

NPA Homework solutions 1

08/10/10

- 1) The neon isotope Ne_{11} has $Z=10$, $N=11$ and therefore $A=21$ [2]
- 2) From the lecture notes and the result of the Hofstadter experiment the nuclear mass density is measured to be $\sim 2 \times 10^{17} \text{ Kg m}^{-3}$. Assuming the neutron star is a giant nucleus, i.e. has the same density, then the mass of $1 \text{ cm}^3 = 0.01^3 \text{ m}^3 = 10^{-6} \text{ m}^3$ is simply $2 \times 10^{17} \times 10^{-6} = 2 \times 10^{11} \text{ Kg (!)}$ [2]

No marks awarded if you needed to look up the mass/size of a neutron star since you didn't read the question properly. Answers within a factor of ~ 4 are acceptable.

3) $17.999160 \text{ u} = 17.999160 \times 931.502 \text{ MeV}/c^2$
 $= 16766.25 \text{ MeV}/c^2$ [2]

The sum of nucleon masses = $8 \times 938.280 + 10 \times 939.573 = 16901.97 \text{ MeV}/c^2$ [2]

The difference of $135.72 \text{ MeV}/c^2$ is equivalent to about 15% of a neutron mass, or about 1% of the complete nuclear mass [0]

The mass is converted to binding energy to keep the nucleus together and is manifested in the field that keeps the nucleus bound (i.e. the strong nuclear force). [4]

a) Mass difference per nucleon = $135.72 \text{ MeV}/c^2 / 18 = 7.54 \text{ MeV}/c^2$ [1]

Thus mass difference for ^{235}U is $7.54 \times 235 = 1771.9 \text{ MeV}/c^2$ [1]

No marks if units are incorrect!

b) The mass difference can be expressed as an energy by multiplying by c^2
 $1771.9 \text{ MeV}/c^2 \times c^2 = 1771.9 \text{ MeV}$ [2]

c) 10% of this energy is 177.2 MeV

1 mole has Avogadro's number of nuclei, N_A

Total molar energy release = $N_A \times 177.2 \text{ MeV} = 1.1 \times 10^{26} \text{ MeV}$ [4]

d) In Joules: = $1.1 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{-6} \text{ J} = 1.8 \times 10^{13} \text{ J/mol}$ [2]

This compares to $1.3 \times 10^6 \text{ J/mol}$ from ethanol, i.e. 10^7 difference!

4)

a) potential $V(r)$ is the Coulomb potential

$$V(r) = \frac{z_1 z_2 e^2}{4\pi \epsilon_0 r} \quad z_1 = z_2 = 1$$

$$r = 1 \text{ fm}$$

from Krane $\frac{e^2}{4\pi \epsilon_0} = 1.44 \text{ MeV fm}$

$$\therefore V = \underline{\underline{1.44 \text{ MeV}}} \quad (2)$$

b) $z_1 = 79 \quad z_2 = 2 \quad r = 10 \text{ fm}$

$$\therefore V = 1.44 \times \frac{z_1 z_2}{10} = \frac{\cancel{114} \text{ MeV}}{\underline{\underline{22.8 \text{ MeV}}}} \quad (2)$$

5) Of course, there is no difference in decay rates, they are all the same! [2]

To convince yourselves you can compare the typical energy of nuclear processes ($\sim 1\text{-}10 \text{ MeV}$) to :

- the typical thermal kinetic energy of gases at std temp/pressure
- as above but for, lets say, factor 100 higher pressure
- as above but for, lets say, factor 100 higher temperature
- the gravitational potential energy between two nuclei with surfaces touching, or even the gravitational potential between a nucleus at Earth's surface and the Earth itself.

All of these energies are far below the $\sim \text{MeV}$ scale. We will see later that nuclear reactions can only occur when sufficient energy is supplied to cause a transition from from nuclear state to another, for example an excitation of a nucleon to a higher energy level (just like for atomic electrons). If insufficient energy is supplied then the transition is less likely to occur.

6) I forgot that the answer is actually given in the lecture notes! In general the time over which a radio-isotope can be used for dating is no more than about 10 half-lives since over this period the number of active nuclei will have reduced to 0.01% = $(\frac{1}{2})^{10}$ of their original number making detection difficult unless the sample is very large. Thus the answer is about 50,000 years or any number within ~ factor 2. [2]

⁴¹Ar is unsuitable for several reasons:

a) the half life is far too short! It is around 100 mins only. [2]

b) Ar is inert (noble gas) and so is not readily taken up by biological organisms. [2]