Semiconductors. Carrier mobility

Mobilty, the drift velocity per unit electric field, $\mu = |v_D|/E$, for electrons can be expressed as

$$\mu_e = \frac{e\tau_e}{m_e^*}$$ and for holes $$\mu_h = \frac{e\tau_h}{m_h^*}$$

We can see that it is explicitly related to the scattering time and therefore should depend on various scattering mechanisms. The most important scattering processes involve interactions of electrons (or holes) with lattice vibrations (or phonons) and with impurity atoms. Both processes are temperature dependent and electron-phonon scattering dominates at high T while impurity-related scattering become important at low T. The overall scattering time can therefore be expressed as:

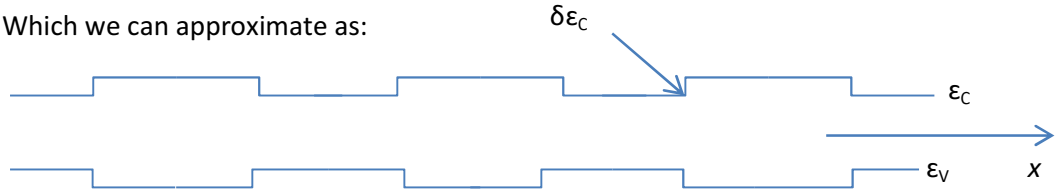
$$\frac{1}{\tau} = \frac{1}{\tau_{ph}} + \frac{1}{\tau_i}$$

Let's examine these processes in some details to understand the origins of the scattering mechanisms and their contribution to mobility and ultimately to conductivity. We shall first consider electron-phonon scattering in a semi-classical approximation.

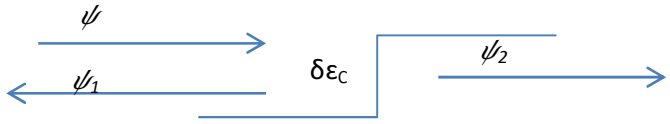
Electron-phonon scattering

We saw that in general longitudinal and transverse vibrations are possible in a crystal lattice. It is longitudinal vibrations that will be contributing to the scattering. We observe that such a vibration would result in alternating regions of compression and extension of the lattice. If we also recollect that the electronic structure of the lattice is defined by the atomic positions and interatomic interactions we conclude that such vibrations should lead to the altering electronic structure:

Which we can approximate as:



And since we can represent an electron in a conduction band by a free particle wave function the problem is reduced to a scattering from a potential step:



The incident wave is $\psi(x) = Ae^{ik_0x}$, reflected wave is then $\psi_1(x) = Be^{-ik_1x}$, and transmitted wave is $\psi_2(x) = Ce^{ik_2x}$. The energy of the incident and transmitted electron are then:

$$\varepsilon_0 = \frac{\hbar^2 k_0^2}{2m_e^*} \text{ and } \varepsilon_1 = \varepsilon_0 - \delta\varepsilon_C = \frac{\hbar^2 k_1^2}{2m_e^*} \text{ and then } -\delta\varepsilon_C = \frac{\hbar^2}{2m_e^*}(k_1^2 - k_0^2)$$

Since the wavefunction and its derivative must be continuous at the barrier (*x*=0), we may write:

$$\psi_0(0) + \psi_1(0) = \psi_2(0)$$

and

$$\psi_0'(0) + \psi_1'(0) = \psi_2'(0)$$

These conditions will result in:

$$\frac{B}{A} = \frac{k_0 - k_1}{k_0 + k_1} \text{ and } \frac{C}{A} = \frac{2k_0}{k_0 + k_1}$$

Resulting in reflection and transmission probabilities *R* and *T*:

$$R = \frac{\psi_1^*\psi_1}{\psi_0^*\psi_0} = \left(\frac{k_0 - k_1}{k_0 + k_1}\right)^2 \text{ and } T = 1 - R = \frac{4k_0k_1}{(k_0 + k_1)^2}$$

Now, if the step $\delta\varepsilon_C$ is small then $k_1 \cong k_0$ and hence $R \cong \left(\frac{m_e^*\delta\varepsilon_C}{2\hbar^2 k_0^2}\right)^2$ while $\delta\varepsilon_C$ can be related to the volume change as:

$$-\delta\varepsilon_C = const \times \frac{\delta V}{V_0}$$

Where *K* is a *deformation potential constant*. We need to link *R* and temperature. This can be done if we rewrite above as:

$$\delta\varepsilon_C^2 = const \times \left(\frac{\delta V}{V_0}\right)^2$$

and link volume and temperature. A longitudinal wave passing through crystal and resulting in the volume change should also exert local pressure δP. The local energy associated with this is then proportional to $\delta P \times \delta V$ and since the source of the energy is thermal the magnitude must be proportional to $k_B T$ or:

$$\delta\varepsilon_C \sim \delta P \times \delta V \sim k_B T$$

We are almost there except it would be good if we can exclude pressure explicitly by introducing the compressibility *K*:

$$K = \frac{1}{V_0}\frac{\delta V}{\delta P}$$

and expressing δP in terms of *K* to obtain:

$$\delta\varepsilon_C \sim \delta P \times \delta V = \frac{(\delta V)^2}{KV_0} \sim k_B T \text{ and hence}$$

$$\frac{(\delta V)^2}{V_0^2} \sim \frac{Kk_B T}{V_0} \text{ and } \delta\varepsilon_C^2 = const \times \frac{Kk_B T}{V_0}$$

We can now substitute this result into the expression for reflection *R* to obtain:

$$R \cong \left(\frac{m_e^* \delta\varepsilon_C}{2\hbar^2 k_0^2}\right)^2 = \frac{m_e^{*2}}{2\hbar^4 k_0^4} \times const \times \frac{Kk_B T}{V_0}$$

Now, the probability of a reflection (scattering) in a distance δx travelled by an electron is $\delta x/\lambda_e$ where λ_e is the electron mean free path. The volume V_0 over which we evaluate our disturbance is of the order of λ^3, where λ is the phonon wavelength. Then we can equate λ/λ_e (probability of scattering over the length λ) to *R* above to obtain:

$$\frac{\lambda}{\lambda_e} = R \rightarrow \lambda_e \cong \frac{2\lambda\hbar^4 k_0^4 V_0}{m_e^{*2} K k_B T} \text{ and since} V_0 = \lambda^3 \text{ and } k_0 = \frac{2\pi}{\lambda} \text{ we obtain } \lambda_e \cong \frac{2\times 16\pi^4\hbar^4}{m_e^{*2} K k_B T} = const \times T^{-1}$$

If we now recollect that $\tau_e = \lambda_e/\bar{v}$, where $\bar{v} \sim \sqrt{kT/m}$ we then obtain $\tau_e \cong const \times T^{-3/2}$ which will reflect the temperature-dependence of the electron-phonon scattering component.

A brief analysis of the scattering on charge impurities.

Assume that a carrier is scattered when its potential energy in the field of the scatterer is similar to its kinetic energy. The Coulombic potential at distance *r*

$$V \propto \frac{1}{r}$$

The kinetic energy is thermal so $\varepsilon \propto T$

Therefore we can define the effective radius of the scatterer as

$$r_s \propto \frac{1}{T}$$

Hence we get a scattering cross-section

$$\pi r_s^2 \propto T^{-2}$$

The rate at which the carrier encounters scatterers is proportional to the carrier thermal velocity

$$v_{\text{thermal}} \propto \sqrt{T}$$

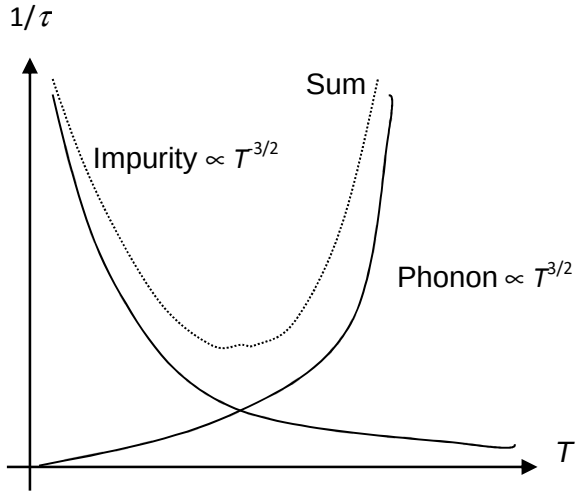
so overall $p_{\text{scatt}} \propto T^{-3/2}$ and the scattering time is a reverse of that hence $\tau_i \cong const \times T^{3/2}$

The overall effect is the sum of the two resulting in:

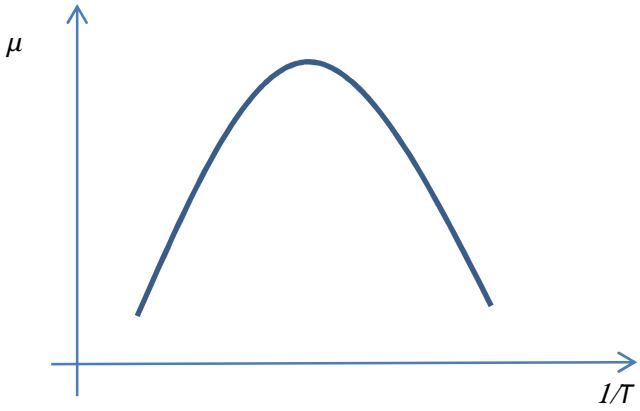
So considering charged impurity scattering and phonon scattering

$$\frac{1}{\tau} = \frac{1}{\tau_{\mathrm{Im}\,p}} + \frac{1}{\tau_{Phonon}}$$

(Remember that the probability of scattering is equal to $1/\tau$)

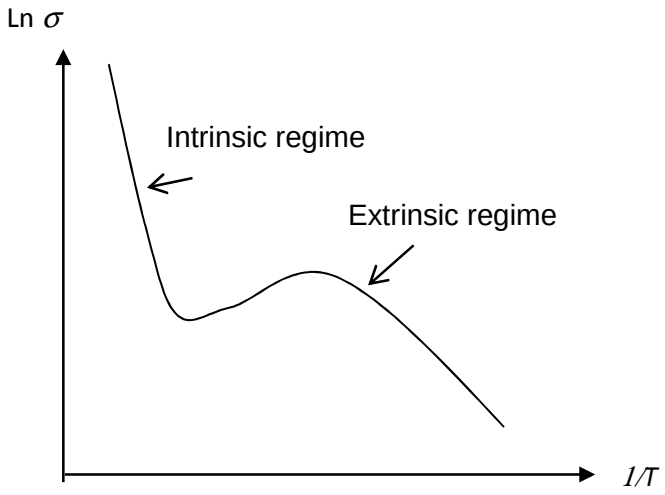


Or for μ



Mobility peaks at intermediate temperatures – typically 100-200 K

To find the conductivity we need to multiply by the number of carriers giving the result in the following graph


Ln σ
Intrinsic regime
Extrinsic regime
1/T