Nuclear Physics and Astrophysics

PHY-302

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Lecture 6 - Interaction Cross section

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Cross Section Definition

Rate at which interactions occur depends on two pieces:

- number of particles in your experiment - particle fluxes / target density
- intrinsic physics describing reaction between 2 particles = cross section

Think of cross section as proportional to the probability for a reaction to occur
It is quantified in units of area - effective area presented by target to beam
Nucleus has radius ~ 6 fm radius

area ~ 100 fm^2

but cross section for neutron capture can be up to 10^8 fm^2 !!!

A cross section is NOT a geometrical area!
It quantifies rate of a reaction independent of your experiment

Cross Section Definition

Consider two colliding beams

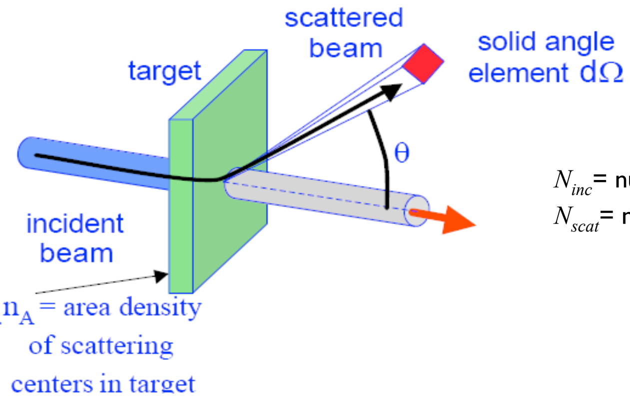
A = beam spot area
Flux of particles is Φ

$\Phi_1 = N_1/t$ and $\Phi_2 = N_2/t$
what is the interaction rate R_{int}?

interaction rate:

$$R_{int} \propto \frac{\Phi_1 \Phi_2}{A} \rightarrow \mathcal{L}$$
$$= \sigma \mathcal{L}$$

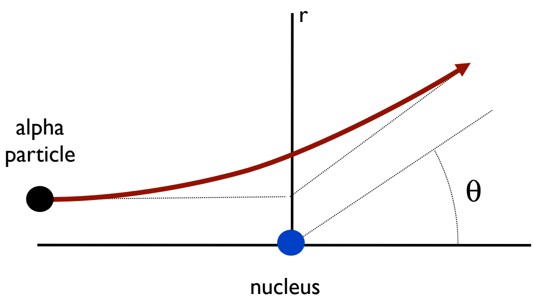
$\mathcal{L}$ = Luminosity [cm^{-2} s^{-1}]
no. of particles per unit area per unit time.
Depends only on design of your experiment
σ = constant of proportionality
depends on the fundamental physics only!



N_{inc}= number incident particles
N_{scat}= number scattered particles into solid angle $d\Omega$

$$N_{scat}(\theta) \propto N_{inc} \cdot n_A \cdot \mathrm{d}\Omega \rightarrow \mathcal{L}$$
$$= \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \cdot N_{inc} \cdot n_A \cdot \mathrm{d}\Omega$$

Scattering Cross Sections



Nuclear and particle physicists perfom scattering experiments to deduce internal structure
They measure the probability of a collision between two particles
This is quantified as a cross Section

Total Cross Section is the cross section for a collision with any possible outcome: a+X → ??

Partial Cross Section is the cross section for a particular outcome: a+X → b+Y

Differential Cross Section is the partial cross section per unit range of a given continuous variable (e.g. angle) at a specified value of that variable (e.g. at 10.5°) : a+X → b+Y(θ= 10.5°)

Measurement of Nuclear Lifetimes

Most direct way to measure lifetime / decay const:

Measure exponential decay vs time

$N(t) = N_0 e^{-\lambda t}$

But it is not possible to directly count N(t) so, measure activity instead

Activity (A) = $\lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$ i.e. activity decays exponentially

Plotting activity (A) vs time (t) on semi-log scale gives λ directly

$\ln(A) = \ln(A_0) - \lambda t$ straight line with slope λ

Easy to do for t ~ mins - hours

Need to choose sampling time carefully:
- too long - material will have decayed away
- too short - statistical uncertainties will be too large

For very short lifetimes need fast electronics to count decays in time interval

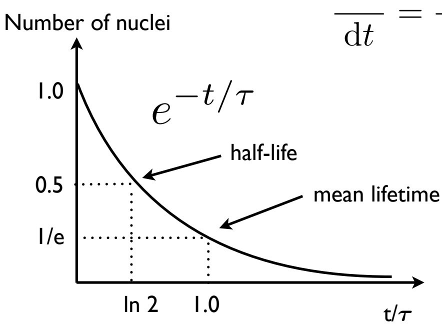
Worked Example of Decay Rate Calculation

For N nuclei present at time t, the number dN decaying in time dt is proportional to N

$$dN = -\lambda \cdot N \cdot dt$$

λ is the decay constant

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N \longrightarrow \boxed{N = N_0 e^{-\lambda \cdot t}}$$



$$\boxed{\tau = \frac{1}{\lambda}}$$

Mean lifetime is inverse of decay constant

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

Half life is time for half of the nuclei to decay

Multi-modal decays: decays in which one parent can decay via several channels simultaneously: $\lambda = \sum \lambda_i$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda_1 N - \lambda_2 N \longrightarrow N(t) = N_0 e^{-(\lambda_1 + \lambda_2)t}$$

Radioactive Decay Example

$$^{234}_{92}\mathrm{U} \;\rightarrow\; ^{230}_{90}\mathrm{Th}+\alpha \;\rightarrow\; ^{226}_{88}\mathrm{Ra}+\alpha$$

For chain of decay reactions: $N_1 \rightarrow N_2 \rightarrow N_3$

then $\mathrm{d}N_2 = \lambda_1 N_1 \mathrm{d}t - \lambda_2 N_2 \mathrm{d}t$

try solution: $N_2(t) = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t}$

with boundary conditions

with boundary conditions	$N_1(t=0) = N_0$	Initial sample is pure ^{234}U
	$N_2(t=0) = 0$	No ^{230}Th at time t=0
	$N_3(t=0) = 0$	No ^{226}Ra at time t=0

thus $N_2(t) = N_0 \dfrac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} + e^{-\lambda_2 t})$

hint: maximum production is reached when rate of change = 0

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Radioactive Decay Example

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2}$$

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad N_1 = N_0 e^{-\lambda_1 t}$$

$$\frac{dN_2}{dt} = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 N_2$$

trial solution: $N_2 = A e^{-\lambda_1 t} + B e^{-\lambda_2 t}$

differentiating: $\dfrac{dN_2}{dt} = -\lambda_1 A e^{-\lambda_1 t} - \lambda_2 B e^{-\lambda_2 t}$

but also: $\dfrac{dN_2}{dt} = \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_2 (A e^{-\lambda_1 t} + B e^{-\lambda_2 t})$

equating coeff's for $e^{-\lambda_1 t}$: $-\lambda_1 A = \lambda_1 N_0 - \lambda_2 A$

$$\therefore A = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}$$

for $t=0 \quad N_2=0$

$$0 = A + B \quad \Rightarrow \quad A = -B$$

$$\therefore \quad N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

If $\lambda_1 \gg \lambda_2 \quad \Longrightarrow \quad e^{-\lambda_1 t} \ll e^{-\lambda_2 t}$

$$\Longrightarrow \quad e^{-\lambda_1 t} \simeq 0 \quad \text{for all } t$$

i.e. N_1 decays very quickly compared to N_2
so in short time all N_1 has decayed to N_2

$$\therefore \quad N_2 \simeq N_0 \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t}$$