Nuclear Physics and Astrophysics

PHY-302

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Lecture 3 - Radioactivity

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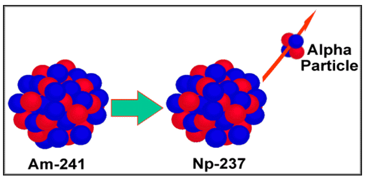
Radioactive Decay

Nuclear Decay

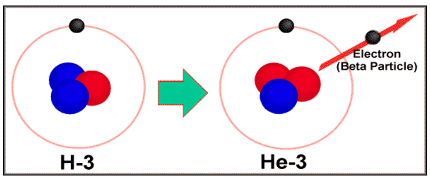
What is nuclear decay? Spontaneous transition from one state to another $X \Rightarrow Y + \ldots$

α decay: emission of ^{4}He nucleus



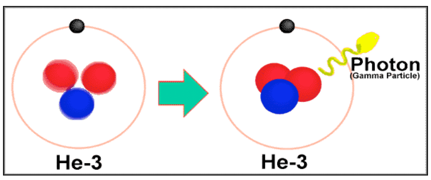
Large mass
double + charge
low speed =
highly ionising

β decay: emission of electron / positron



Converts between neutrons and protons
Ionising radiation

γ decay: emission of photon

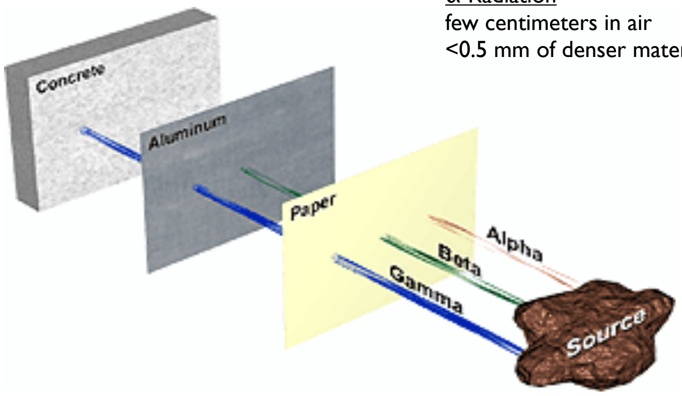


Can ionise atoms causing damage

We will return to these in much more detail in future lectures.
For now we concentrate on properties common to all types of decay

Radiation Stopping Distances

Each radiation type has its own degree of penetration in different materials



α Radiation
few centimeters in air
<0.5 mm of denser material e.g. water, skin, paper

γ Radiation
typically 4 m of water
~ 2m concrete
~ 0.5m lead

β Radiation
typically 3 m of air
few centimeters water
thin layer of glass / metal

Decays are statistical: cannot predict when any particular nucleus will decay

Process follows Poisson Statistics

For N nuclei present at time t, the number dN decaying in time dt is proportional to N

$$\mathrm{d}N = -\lambda N \mathrm{d}t$$

λ is the decay constant

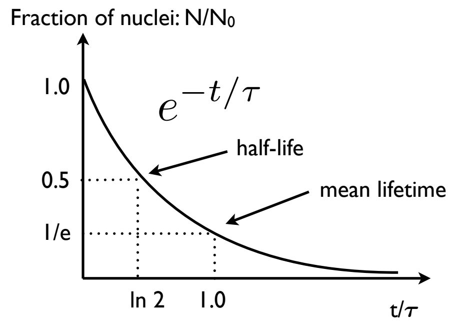
$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N \longrightarrow \boxed{N = N_0 e^{-\lambda \cdot t}}$$

$$\boxed{\tau = \frac{1}{\lambda}}$$

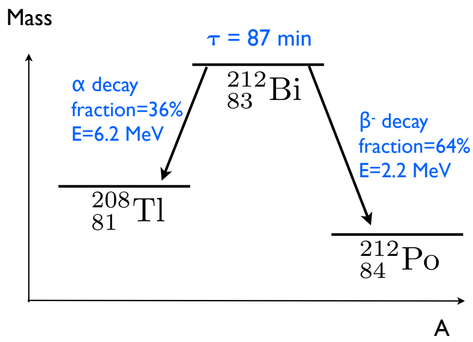
Mean lifetime is inverse of decay constant

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

Half life is time for half of the nuclei to decay



Multimodal Decays



Unstable nuclei can often decay via 2 or more modes e.g. α and β decay modes
Such nuclei have multimodal decays
Each decay mode is random and independent of the other modes
Each mode has it's own transition probability i.e. it's own λ or τ

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda_1 N - \lambda_2 N \quad \longrightarrow \quad N(t) = N_0 e^{-(\lambda_1+\lambda_2)t}$$

The total decay constant is the sum of partial decay constants

$$\lambda = \lambda_1 + \lambda_2 \qquad \text{or} \qquad \lambda = \sum \lambda_i$$

Radioactive Production and Sequential Decay

Radioactive material often produced by exposure to neutron flux
Neutron absorbed by nucleus - often followed by β decay e.g.

$$^{23}_{11}\mathrm{Na} + n \;\rightarrow\; ^{24}_{11}\mathrm{Na} \;\rightarrow\; ^{24}_{12}\mathrm{Mg} + e^- + \bar{\nu}_e$$

Rate of change of ^{24}Na nuclei

Differential equation for change in number of ^{24}Na nuclei:
where p is production rate of ^{23}Na (contributes with + sign)

$$\boxed{\frac{\mathrm{d}N}{\mathrm{d}t} = p - \lambda N}$$

Materials often have decay chains: parent produces daughter products which are also radioactive

For a decay chain $N_1 \rightarrow N_2 \rightarrow N_3$
production/decay rates are given by

$$\boxed{\frac{\mathrm{d}N_1}{\mathrm{d}t} = -\lambda_1 N_1}$$

$$\boxed{\frac{\mathrm{d}N_2}{\mathrm{d}t} = -\lambda_2 N_2 + \lambda_1 N_1}$$

Radioactive Decay

$$^{234}_{92}\mathrm{U} \rightarrow {}^{230}_{90}\mathrm{Th} + \alpha \rightarrow {}^{226}_{88}\mathrm{Ra} + \alpha$$

For chain of decay reactions: $N_1 \rightarrow N_2 \rightarrow N_3$

then $\mathrm{d}N_2 = \lambda_1 N_1 \mathrm{d}t - \lambda_2 N_2 \mathrm{d}t$

try solution: $N_2(t) = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t}$

$N_2(t) = N_0 \frac{\lambda_1}{\lambda_2 - \lambda_1}(e)$

with boundary conditions

$N_1(t=0) = N_0$	Initial sample is pure ^{234}U
$N_2(t=0) = 0$	No ^{230}Th at time t=0
$N_3(t=0) = 0$	No ^{226}Ra at time t=0

thus $N_2(t) = N_0 \dfrac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$

hint: maximum production is reached when rate of change = 0

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Radioactive Decay

- Decay Law is a probability relation - need to take into account possible statistical fluctuation in measurements
- Decay time is independent of real time at t=0, i.e. cannot tell age of sample

Activity A , number of decays per second
$$\begin{aligned} A &= -\frac{\mathrm{d}N}{\mathrm{d}t} \\ &= -\lambda N(t) \\ &= \lambda N_0 e^{-\lambda t} \end{aligned}$$

How radioactive is a sample?:

Becquerel (Bq) : modern SI unit
A radioactive sample with an activity of 1 Bq has an average of 1 decay per s.

Activity of a sample is affected by how much you have AND how old it is
more material = more activity older sample = less activity

Other units attempt to quantify the hazard posed by radioactive sources
e.g. Sieverts, Grays - will define these later in discussion of medical applications

Natural Radioactivity

Earth formed ~ 4.5 billion years ago

Material rich in Fe, C, O, Si

Heavier elements created in process of Nucleosynthesis from supernovae
will be covered in more detail later

Only few elements have half-lives similar to age of earth

Element	Nucleus	Half-life /years
Thorium	^{232}Th	1.41×10^{10}
Neptunium	^{237}Np	2.14×10^{6}
Uranium	^{238}U	4.47×10^{9}
Actinium	^{235}U	7.04×10^{8}

These are not the only sources of natural radioactivity
shorter half-life sources continuously formed on earth today
^{14}C is a useful example

Organic matter absorbs CO_2 from atmosphere

In air stable ^{12}C is most abundant (98.89%)

^{13}C is remaining 1.11%

Radioactive ^{14}C exists in 1 atom in 10^{12} of ^{12}C

Produced in upper atmosphere by cosmic rays:

$$^{14}N + n \rightarrow {}^{14}C + p$$

Half-life is 5730y

When organism dies it stops acquiring ^{14}C - no longer in equilibrium state

Remaining ^{14}C decays as usual

Age of samples determined by measuring specific activity (activity per gram)

$$\begin{aligned} \text{Activity } A &= -\frac{\mathrm{d}N}{\mathrm{d}t} \quad \text{(rate at which decays occur)} \\ &= -\lambda N(t) \\ &= \lambda N_0 e^{-\lambda t} \end{aligned}$$

Can also use mass spectrometers to determine ^{14}C ratios

Radio-carbon Dating

A sample of wood used in constructing an ancient shelter yields 2.1 ^{14}C decays per minute

A similar sample of wood cut recently from a tree yields 5.3 decays per minute

How old is the archeological sample?

$$N = N_0 e^{-\lambda t} \qquad\qquad t_{1/2} = \frac{\ln 2}{\lambda} = 5730 \text{ years}$$

consider the activity of sample when it died (t=0) and now (t=T)

$$\frac{\Delta N}{\Delta t}|_{t=0} = A_0 = \lambda N_0 \qquad \text{and} \qquad \frac{\Delta N}{\Delta t}|_{t=T} = A_T = \lambda N(T) = \lambda N_0 e^{-\lambda T}$$

organism comes out of
^{14}C equilibrium when it dies

$$e^{\lambda T} = \frac{A_0}{A_T} \longrightarrow T = \frac{1}{\lambda} \ln \frac{A_0}{A_T}$$

$$= \left(\frac{t_{1/2}}{\ln 2}\right) \ln \frac{A_0}{A_T}$$

$$= 7653 \text{ years}$$

Radio-carbon Dating

Technique assumes constant production of ^{14}C over 50,000 years

- tested by counting tree rings and historical records
- not used for samples more than ~ 50,000 years old; count rate too low
- can use other decay chains in similar manner - different half-life / assumptions

However, in last century fossil fuel burning increases stable carbons in air

Also atmospheric atomic bomb tests increase ^{14}C in air