Nuclear Physics and Astrophysics

PHY-302

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Lecture 5 - Quantum Statistics & Kinematics

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Nuclear Reaction Types

Nuclear reactions are often written as:

$$a + X \rightarrow Y + b$$

for accelerated projectile a colliding with (usually stationary) target X

producing Y and b reaction products

Alternatively this is written as X(a,b)Y

Smaller nuclei / particles written inside brackets

For example: ^{235}U (n,γ)^{236}U

Types of Nuclear Reaction

- Scattering Processes - X and Y are the same (elastic if Y and b are in ground state)
- Knockout Reactions - if extra particles are removed from the incident nucleus
- Transfer Reactions - where 1 or 2 nucleons are exchanged between projectile and target

Nuclear Decay Kinematics

Energy level diagrams often used to illustrate radiative transitions between states

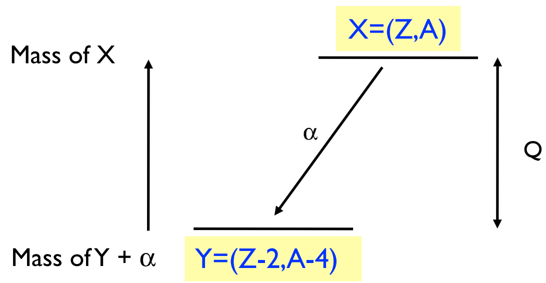
Much used in atomic physics

Make use of this in nuclear decay transitions

But, decays may occur to different nuclei

Example

- Parent nucleus X decays to daughter Y via emission of α−particle
- Daughter is written horizontally displaced
- Vertical height is energy difference in MeV known as the Q value
- Excited nuclear states (isomers) have larger mass (i.e. less binding energy)
- Written as * to indicate excited state



$$Q = [M(Z,A) - M(Z-2,A-4) - M(2,4)]\,c^2$$

The available kinetic energy goes to the α-particle and recoil of daughter

If Q>0 then α-decay is possible (but may be forbidden for other reasons)

Nuclear Decay Kinematics

In general Q-value for any reaction can be defined as:

$$Q = \left[\sum_i M_i(Z,A)c^2 - \sum_f M_f(Z,A)c^2 \right]$$

summing over all initial state particles i, and final state particles f

Q is net energy released (or required) in the reaction

If a particle is left in an excited state e.g. Y*, Q must include mass-energy of this state

$$Q' = [M_X - M_\alpha - M_{Y^*}]\,c^2 = Q - E$$

since $M_{Y^*}c^2 = M_Y c^2 + E$

Q-value may be positive, negative or zero

Q>0 exothermic
nuclear mass / binding energy is released as kinetic energy

Q<0 endothermic
initial kinetic energy is converted into nuclear mass or binding energy

Energy-Momentum Conservation

Conservation of total relativistic energy-momentum for our basic reaction gives:

a + X → Y + b

$$M_X c^2 + T_X + M_A c^2 + T_A = M_Y c^2 + T_Y + M_B c^2 + T_B$$

where T are kinetic energies
Reaction Q value is defined by analogy with radioactive decays:

$$\begin{aligned} Q &= (M_i - M_f)c^2 \\ &= T_f - T_i \end{aligned}$$

Conservation Laws

When analysing nuclear reactions we apply conservation laws:

Energy-Momentum:
Usually used to determine energy of excited states

Charge:
The total electric charge is conserved in reactions

Proton and Neutron Number
Applied in low energy processes where quark exchange takes place

Angular Momentum
Spin of initial particles and orbital angular momentum is conserved and used to deduce spins of final state nucleons

Parity
Spatial inversion symmetry also valid for the strong nuclear force

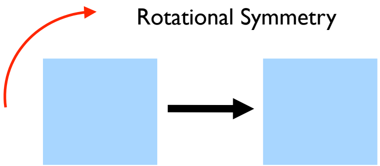
Time
Time reversal symmetry, most physics laws are indep of time running forwards or backwards

some of these are not exact, or cannot be applied universally

Symmetries

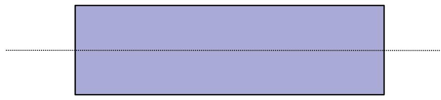
Many forces and phenomena in nature exhibit symmetries
An object has a symmetry if an operation/transformation leaves it invariant

Rotational Symmetry



Squares rotated 90° remain unchanged
transformation is $\theta \rightarrow \theta + 90°$

Reflection Symmetry = Parity



Rectangle reflected about axis is invariant
transformation is $x \rightarrow -x$

Mass symmetry

$$F = G\frac{m_1 m_2}{r^2} = G\frac{m_2 m_1}{r^2}$$

Newtons Law is symmetric about
transformations of m_1 and m_2

(Not really a mass symmetry but the associative property of multiplication)

Time symmetry

Reflections about the t=0 axis are invariant
transformation is $t \rightarrow -t$

Parity

Parity is simply a reflection about a given axis in space

In 3d we reflect object about all three Cartesian axes

$x \rightarrow -x$

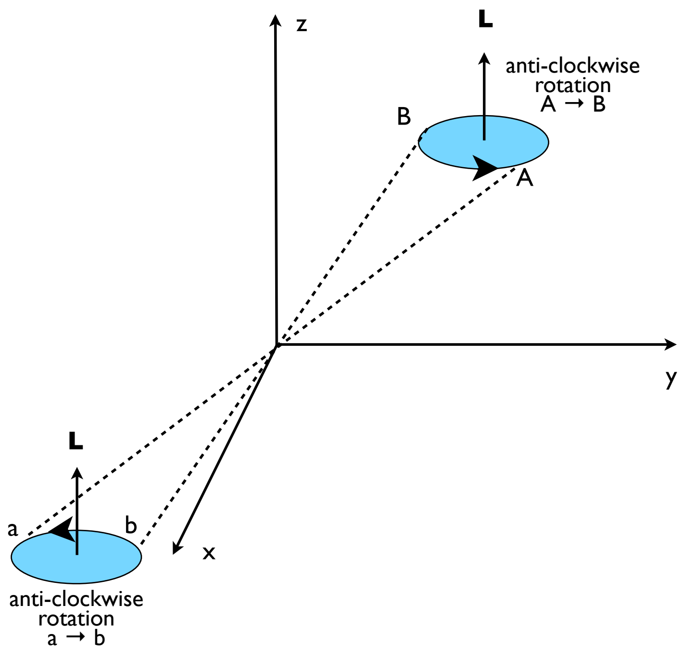
$y \rightarrow -y$

$z \rightarrow -z$

$A \rightarrow a$

$B \rightarrow b$

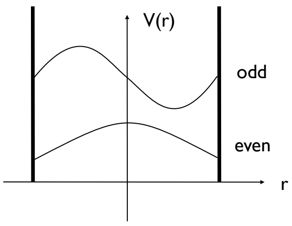
Some quantities do not flip sign:
L = spin or angular momentum
This remains the same after a parity inversion



Parity

In terms of wave-functions of a state in a parity invariant potential:

$$V(r) = V(-r) \qquad \text{then: } |\psi(r)|^2 \equiv |\psi(-r)|^2 \;\rightarrow\; \psi(-r) = \pm\psi(r)$$



If V(r) is unchanged then resulting stationary states must be even or odd parity

$$\psi(-r) = +\psi(r) \quad \text{even}$$
$$\psi(-r) = -\psi(r) \quad \text{odd}$$

$$V(r) \neq V(-r) \quad \text{then} \quad |\psi(r)|^2 \neq |\psi(-r)|^2$$

Quantum Statistics

Before continuing, we need to understand some quantum statistical effects for indistinguishable particles

Consider 2 electrons in Helium atom at positions r_1 and r_2 and in states Ψ_A and Ψ_B
Choose combined wave function to be:

$$\psi_{AB} = \psi_A(r_1) \cdot \psi_B(r_2)$$

Interchanging the two electrons we get:

$$\psi_{BA} = \psi_B(r_1) \cdot \psi_A(r_2)$$

But, if a measurement could detect the interchange then they are not indistinguishable

Therefore this 2-particle combined wave function is no good!

Quantum Statistics

For truly indistinguishable particles, probability densities <u>must</u> be invariant to exchange
i.e. ψ_{AB} can differ by sign only from ψ_{BA}

Wave-functions are symmetric if $\psi_{AB} = +\psi_{BA}$

Wave-functions are anti-symmetric if $\psi_{AB} = -\psi_{BA}$

Experiments show that particles with integer spin (0,1,2...) have symmetric wave-functions

For half integer spin particles ($^1/_2$, $^3/_2$, $^5/_2$...) wave-functions are anti-symmetric

Spin = intrinsic angular momentum of a particle
measured in units of ħ

Instead, for two particle systems take wave function of the form:

bosons : integer spin +
fermions: half integer spin −

$$\psi_{AB} = \frac{1}{\sqrt{2}}[\psi_A(r_1)\psi_B(r_2) \pm \psi_B(r_1)\psi_A(r_2)]$$

Pauli Exclusion Principle

A special case exists for identical quantum states A and B

Anti-symmetric combined wave function vanishes i.e. probability density = 0 everywhere

$$\psi_{AB} = \frac{1}{\sqrt{2}}[\psi_A(r_1)\psi_B(r_2) \pm \psi_B(r_1)\psi_A(r_2)]$$

$$\text{if} \quad \psi_A = \psi_B \quad \text{then} \quad \psi_{AB} \equiv 0$$

This is the Pauli Exclusion Principle:

<u>Two identical fermions cannot exist in the same quantum state</u>

This determines the filling of atomic electrons in shells

Also nucleons in nuclear 'orbitals'

Will play a crucial role in describing nucleon behaviour in nuclei

Nucleon-Nucleon Scattering

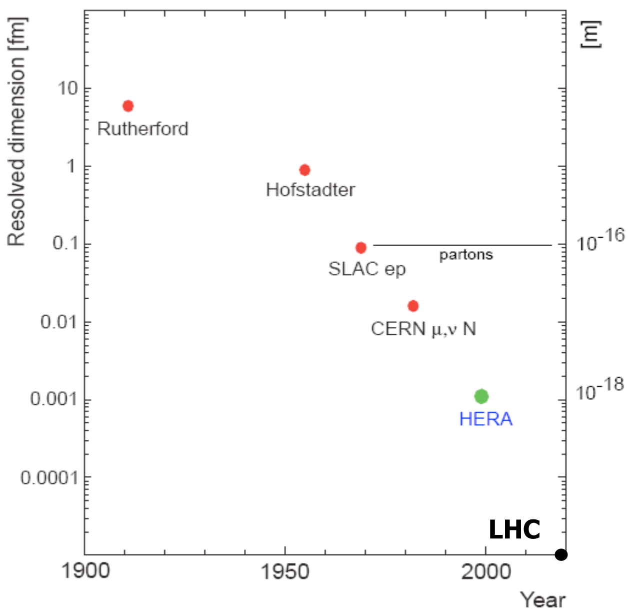
Back to the point.

In order to understand nuclear forces we perform nucleon-nucleon scattering experiments (just like Rutherford / Hofstadter)

Understanding nucleon-nucleon interactions is relevant to all phenomena covered in this course

Also essential for applied phenomena e.g. medical treatment & solar fusion

Such scattering experiments still performed today at much higher energies by particle physicists - e.g. looking at structure within a proton



Similar scattering experiments continue as particle physics experiments resolve structure deep within nucleons

quarks & gluons

resolved scale ~ $\lambda = hc/E$

$h \sim 4 \times 10^{-15}$ ev s

$c \sim 3 \times 10^{8}$ ms^{-1}

$E \sim 10^{13}$ eV (10 TeV)

$\lambda \sim 10^{-20}$ m !!!