PHY 319 2011 These ones [1]

A1) a) $$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

b) V(x) ONLY i.e. TIME INDEPENDENT

c) $$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

5

A2) a) $$P_{ab}(t) = \int_a^b |\Psi(x,t)|^2 dx = \int_a^b \Psi^*(x,t)\Psi(x,t)\,dx$$

b) $$\int_{-\infty}^{+\infty} \Psi^*(x,t)\Psi(x,t)\,dx = 1$$

5

A3) a) $$\langle q\rangle = \int_{-\infty}^{+\infty} \Psi^*(x,t)\hat{Q}\Psi(x,t)\,dx$$

b) $$\Delta q = \sqrt{\langle q^2\rangle - \langle q\rangle^2}$$

c) $$\hat{X} = x \qquad \hat{p} = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{X}^2 = x^2 \qquad \hat{p}^2 = -\hbar^2\frac{\partial^2}{\partial x^2}$$

10

d) $$\hat{Q}\Psi(x,t) = q\Psi(x,t)$$

[2]

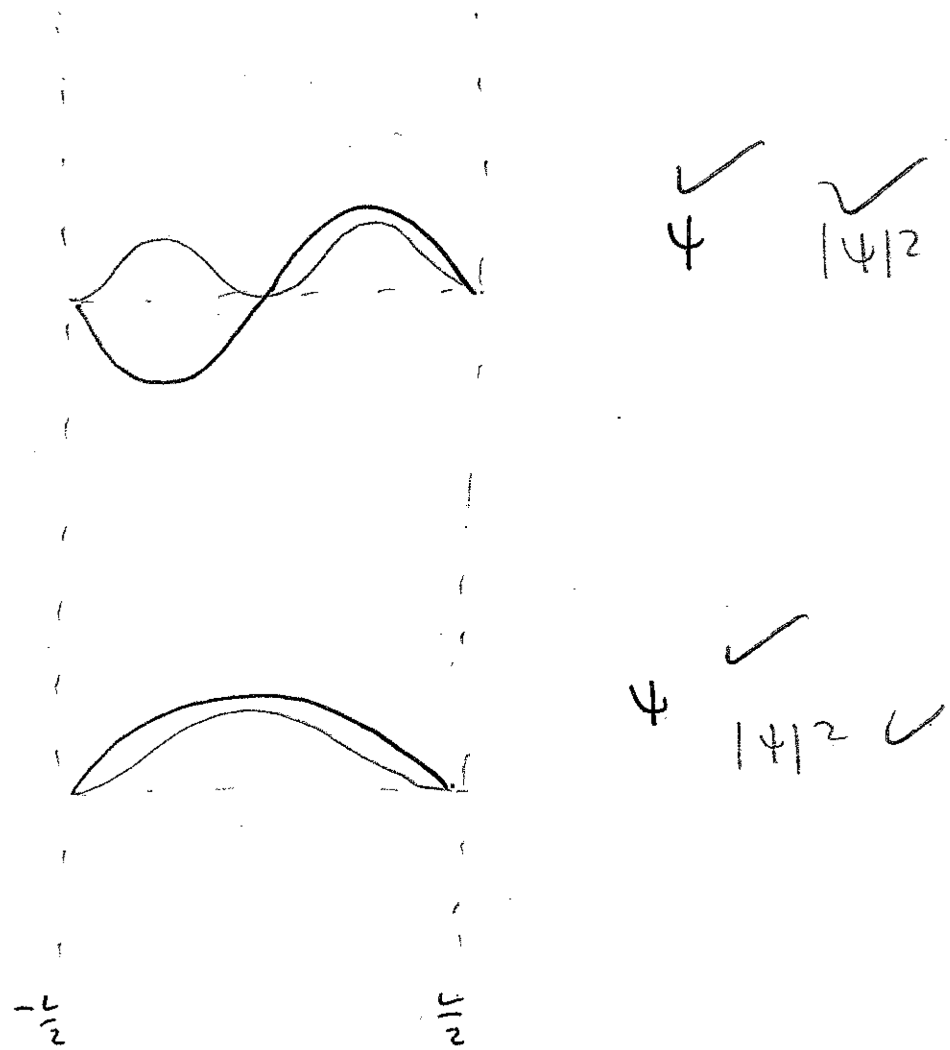
A4) a) IF $V(-x) = V(x)$

THEN WE HAVE DEFINITE PARITY EIGENSTATES

EITHER $\psi(-x) = \psi(x)$ EVEN PARITY

OR $\psi(-x) = -\psi(x)$ ODD PARITY

b)



$\overline{5}$

A5). IF $l = 2$

THEN $m_l = -2, -1, 0, 1, 2$

5

A6) $$\int_{-\infty}^{+\infty} \Psi^* \Psi \, dx = 1 \Rightarrow \int_{-a}^{a} N \cdot N \, dx = 1$$

$$N^2 \int_{-a}^{a} dx = 1$$

$$N^2 [x]_{-a}^{+a} = 1 \Rightarrow 2aN^2 = 1$$

$$\Rightarrow N = \frac{1}{\sqrt{2a}}$$

5

A7) $$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\langle x \rangle = \int_{-a}^{a} N x N \, dx = \frac{1}{2a} \int_{-a}^{a} x \, dx = 0$$

$$\langle x^2 \rangle = \int_{-a}^{a} N x^2 N \, dx = \frac{1}{2a} \int_{-a}^{a} x^2 \, dx = \frac{1}{2a} \left[\frac{x^3}{3} \right]_{-a}^{a}$$

$$\langle x^2 \rangle = \frac{1}{2a} \cdot \frac{2a^3}{3} = \frac{a^2}{3} \quad \therefore \Delta x = \sqrt{\frac{a^2}{3} - 0} = \frac{a}{\sqrt{3}}$$

5

A8) $$P_{-\Delta x \; +\Delta x} = \int_{-\Delta x}^{+\Delta x} |\Psi|^2 dx$$

$$= \int_{-\frac{a}{\sqrt{3}}}^{+\frac{a}{\sqrt{3}}} N^2 dx = \frac{1}{2a} \int_{-\frac{a}{\sqrt{3}}}^{\frac{a}{\sqrt{3}}} dx$$

$$P = \frac{1}{2a} \left[x \right]_{-\frac{a}{\sqrt{3}}}^{\frac{a}{\sqrt{3}}} = \frac{1}{2a} \cdot \frac{2a}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

5

A9) $\Psi(x,0)$ CONTAINS "SHARP CORNERS"

THAT IS $\frac{\partial \Psi}{\partial x}$ IS UNDEFINED AT $\pm a$

SO : PROBLEM SUBSTITUTING INTO TDSE

~~ALSO~~ ALSO $\hat{p} = -i\hbar \frac{\partial}{\partial x}$

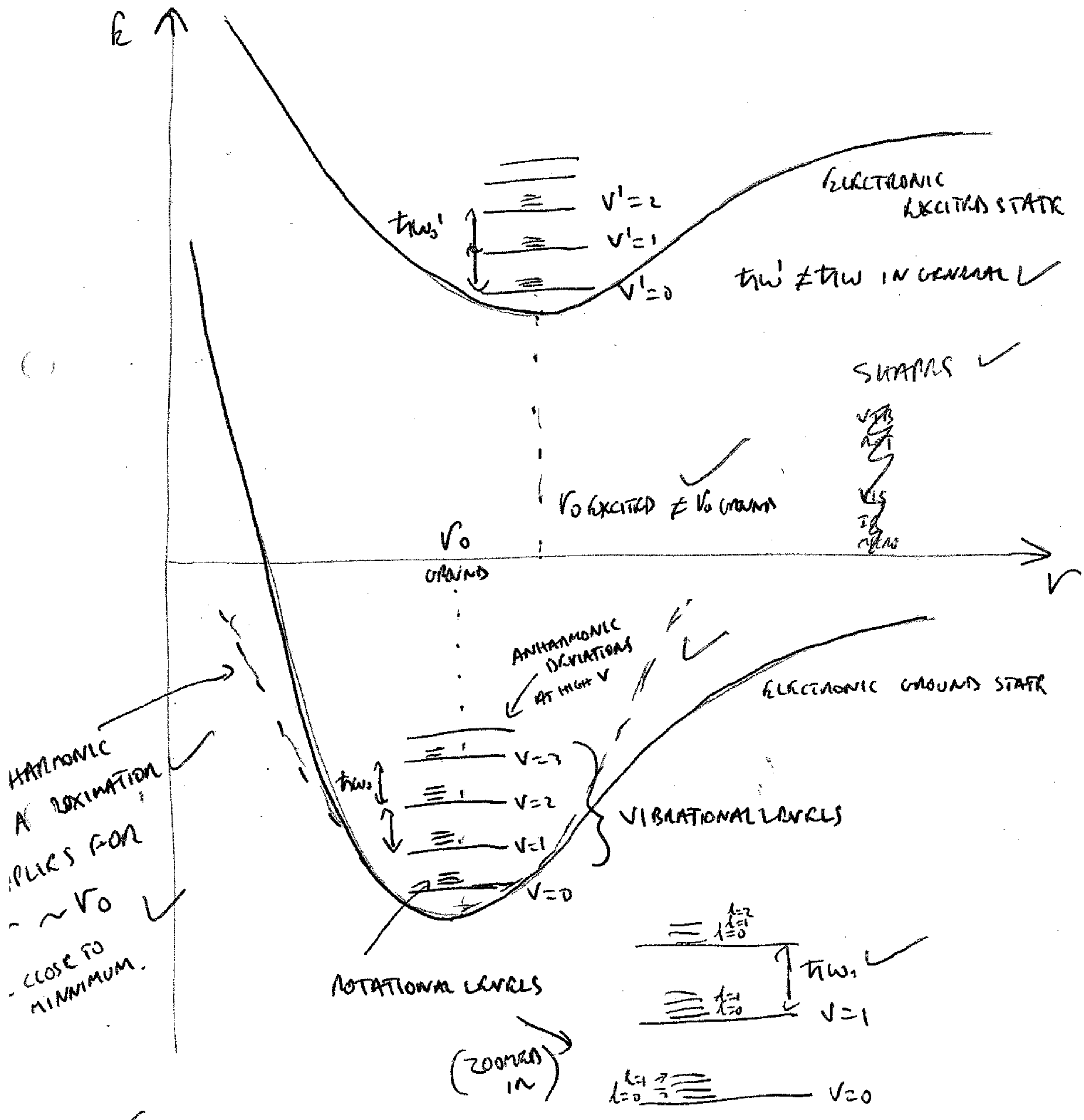
THERE IS A PROBLEM OPERATING USING $\hat{p}$

CAN'T CALCULATE $\langle p \rangle$, FOR EXAMPLE

5

ALSO, DOUBLE VALUED AT $\pm a$

B1.) a)



ELECTRONIC TRANSITIONS UV-VISIBLE $\sim eV$

VIBRATIONAL " IR $\sim \frac{1}{10} eV$

ROTATIONAL " MICROWAVE $\sim \frac{1}{100} eV$

MAX 10

B1) b) i) $E_0 = \frac{\hbar\omega_0}{2}$ $E_1 = \frac{3\hbar\omega_0}{2}$ 2

ii) $\Psi = \frac{1}{\sqrt{2}}\left\{\Psi_0 - \Psi_1\right\}$

$|\Psi|^2 = \Psi^* \Psi$

$\Psi_0(x,t) = \left(\frac{a}{\pi}\right)^{\frac{1}{4}} e^{\frac{-ax^2}{2}} e^{\frac{-iE_0t}{\hbar}}$

$\Psi_1(x,t) = \sqrt{2}\left(\frac{a^3}{\pi}\right)^{\frac{1}{4}} x e^{\frac{-ax^2}{2}} e^{\frac{-iE_1t}{\hbar}}$

$$|\Psi|^2 = \frac{1}{\sqrt{2}}\left(\Psi_0^* - \Psi_1^*\right) \cdot \frac{1}{\sqrt{2}}\left(\Psi_0 - \Psi_1\right)$$

$$= \frac{1}{2}\left\{\Psi_0^*\Psi_0 - \Psi_0^*\Psi_1 - \Psi_0\Psi_1^* + \Psi_1^*\Psi_1\right\}$$

$$= \frac{1}{2}\left\{|\Psi_0|^2 + |\Psi_1|^2 - \left(\Psi_0^*\Psi_1 + \Psi_1^*\Psi_0\right)\right\}$$

$$= \frac{1}{2}\left\{\sqrt{\frac{a}{\pi}}e^{-ax^2} + 2\sqrt{\frac{a^3}{\pi}}x^2e^{-ax^2} - 2\,\mathrm{Re}\,\Psi_1^*\Psi_0\right\}$$

$$= \frac{1}{2}\left\{\sqrt{\frac{a}{\pi}}e^{-ax^2} + 2\sqrt{\frac{a^3}{\pi}}x^2e^{-ax^2} - 2\,\mathrm{Re}\left[\sqrt{2}\left(\frac{a}{\pi}\right)^{1/4}\left(\frac{a^3}{\pi}\right)^{1/4} x e^{-ax^2} e^{\frac{it(E_1-E_0)}{\hbar}}\right]\right\}$$

$$= \frac{1}{2}\left\{\sqrt{\frac{a}{\pi}}e^{-ax^2} + 2\sqrt{\frac{a}{\pi}}\,a x^2 e^{-ax^2} - 2\sqrt{2}\left(\frac{a^4}{\pi^2}\right)^{1/4} x e^{-ax^2}\cos\left((E_1-E_0)\frac{t}{\hbar}\right)\right\}$$

$$= \frac{e^{-ax^2}}{2}\left\{ \sqrt{\frac{a}{\pi}} + 2a\sqrt{\frac{a}{\pi}}x^2 - 2\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a}{\pi}}\,x\cos\left((E_1-E_0)\frac{t}{\hbar}\right)\right\}$$

$$= e^{-ax^2}\sqrt{\frac{a}{\pi}}\left\{ \frac{1}{2} + ax^2 - \sqrt{2}\sqrt{a}\;x\cos\left((E_1-E_0)\frac{t}{\hbar}\right)\right\}$$

$E_1 - E_0 = \hbar\omega_0$

$$= \sqrt{\frac{a}{\pi}}\,e^{-ax^2}\left\{ \frac{1}{2} + ax^2 - \sqrt{2}\sqrt{a}\,x\cos\left(\frac{\hbar\omega_0 t}{\hbar}\right)\right\}$$

so

$$|\Psi(x,t)|^2 = \sqrt{\frac{a}{\pi}}\,e^{-ax^2}\left\{ \frac{1}{2} + ax^2 - \sqrt{2}\sqrt{a}\,x\cos(\omega_0 t)\right\}$$

7

iii) $$\langle x\rangle = \int_{-\infty}^{+\infty}\Psi^* x\,\Psi\,dx = \int_{-\infty}^{+\infty} x\,\Psi^*\Psi\,dx$$

$$\langle x\rangle = \int_{-\infty}^{+\infty} x\,|\Psi|^2\,dx = \int_{-\infty}^{+\infty} x\sqrt{\frac{a}{\pi}}\,e^{-ax^2}\left\{\frac{1}{2} + ax^2 - \sqrt{a}\,x\cos\omega_0 t\right\}dx$$

$$\langle x\rangle = \int_{-\infty}^{+\infty} \frac{x}{2}\sqrt{\frac{a}{\pi}}\,e^{-ax^2} + a\sqrt{\frac{a}{\pi}}\,x^3 e^{-ax^2} - \sqrt{a}\,x^2 e^{-ax^2}\sqrt{\frac{a}{\pi}}\cos\omega_0 t\;dx$$

B1) b) iii) CONTD

$$\langle x \rangle = \frac{1}{2}\sqrt{\frac{a}{\pi}}\int_{-\infty}^{+\infty} x e^{-ax^2}\,dx + a\sqrt{\frac{a}{\pi}}\int_{-\infty}^{+\infty} x^3 e^{-ax^2}\,dx - \sqrt{\frac{a}{\pi}}\sqrt{2a}\cos\omega_0 t\int_{-\infty}^{+\infty} x^2 e^{-ax^2}\,dx$$

$\shortparallel$ 0 $\qquad$ $\shortparallel$ 0

ODD FUNCTIONS,
INTEGRATED SYMMETRICALLY ABOUT 0

$$\langle x \rangle = \sqrt{\frac{a}{\pi}}(-\sqrt{2a})\cos\omega_0 t\int_{-\infty}^{+\infty} x^2 e^{-ax^2}\,dx = \sqrt{\frac{a}{\pi}}(-\sqrt{2a})\cos\omega_0 t \cdot \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

$$\langle x \rangle = \sqrt{\frac{2a^2}{\pi}} \cdot \sqrt{\frac{\pi}{a^3}} \cdot \frac{1}{2} \cdot -\cos\omega_0 t$$

$$\langle x \rangle = \frac{-1}{2\cancel{\sqrt{2a}}\sqrt{2a}}\cos\omega_0 t$$

$\therefore$ FREQUENCY ω_0 AND AMPLITUDE $\frac{1}{2\sqrt{a}}$ $\frac{1}{\sqrt{2a}}$

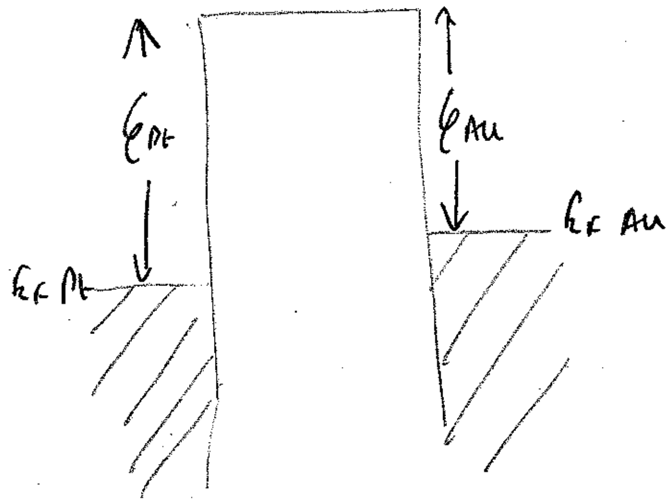
6

$\frac{TOT}{28}$

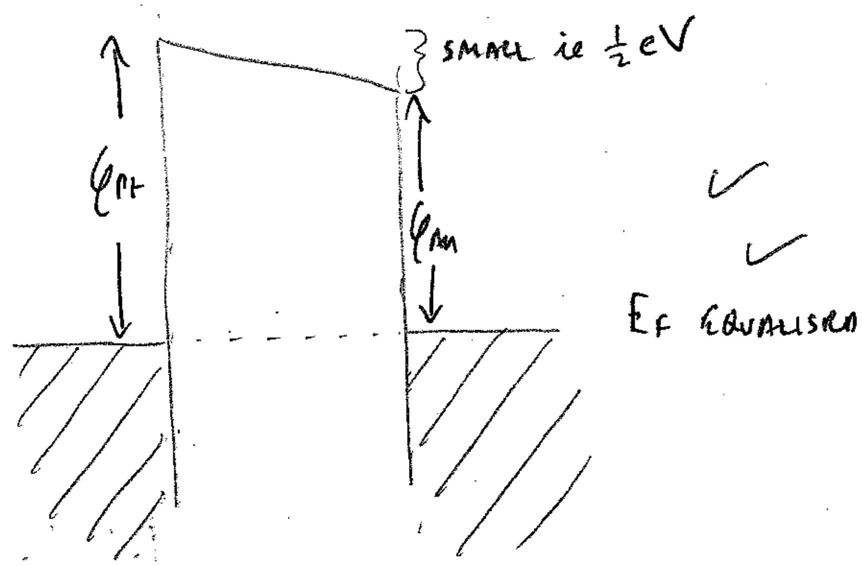
TIP - SUBSTRATE DISTANCE

B2) a)

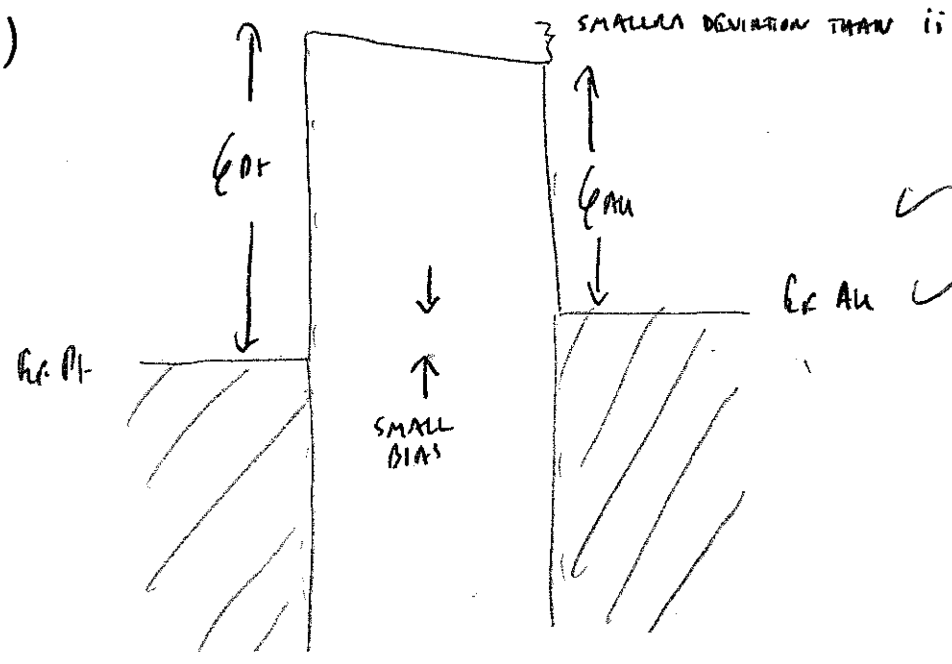
i)



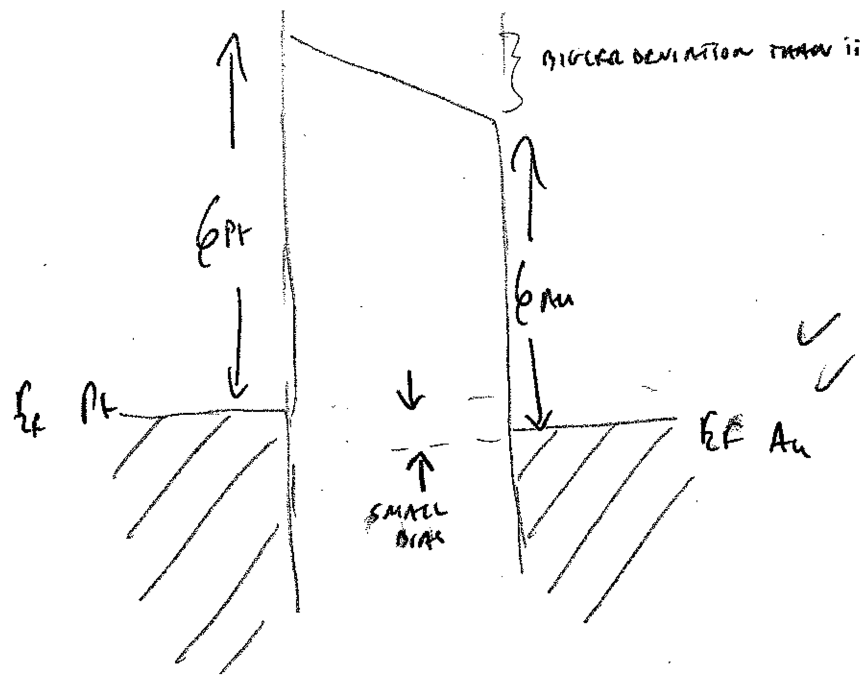
ii)



iii)



B2) a) iv)



8

b)

$$T = \frac{4\kappa^2 k^2}{(\kappa^2+k^2)^2 \sinh^2(\kappa L) + 4\kappa^2 k^2}$$

$$\kappa = \sqrt{\frac{2m(V_0-E)}{\hbar}}$$

$$k = \sqrt{\frac{2mE}{\hbar}}$$

USING $\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

$$T = \frac{4\kappa^2 k^2}{\frac{1}{4}(\kappa^2+k^2)\left(e^{\kappa L} - e^{-\kappa L}\right)^2 + 4\kappa^2 k^2}$$

FOR STRONGLY FORBIDDEN CASE $\kappa L \gg 1$

B2) b) CONTD.

NOW $$T = \frac{16\,\kappa^2 k^2}{(\kappa^2+k^2)^2(e^{\kappa L}-e^{-\kappa L})^2 + 16\,\kappa^2 k^2}$$

$$T = \frac{16\,\kappa^2 k^2}{(\kappa^2+k^2)^2(e^{2\kappa L}+e^{-2\kappa L}-2) + 16\,\kappa^2 k^2}$$

AND IF $\kappa L \gg 1$

THEN $$T \simeq \frac{16\,\kappa^2 k^2}{(\kappa^2+k^2)^2\, e^{2\kappa L}}$$

SINCE FIRST EXPONENTIAL WILL DOMINATE

SO $$T \simeq \frac{16\,\kappa^2 k^2\, e^{-2\kappa L}}{(\kappa^2+k^2)^2}$$

$$\kappa^2 = \frac{2m(V_0-E)}{\hbar^2}$$

& $$k^2 = \frac{2mE}{\hbar^2}$$

$$T \simeq \frac{16 \cdot 2m\left(\frac{V_0-E}{\hbar^2}\right) \cdot \frac{2mE}{\hbar^2}\, e^{-2\kappa L}}{\left(\frac{2m(V_0-E)}{\hbar^2} + \frac{2mE}{\hbar^2}\right)^2}$$

$$T \simeq \frac{16\,E(V_0-E)\, e^{-2\kappa L}}{(V_0-E+E)^2} \simeq \frac{16\,E(V_0-E)\, e^{-2\kappa L}}{V_0^2}$$

8

B2) c) $E < V_0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = \frac{2m(V_0-E)}{\hbar^2}\psi$$

WITH $K = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$

$$\psi(x) = Ae^{Kx} + Be^{-Kx}$$

$E > V_0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{2m(E-V_0)}{\hbar^2}\psi$$

$q = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

$$\psi(x) = Ae^{iqx} + Be^{-iqx}$$

SO, JUST SUBSTITUTE $K = iq$ WITH $q = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

B2) c) CONT'D

USING $$T = \frac{16\kappa^2 k^2}{(\kappa^2+k^2)^2(e^{\kappa L}-e^{-\kappa L})^2 + 16\kappa^2 k^2}$$ AND $\kappa = iq$

$$T = \frac{16\ i^2 q^2 k^2}{(k^2-q^2)^2(e^{iqL}-e^{-iqL})^2 - 16q^2k^2}$$

NOW.. $\frac{e^{i\theta}-e^{-i\theta}}{2i} = \sin\theta$

$$= \frac{-16q^2k^2}{(k^2-q^2)^2 \cdot 4i^2 \sin^2(qL) - 16q^2k^2}$$

$$= \frac{-16q^2k^2}{-4(k^2-q^2)^2\sin^2(qL) - 16q^2k^2}$$

$$T = \frac{4q^2k^2}{(k^2-q^2)^2\sin^2(qL) + 4q^2k^2}$$ (NEAT)

9

C.F. EXPRESSION GIVEN IN QUESTION

TOT
25

B3) a)

i) n : PRINCIPAL QUANTUM NUMBER, DETERMINES THE ENERGY IN ABSENCE OF EXTERNAL FIELDS, INTEGER

$n = 1, 2, 3 \ldots$

l : ORBITAL ANGULAR MOMENTUM QUANTUM NUMBER, DETERMINED BY n, INTEGER
(DETERMINES DEGENERACY OF ENERGY LEVELS)

$l = 0, 1, 2 \ldots (n-1)$

M_l : ORBITAL ANGULAR MOMENTUM MAGNETIC QUANTUM NUMBER, DETERMINED BY l, INTEGER,
(ALSO DETERMINES DEGENERACY)

$M_l = -l \ldots -1, 0, 1, \ldots +l$

M_s : SPIN ANGULAR MOMENTUM MAGNETIC QUANTUM NUMBER
SINCE ELECTRONS HAVE SPIN $= \frac{1}{2}$ $(s = \frac{1}{2})$
ONLY 2 VALUES ALLOWED

$M_s = \pm \frac{1}{2}$

B3) a) ii)

$$|\underline{L}|^2 = \hbar^2 l(l+1)$$ ✓

$$L_z = \hbar m_l$$ ✓

10

b) COULOMBIC, THEREFORE RADIAL

$$q_p = +e \quad , \quad q_e = -e$$

$$\therefore \quad V(r) = \frac{-e^2}{4\pi\varepsilon_0 r}$$

c)

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}\right] + V(r)\psi = E\psi$$

SUBSTITUTE $\psi = RY$

$$\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial RY}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial RY}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 RY}{\partial\varphi^2}\right] \cancel{+VRY=E}$$

$$= ERY - VRY$$

B 3) c) CONTD

$$-\frac{\hbar^2}{2m}\left[\frac{Y}{r^2}\frac{d\left(r^2\frac{dR}{dr}\right)}{dr} + \frac{R}{r^2\sin\theta}\frac{\partial\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)}{\partial\theta} + \frac{R}{r^2\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right] = ERY - VRY$$

$\div Y$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{d\left(r^2\frac{dR}{dr}\right)}{dr} + \frac{R}{Yr^2\sin\theta}\frac{\partial\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)}{\partial\theta} + \frac{R}{Yr^2\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right] = ER - VR$$

$\times r^2$

$$-\frac{\hbar^2}{2m}\left[\frac{d\left(r^2\frac{dR}{dr}\right)}{dr} + \frac{R}{Y\sin\theta}\frac{\partial\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)}{\partial\theta} + \frac{R}{Y\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right] = r^2(ER - VR)$$

$\div R$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{R}\frac{d\left(r^2\frac{dR}{dr}\right)}{dr} + \frac{1}{Y\sin\theta}\frac{\partial\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)}{\partial\theta} + \frac{1}{Y\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right] = \frac{r^2(ER - VR)}{R}$$

$$\frac{1}{R}\frac{d\left(r^2\frac{dR}{dr}\right)}{dr} + \frac{1}{Y\sin\theta}\frac{\partial\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)}{\partial\theta} + \frac{1}{Y\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2} = \frac{-2mr^2(ER - VR)}{\hbar^2 R}$$

$$\underbrace{-\left(\frac{1}{Y\sin\theta}\frac{\partial\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)}{\partial\theta} + \frac{1}{Y\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right)}_{\text{FUNCTIONS OF } \theta,\phi} = \underbrace{\frac{1}{R}\frac{d\left(r^2\frac{dR}{dr}\right)}{dr} + \frac{2mr^2}{\hbar^2}(E - V)}_{\text{FUNCTIONS OF } r}$$

$= \text{CONSTANT}$ SAY $= l(l+1)$

B3 b)

$$Y_l^{m_l} = -\frac{1}{2}\sqrt{\frac{3}{2\pi}} \sin\theta \, e^{i\phi}$$

linear in $\sin\theta$ (or $\cos\theta$) $\therefore l = 1$ ✓

$Y = \Theta\Phi$ ✓ & $\Phi = e^{im_l\phi}$

$\therefore m_l = 1$ ✓

MAX 3

c) $\hat{L}_z = -i\hbar \frac{\partial}{\partial\phi}$

$\hat{L}_z\psi = \hbar m_l \psi$ ✓ $\quad \psi = R\Theta\Phi$ ✓

or $Y = \Theta e^{i\phi}$

$$\hat{L}_z\psi = -i\hbar \frac{\partial R\Theta e^{i\phi}}{\partial\phi}$$

$$= -i\hbar R\Theta \frac{\partial e^{i\phi}}{\partial\phi} \checkmark$$

$$= -i\hbar R\Theta \cdot i\Phi \checkmark$$

$$\hat{L}_z\psi = 1\hbar R\Theta\Phi = m_l\hbar\psi \therefore m_l = 1 \checkmark$$

B3) c) CONTD

$$\hat{L}^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial \sin\theta \frac{\partial}{\partial\theta}}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)$$

$$\hat{L}^2\psi = \hbar\, l(l+1)\psi \checkmark$$

$$\hat{L}^2 R\Theta\Phi = \hbar\, l(l+1) R\Theta\Phi$$

$$= -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial \sin\theta \frac{\partial R\Theta\Phi}{\partial\theta}}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 R\Theta\Phi}{\partial\varphi^2}\right) \checkmark$$

$$= -\hbar^2\left(\frac{R\Phi}{\sin\theta}\frac{\partial \sin\theta \frac{\partial \sin\theta}{\partial\theta}}{\partial\theta} + \frac{R\Theta}{\sin^2\theta}\frac{\partial^2 e^{i\varphi}}{\partial\varphi^2}\right) \checkmark$$

$$= -\hbar^2\left(\frac{R\Phi}{\sin\theta}\frac{\partial \sin\theta\cos\theta}{\partial\theta} + \frac{R\Theta}{\sin^2\theta}\, i^2\,\Phi\right)$$

$\checkmark$

$$= -\hbar^2\left(\frac{R\Phi}{\sin\theta}(\cos^2\theta - \sin^2\theta) - \frac{R\Theta}{\sin^2\theta}\Phi\right)$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$\checkmark$

$$= -\hbar^2\left(\frac{R\Phi}{\sin\theta}(1 - 2\sin^2\theta) - \frac{R\Theta\Phi}{\sin^2\theta}\right.$$

$$= -\hbar^2\left(\frac{R\Phi\cancel{\Theta}}{\sin\theta}(1 - 2\sin^2\theta) - \frac{R\Theta\Phi}{\sin^2\theta}\right)$$

18

$$= -\hbar^2 \left(\frac{R\Theta\cancel{\Phi}}{\cancel{\sin^2\theta}} - 2\frac{R\Theta\Phi \sin^2\theta}{\sin^2\theta} - \frac{R\Theta\cancel{\Phi}}{\cancel{\sin^2\theta}} \right)$$

✓

$$= 2\hbar^2 R\Theta\Phi$$

$$\hat{L}^2 \Psi = 2\hbar^2 \Psi \qquad \therefore 2 = l(l+1)$$

$$\therefore l = 1$$

✓

$$l^2 + l - 2 = 0$$

$$(l+2)(l-1) = 0$$

$$l = 1, \quad l = -2$$

THIS ONE SINCE l POSITIVE INTEGER.

$\overline{12}$

TOT

$\overline{25}$

B4) a)

REGION (1) $\quad V = 0$

$$\therefore \quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + 0 = E\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi \qquad \text{let } k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

INCIDENT $\qquad$ REFLECTED

REGION (2) $\quad V = -V_0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{2m(E+V_0)}{\hbar^2}\psi \qquad \text{let } q = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

$$\frac{d^2\psi}{dx^2} = -q^2\psi$$

$$\psi(x) = Ce^{iqx} + De^{-iqx}$$

TRANSMITTED $\qquad$ NO SOURCE AT $x \to +\infty$

$$\psi(x) = Ce^{iqx} \qquad \therefore D = 0$$

5

B4) b)

$$\Psi_{(x,t)} = \psi_{(x)} e^{-\frac{iEt}{\hbar}}$$

IN ①

$$\Psi_{\text{INCIDENT}} = A e^{ikx} e^{-\frac{iEt}{\hbar}} = A e^{i\left(kx - \frac{Et}{\hbar}\right)}$$

$$\Psi_{\text{REFLECTED}} = B e^{-ikx} e^{-\frac{iEt}{\hbar}} = B e^{i\left(-kx - \frac{Et}{\hbar}\right)}$$

IN ②

$$\Psi_{\text{TRANSMITTED}} = C e^{iqx} e^{-\frac{iEt}{\hbar}} = C e^{i\left(qx - \frac{Et}{\hbar}\right)}$$

$$j_{\text{INCIDENT}} = \frac{1}{m} \mathrm{Re}\left[A^* e^{-i\left(kx - \frac{Et}{\hbar}\right)} \cdot -i\hbar \frac{\partial}{\partial x} A e^{i\left(kx - \frac{Et}{\hbar}\right)} \right]$$

$$= \frac{1}{m} \mathrm{Re}\left[|A|^2 \cdot -i^2 k\hbar \cdot e^0 \right]$$

$$j_{\text{INCIDENT}} = |A|^2 \frac{\hbar k}{m}$$

SIMILARLY

$$j_{\text{REFLECTED}} = |B|^2 \left(-\frac{\hbar k}{m}\right)$$

$$j_{\text{TRANSMITTED}} = |C|^2 \frac{\hbar q}{m}$$

5

B4) c) $R = \left|\frac{j_R}{j_I}\right| = \left|\frac{B}{A}\right|^2$

MUST APPLY BOUNDARY CONDITIONS AT $x=0$

ψ CONTINUOUS $\Rightarrow$ $Ae^{ikx} + Be^{-ikx} = Ce^{iqx}$

$\Rightarrow$ $A + B = C$ ① AT $x=0$

$\frac{d\psi}{dx}$ CONTINUOUS $\Rightarrow$ $ikAe^{ikx} - ikBe^{ikx} = iqCe^{iqx}$

$\Rightarrow$ $ikA - ikB = iqC$ AT $x=0$

$kA - kB = qC$

or $\frac{k}{q}A - \frac{k}{q}B = C$ ②

$A - \frac{k}{q}A + B + \frac{k}{q}B = 0$ ① − ②

$qA - kA + qB + kB = 0$

$A(q-k) + B(q+k) = 0$

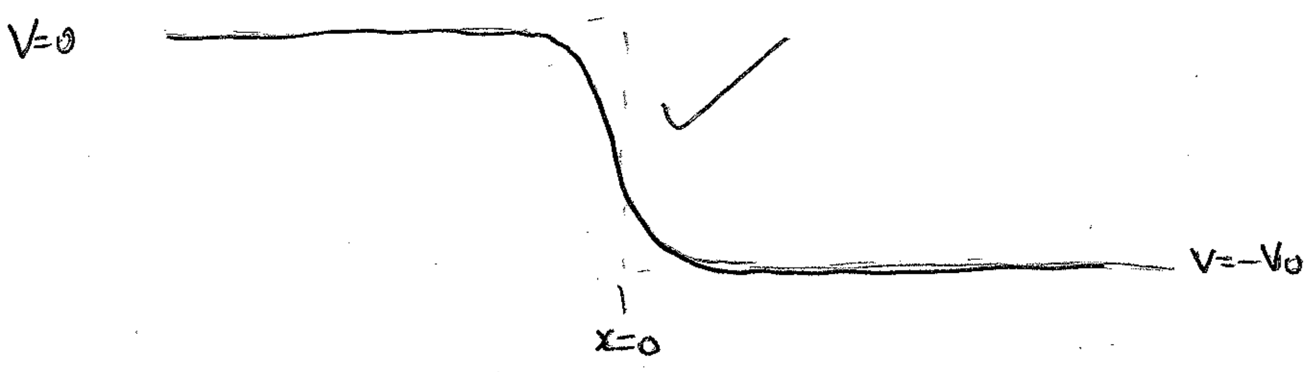
$B(q+k) = A(k-q)$

$\frac{B}{A} = \frac{k-q}{k+q}$ $\therefore R = \left(\frac{k-q}{k+q}\right)^2$

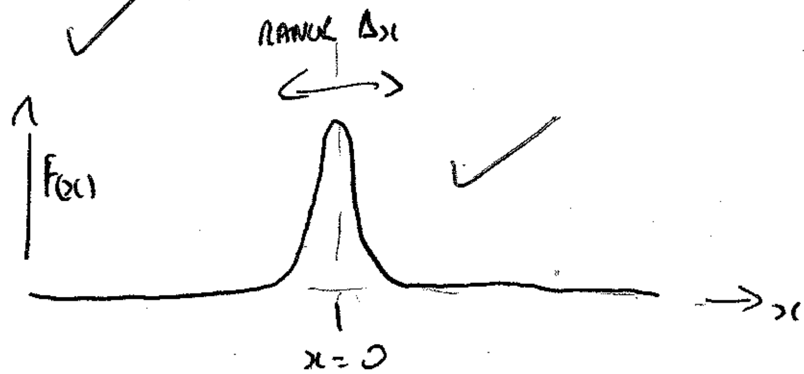
7

B4) d)

i)



$$F = -\frac{dV}{dx}$$



THE RANGE OVER WHICH THE FORCE ACTS, Δx, IS SMALL COMPARED TO THE WAVELENGTH $\lambda = \frac{\hbar}{k}$ OF THE MATTER WAVE.

4 max

B4) d) ii) SOLUTIONS WOULD BE

IN (2) $\psi(x) = A e^{-iqx} + B e^{iqx}$

INCIDENT REFLECTED

IN (1) $\psi(x) = C e^{-ikx}$

SO REFLECTION COEFFICIENT IS $\left|\frac{B}{A}\right|^2$

BOUNDARY CONDITIONS GIVE $A + B = C$

AND $-qA + qB = -kC$

OR $\frac{q}{k}A - \frac{q}{k}B = C$

SO $A - A\frac{q}{k} + B + B\frac{q}{k} = 0$

$$A(k-q) + B(k+q) = 0$$

$$\frac{B}{A} = \frac{q-k}{k+q}$$

$$\left|\frac{B}{A}\right|^2 = \left(\frac{q-k}{q+k}\right)^2 = \frac{|q-k|^2}{(q+k)^2} = \frac{|k-q|^2}{(k+q)^2} = \left(\frac{k-q}{k+q}\right)^2$$

SO, REFLECTION COEFFICIENT IS UNCHANGED

OR $k \to -k$, $q \to -q$ $\left(\frac{k-q}{k+q}\right)^2$ UNCHANGED

4

TOTAL 28