] v^r + \cot\theta v^\theta\Big) = 0, \end{aligned} \tag{10}$$

- So, for convenience we define a spatial metric perturbation as:

$$3\psi = \delta g^k_k = C_{rr} + C_{\theta\theta} + C_{\phi\phi}\,, \tag{11}$$

Conserved Quantities in Perturbed LTB

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- ψ transformation behaviour:

$$3\tilde{\psi} = 3\psi + \left[\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y}\right]\delta t + \left[\frac{X'}{X} + 2\frac{Y'}{Y}\right]\delta r + \partial_i \delta x^i + \delta\theta \cot\theta\,, \quad (12)$$

- Gauge choices; uniform density:

$$\delta t\Big|_{\delta\tilde{\rho}=0} = -\frac{1}{\dot{\bar{\rho}}}\left[\delta\rho + \bar{\rho}'\delta r\right]\,. \quad (13)$$

and comoving:

$$\delta x^i = \int v^i dt\,. \quad (14)$$

- Gives us gauge invariant **Spatial Metric Trace Perturbation (SMTP)** on comoving, uniform density hypersurfaces:

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- Gives us gauge invariant **Spatial Metric Trace Perturbation (SMTP)** on comoving, uniform density hypersurfaces:

$$\begin{aligned} -\zeta_{\mathbf{SMTP}} &= \psi + \frac{\delta\rho}{3\bar{\rho}} + \frac{1}{3}\Big\{ \left(\frac{X'}{X} + 2\frac{Y'}{Y} + \frac{\bar{\rho}'}{\bar{\rho}}\right)\int v^r dt \\ &+ \partial_r \int v^r dt + \partial_\theta \int v^\theta dt + \partial_\phi \int v^\phi dt \\ &+ \cot\theta \int v^\theta dt \Big\} \end{aligned} \tag{15}$$

Conserved Quantities in Perturbed LTB

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$$
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&+ \partial_r \int v^r \, dt + \partial_\theta \int v^\theta \, dt + \partial_\phi \int v^\phi \, dt \\
&+ \cot\theta \int v^\theta \, dt \Big\}
\end{aligned}
$$

- **SMTP** Evolution Equation:

$$
\dot{\zeta}_{\mathrm{SMTP}} = \frac{H_X + 2H_Y}{3\bar{\rho}} \delta P_{\mathrm{nad}} \,, \quad (16)
$$

- **which is valid on all scales.**

Conclusion and Further Research

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- Research already extended to Lemaitre spacetime, allowing background pressure, ($\dot{\zeta}_{\mathbf{SMTP}} = \frac{H_X + H_Y}{3(\bar{\rho}+\bar{P})}\,\delta P_{\mathrm{nad}}$ - see arXiv:1403.7661).
- Difference in behaviour for $\dot{\zeta}_{\mathbf{SMTP}}$ LTB vs $\dot{\zeta}$ FRW help compare structure formation.
- Wider potential use of **SMTP** in comparing inhomogeneous spacetimes generally with standard FRW model.