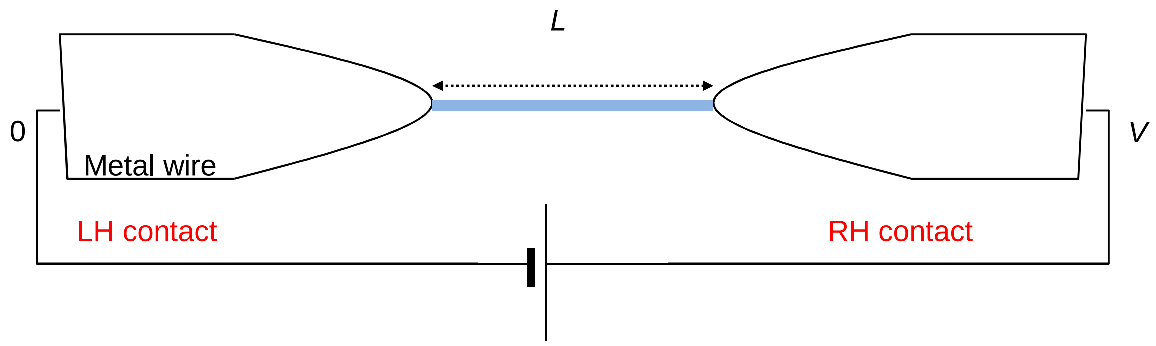
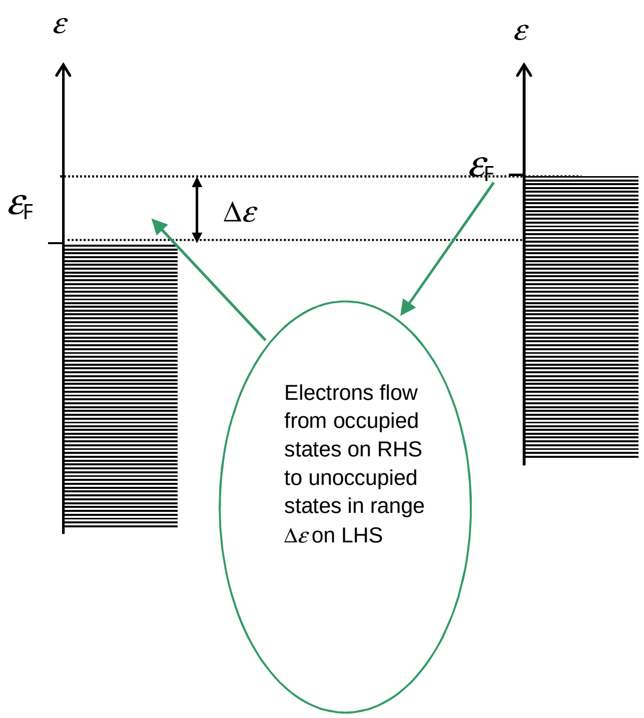
Transport in low-dimensional structures. Landauer formula.

We have so far considered charge transport in materials as a response to an applied electric field. We used essentially a semi-classical approach and included charge concentration and mobility derived from a quantum physics approach. As a system size approaches the limit when quantum effects related to a single charge carrier become important an alternative approach due to Landauer becomes more adequate. Within this approach the current flow viewed as a transmission process - a consequence of the injection of carriers at contacts and probability of the carriers to reach the other end. This approach has been especially useful in nanostructured materials and devices and molecular systems.

Let's consider a system composed of two macroscopic electrodes and of a quantum wire of length *L* and apply voltage *V* between the electrodes (see figure below).



The corresponding band structure diagram will look as follows:



We assume for the moment that the electrons are not subjected to any scattering mechanism (transport in the channel is ballistic) and that the electrons entering the contacts are instantaneously in equilibrium with them. Let the temperature be equal to zero (T=0 K) and the contacts *reflectionless*, i.e. the transmission probability from the contact to contact is equal to unity. The conductance ($G \equiv 1/R$) of the 1D wire is then $G = I/V$. The current through the 1D conductor is then:

$$I = e\int_0^\infty v(k)n(k)f_R(k)dk - e\int_0^\infty v(k)n(k)f_L(k)dk$$

Where $v(k)$ is currier velocity, $n(k)$ is the carrier concentration, $f_R(k)$ is the Fermi-Dirac function and *k* is the momentum. And we can see that in order to proceed we need information about the density of states $n(k)dk$, better still if we can express it in terms of energy ε. We recall from the earlier lectures that:

$$n(k)dk = 2\frac{1}{L}\frac{L}{2\pi}dk = \frac{1}{\pi}dk$$

and the electron's group velocity is

$$v = \frac{p}{m} = \frac{\hbar k}{m}$$

The expression for the current can now be written as:

$$I = e\frac{\hbar}{\pi m}\int_0^\infty k\, f_L(\varepsilon)\frac{dk}{d\varepsilon}d\varepsilon - e\frac{\hbar}{\pi m}\int_0^\infty k\, f_R(\varepsilon)\frac{dk}{d\varepsilon}d\varepsilon = e\frac{\hbar}{\pi m}\int_0^{\varepsilon_F^L} k\frac{dk}{d\varepsilon}d\varepsilon - e\frac{\hbar}{\pi m}\int_0^{\varepsilon_F^R} k\frac{dk}{d\varepsilon}d\varepsilon$$

Recalling that $dk = \frac{m}{\hbar^2 k}d\varepsilon$ and substituting into the above yields:

$$I = e\frac{\hbar}{\pi m}\int_{\varepsilon_F^R}^{\varepsilon_F^L} k\frac{m}{\hbar^2 k}d\varepsilon = e\frac{1}{\pi\hbar}\int_{\varepsilon_F^R}^{\varepsilon_F^L} 1\, d\varepsilon = \frac{e}{\pi\hbar}(\varepsilon_F^L - \varepsilon_F^R) = \frac{2e^2}{h}V$$

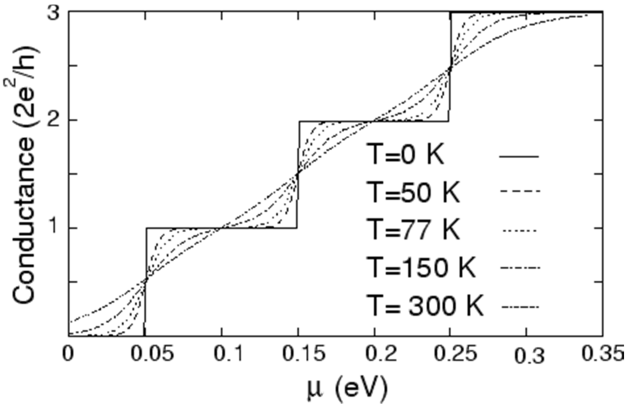
and thus

$$G = \frac{2e^2}{h} = 7.73 \times 10^{-5}\ \text{Ohms}^{-1}$$

and hence *quantum resistance*:

$$\frac{1}{G} = 12.9\ \text{kOhms}$$

The conductance at different temperatures, as a function of the electrochemical potential (μ, or ε_F) of the contact supposing a small voltage is applied between the two contacts. We can observe that as the temperature is increased, the step-like function of the conductance at *T* = 0 Kis smoothed.



All the above assumes reflectionless contacts. The conductance in case of reflecting contacts can be expressed as the *Landauer* formula:

$$G = \frac{2e^2}{h} T(\varepsilon_F)$$

where $T(\varepsilon_F)$ is the transmission probability.