Queen Mary University of London

BSc and MSci Examination

Friday 4th May 2012 14:30 - 17:00

PHY413 Quantum Mechanics B Duration: 2 hours 30 minutes

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Answer ALL questions from Section A and TWO questions from Section B

ONLY NON-PROGRAMMABLE CALCULATORS ARE PERMITTED IN THIS EXAMINATION. PLEASE STATE ON YOUR ANSWER BOOK THE NAME AND TYPE OF MACHINE USED.

COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK WHICH IS NOT TO BE ASSESSED.

IMPORTANT NOTE:
THE ACADEMIC REGULATIONS STATE THAT POSSESSION OF UNAUTHORISED MATERIAL AT ANY TIME WHEN A STUDENT IS UNDER EXAMINATION CONDITIONS IS AN ASSESSMENT OFFENCE AND CAN LEAD TO EXPULSION FROM QMUL.

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EXAM PAPERS CANNOT BE REMOVED FROM THE EXAM ROOM.

Examiners:
Dr. A. Bevan, Dr. T. Kreouzis

SECTION A. Answer all questions.

A1

Write down the time-dependent Schrödinger Equation (TDSE) for a particle of mass m in a 3D potential $V(\underline{r}, t)$. State under what conditions this reduces the the time-independent Schrödinger Equation (TISE), and write down the differential equations that result, and the relationship between the solutions of the TDSE and TISE.

[5]

A2

Define the parity operator $\widehat{P}$ and determine the allowed eigenvalues for an arbitrary wave function Ψ. Explain the significance of each of the possible the eigenvalues.

[5]

A3

(a) Explain the significance of the Born interpretation of the wave function.

(b) Write down the position and momentum operators $\widehat{x}$ and $\widehat{p}$.

(c) Determine the commutator of $\widehat{x}$ with $\widehat{p}$.

(d) Explain the significance of your result.

[5]

A4 Consider the wave function Ψ composed of two distinct eigenstates of the Hamiltonian where

$$\Psi(x, 0) = c_1\psi_1(x) + c_2\psi_2(x). \tag{1}$$

where ψ_n is a properly normalised energy eigenstate with energy eigenvalue E_n.

(a) What is the wave function at some later time t if the system is left undisturbed?

(b) What is the probability associated with obtaining the energy eigenvalue E_1 corresponding to state ψ_1 on measuring the ensemble state.

(c) Having made a measurement of the wave function, and obtained some energy eigenvalue E_n as a result, write down the corresponding wave function of that state at the instant following the measurement.

[5]

A5

Write down the orthonormality condition of a wave function Ψ composed of N independent eigenstates of the Hamiltonian operator.

[5]

A6

(a) Given that $\widehat{L} = \widehat{r} \times \widehat{p}$, determine the angular momentum operators $\widehat{L}_z$ and $\widehat{L}^2$ in terms of Cartesian position and momentum operators, and write down their respective eigenvalues for a spherically symmetric potential.

(b) Write down the commutators $[\widehat{L}_x, \widehat{L}_y]$, $[\widehat{L}_y, \widehat{L}_z]$, and $[\widehat{L}_z, \widehat{L}_x]$, as well as $[\widehat{L}_i, \widehat{L}^2]$, where $i = x, y, z$.

[10]

A7

Write down the generalised Heisenberg uncertainty principle for two quantum operators $\widehat{A}$ and $\widehat{B}$.

[2]

A8

(a) Write down Pauli spin matrices for a spin half particle (for example an electron), and the relationship between these matrices and the corresponding spin operators.

(b) State an experimental result that motivates the concept of intrinsic spin?

[6]

A9

(a) Sketch the wave functions $\psi(x)$ for the ground and first excited states of the infinite square well.

(b) Sketch the corresponding distributions of $|\psi(x)|^2$.

(c) Write down the parity operator eigen values of these states, as well as the corresponding energy eigenvalues.

[7]

SECTION B. Answer two of the three questions in this section.

B1

a) Consider a physical system described by a potential cuboid well of dimension $L_x \times L_y \times L_z$ where $V(x, y, z) = 0$ surrounded by a region of infinite potential. Write down the boundary conditions associated with this situation, and the time-independent Schrödinger equation.

[5]

b) Determine general form of the eigen functions and eigenvalues associated with the infinite well problem given in part **(a)**, and explain the significance of the eigenvalues.

[12]

c) Write down the eigen functions and eigen values of the excited states with $(n_x, n_y, n_z) = (2, 1, 1)$, $(1, 2, 1)$, and $(1, 1, 2)$. Explain what happens when the dimensions of the box are equal $L_x = L_y = L_z = L$ in the context of the eigenvalues.

[8]

B2

a) Write down the Hamiltonian operator for the rigid rotator model of a vibrating molecule, stating assumptions associated with the rotational and vibrational modes, and the consequences these have on the eigenstates and eigen values of the system.

[5]

b) Write down the energy eigenvalues of a rigidly rotating molecule, and indicate the relative magnitudes of the rotational and vibrational contributions. Write down the general form of the wave function describing such a molecule and describe how the form of these eigenvalues leads to selection rules for emission and absorption of radiation.

[10]

c) For a molecule with $l = 1$ write down the eigenstates corresponding to the allowed values of m, compute the corresponding probability density functions in terms of θ and ϕ, and sketch the eigenstates.

[10]

B3

a) Write down the normalised spin eigenstates $\chi_{\pm}$ of $\widehat{S}_z$ and show that these are orthonormal. **[5]**

b) Calculate the eigenstates of $\widehat{S}_x$ and $\widehat{S}_y$, and express your answer in terms of $\chi_{\pm}$. **[10]**

c) Neutral K mesons created in the decay $\phi(1020) \rightarrow K^0\overline{K}^0$ are produced in an entangled quantum state. The wave function for this state is given by

$$\Psi = \frac{1}{\sqrt{2}}(\psi_1\xi_2 - \xi_1\psi_2), \tag{2}$$

where ψ describes a K^0 meson and ξ describes the antiparticle state of a $\overline{K}^0$.

i) A measurement is made on the entangled system at some time after it has been prepared, and the result of detecting the first particle is consistent with a K^0. At the time of this measurement, what was the second particle, what was the probability of this outcome?

ii) What is the wave function before and after the measurement?

iii) The physical states observed to decay via weak interactions are K_L and K_S. These are an admixture of the K^0 and $\overline{K}^0$ as follows:

$$K_S = p\psi - q\xi, \qquad\qquad K_L = p\psi + q\psi. \tag{3}$$

$$\tag{4}$$

After passing a beam of K_L particles through a target, the K^0 and $\overline{K}^0$ ratio of the beam is changed via strong interactions with the target material. Assume that following wave function obtained from such an experiment is

$$\Psi_{after} = \frac{2p\psi}{\sqrt{2}}. \tag{5}$$

What does Ψ_{after} correspond to, and what do you conclude from your result.

[10]