Nuclear Physics and Astrophysics

PHY-302

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Revision Lectures

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Nuclides

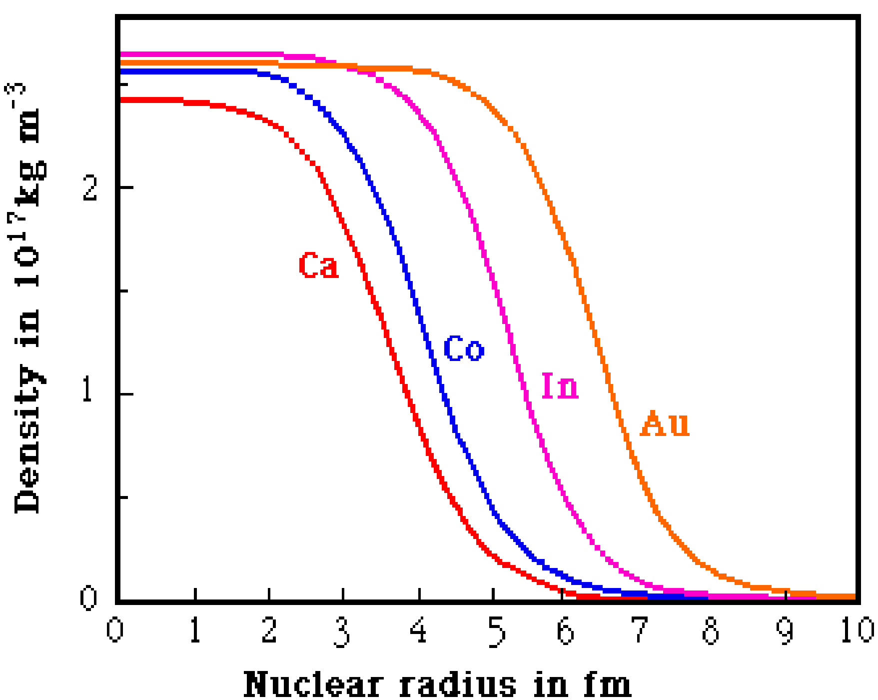
A Nuclide is a particular nucleus and is designated by the following notation:

$$^{A}_{Z}C_{N}$$

$$^{14}_{6}C_{8}$$ 14 nucleons, 6 protons, 8 neutrons

Z = Atomic Number (no. of protons)
A = Atomic Mass Number (no. of nucleons)
A = Z+N (nucleons = nrotons + neutrons)
N = Number of neutrons (sometimes omitted)

Electron - nucleus scattering measures nuclear charge density



constant density $\Rightarrow \dfrac{A}{^4/_3\pi R^3} \sim$ constant

$\Rightarrow$ $R = R_0 A^{1/3}$ and $R_0 \approx 1.2$ fm

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Units In Nuclear Physics

Mass – the atomic mass unit (u) or MeV/c²

Defined so one **atom** of ^{12}C = 12 u

Since $E=mc^2$ we can switch between mass & energy as we please

One mole of ^{12}C has N_A atoms = 6.022×10^{23} atoms

0.012 Kg = $N_A \times 12$ u $\rightarrow$ 1 u = $0.012/(N_A \times 12) = 1.66 \times 10^{-27}$ Kg

Using $E=mc^2$ then, Energy equivalent = $1.66 \times 10^{-27} \times (2.99 \times 10^8)^2$

$= 1.48 \times 10^{-10}$ J

Divide by electron charge to convert to eV = 931.502 MeV

Then 1 u = 931.502 MeV/c^2 or c^2 = 931.502 MeV/u

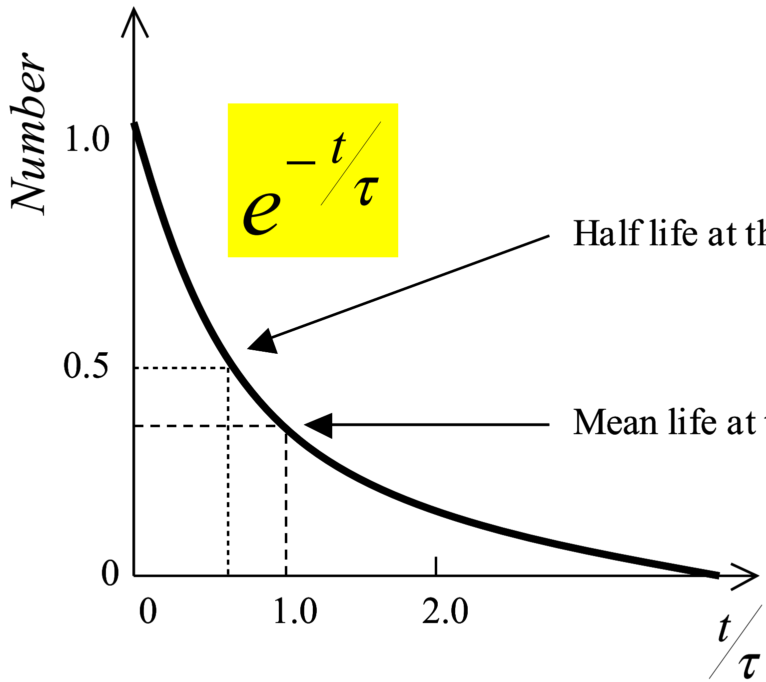
So, mass can be expressed as u, or in MeV/c^2

You should never have to multiply any numerical result by 2.99×10^8 m/s

If you do this, you are probably making an uneccessary step, or a mistake!!!

In Krane appendix C a full table of atomic masses is given.

Radioactive decay is a random statistical process
Individual decays cannot be predicted



$$dN = -\lambda \cdot N \cdot dt \qquad N(t) = N_0 e^{-\lambda t}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Half life is time for half of the nuclei to decay

$$\tau = \frac{1}{\lambda}$$

Mean lifetime is inverse of decay constant

For multi-modal decays:

$$\frac{dN}{dt} = -\lambda_1 N - \lambda_2 N$$

$$N(t) = N_0 e^{-(\lambda_1 + \lambda_2)t}$$

$$\lambda = \sum_i \lambda_i$$

For sequential production & decay:

$${}^{23}_{11}\mathrm{Na} + \mathrm{n} \rightarrow {}^{24}_{11}\mathrm{Na} \rightarrow {}^{24}_{12}\mathrm{Mg} + \mathrm{e}^- + \bar{\nu}_e$$

$$\frac{dN}{dt} = p - \lambda N$$

For a decay chain 1→2→3 production/decay rates are given by

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = -\lambda_2 N_2 - \frac{dN_1}{dt}$$

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Reaction Kinematics

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}X_{N-2} + \alpha $$

$$ m_X c^2 = m_{X'} c^2 + T_{X'} + m_\alpha c^2 + T_\alpha $$

$$ \begin{aligned} Q &= \sum_i M_i(Z,A) - \sum_f M_f(Z,A) \\ &= \sum T_{fin} - \sum T_{init} \\ &= \sum B_{fin} - \sum B_{init} \end{aligned} $$

Q value is net energy release

T = kinetic energy
M = masses

Last equation only true for reactions with constant N and Z
e.g. alpha decay
but not beta decay

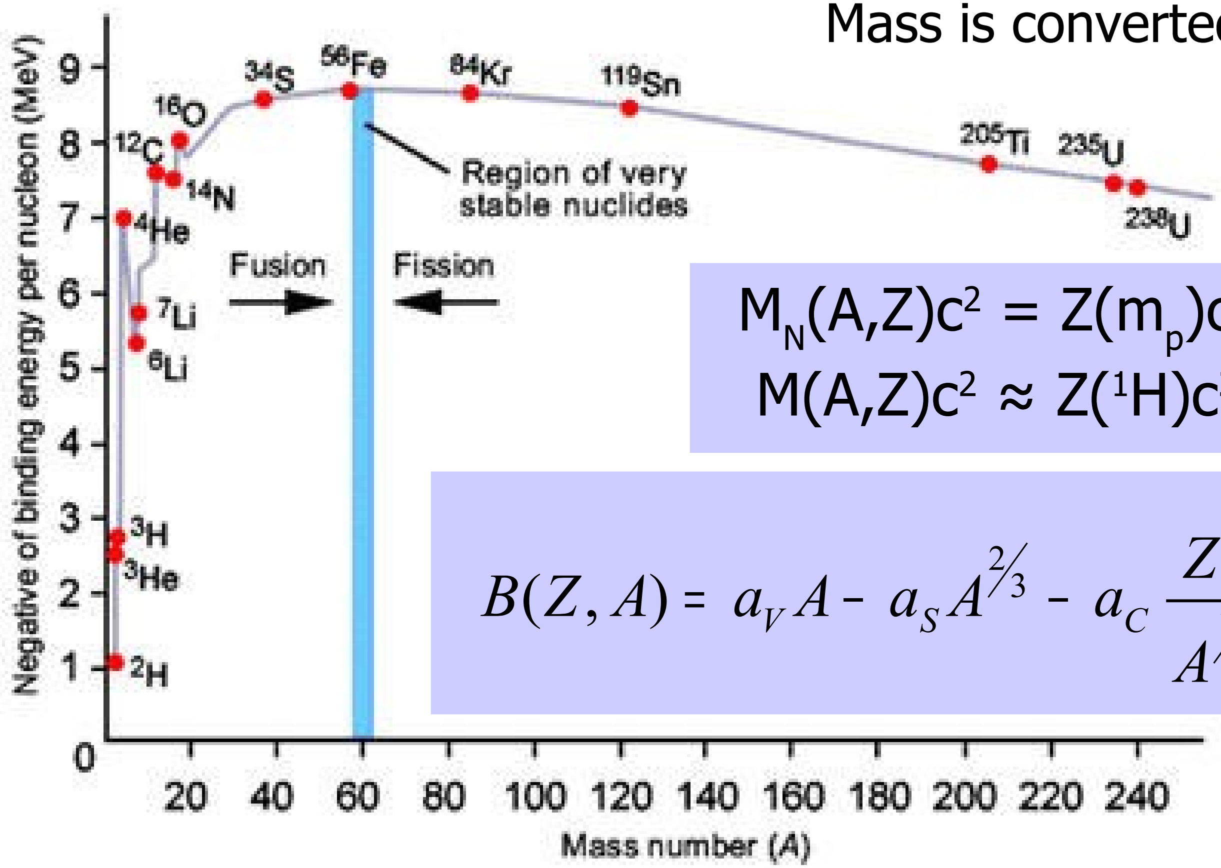
When solving nuclear physics problems try to look at the Q in terms of T and M

In some situations initial T is zero (mother particle is at rest)
Also M (but not T) is a Lorentz invariant - same in all frames
Can help simplify problems

Nucleus mass is always less than sum of nucleon masses!

Difference is the binding energy, B

Mass is converted into energy to bind nucleons



Nuclear masses

Atomic masses

$$M_N(A,Z)c^2 = Z(m_p)c^2 + (A-Z)m_nc^2 - B$$

$$M(A,Z)c^2 \approx Z(^1H)c^2 + (A-Z)m_nc^2 - B$$

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)$$

volume term ∝ volume - each nucleon provides ~ const binding energy

surface term ∝ area - nuclear density ~ const out to surface

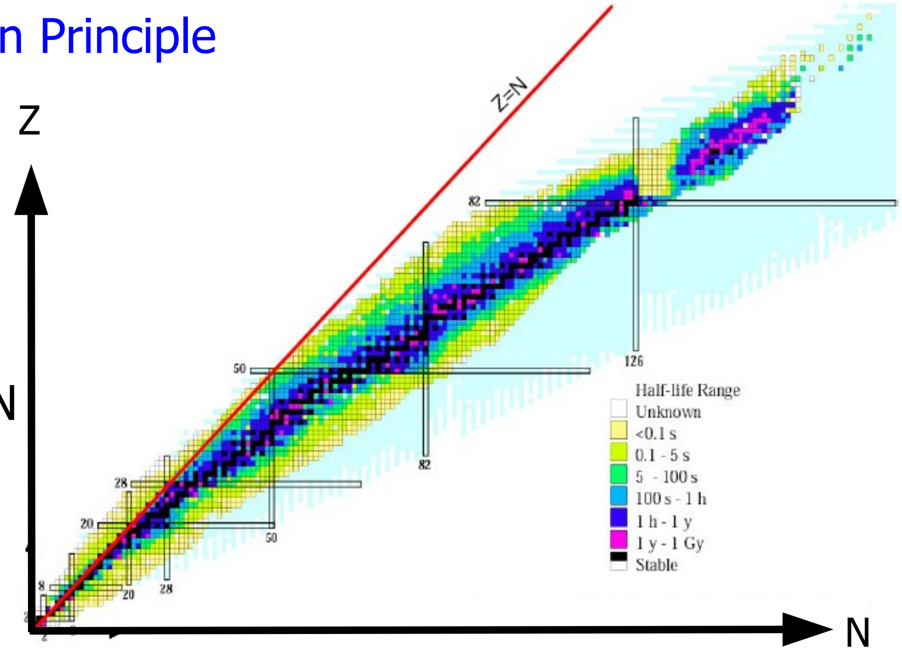
⇒ surface nucleons contribute less B

coulomb term ∝ Z^2/radius - for uniform charge distribution within drop

Asymmetry Term

neutrons & protons are fermions → Paul Exclusion Principle

forbids identical fermions from same QM state
will influence nucleons in potential wells
ΔE is similar for neutron & proton
⇒ for fixed A, energy is minimised by having Z=N



Pairing Term

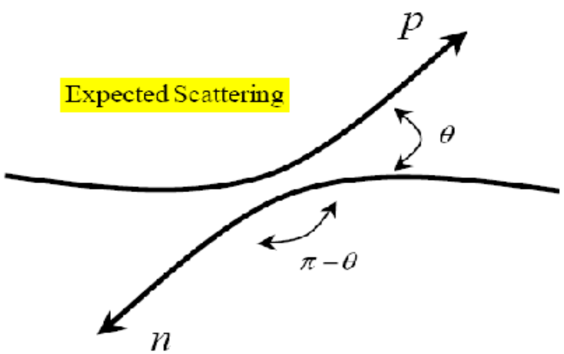
Contribution to binding energy is

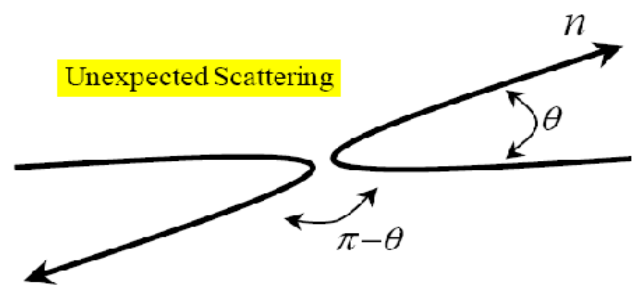
$$\delta(Z, A) = a_P / A^{1/2}$$

We add a paring term the equation

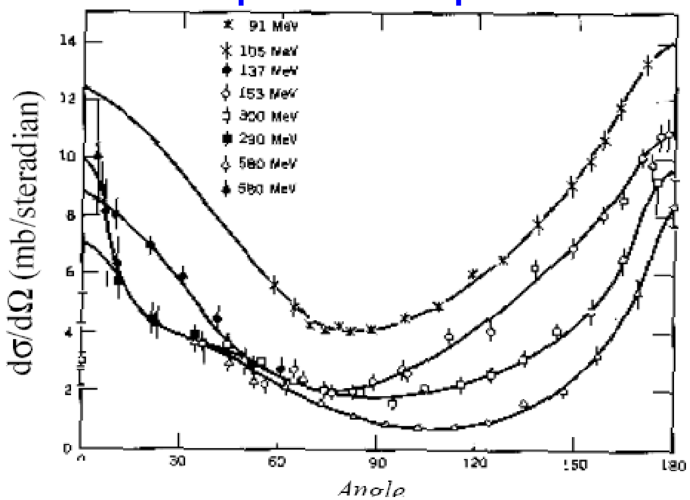
- For odd A nuclei (δ=0)
  - Z even, N odd
  - Z odd, N even
- For A even
  - Z odd, N odd (–δ)
  - Z even, N even (+δ)

Nucleon Scattering





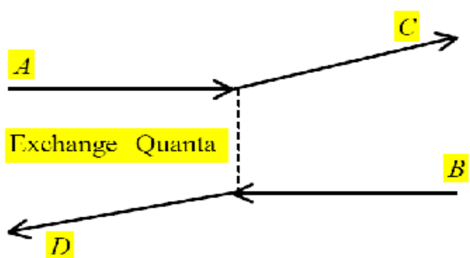
$n + p \rightarrow n + p$



- similar thing happens with nn or pp scattering
- NOT like Rutherford's results!
- nn / pp scattering particles are indistinguishable!

The Exchange Model

When scattering n-p at high energies strong peak in cross-section at 0°
Also strong peak seen at 180° - not explained by standard elastic processes

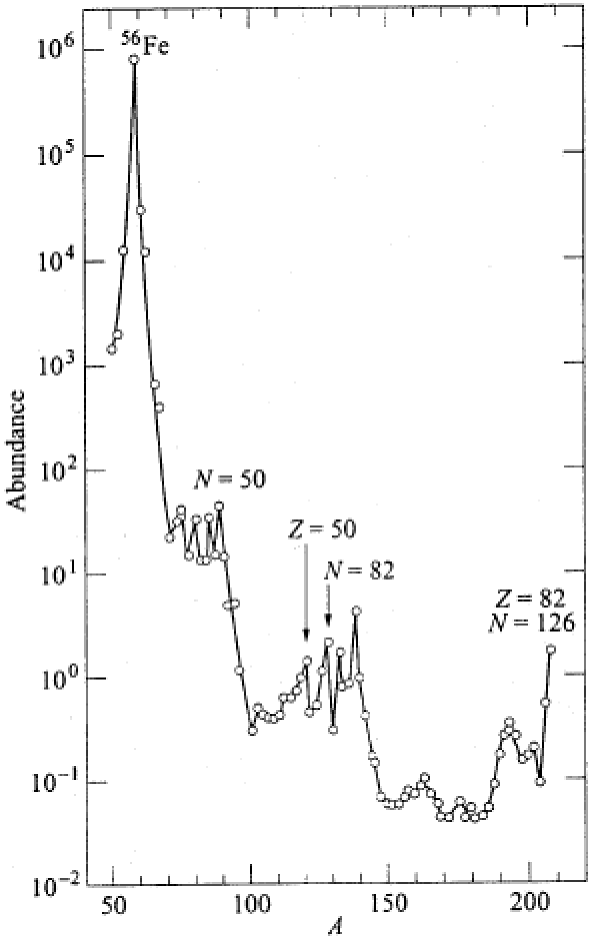


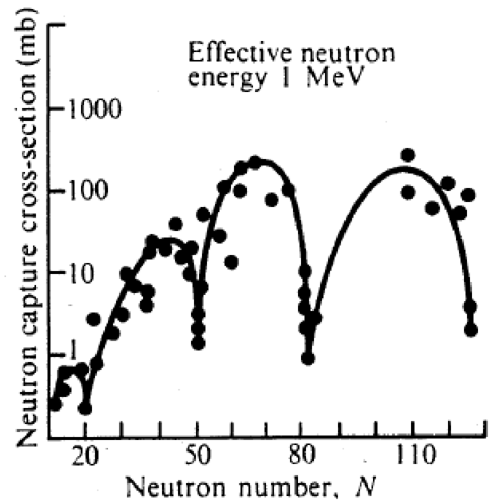
Violation of energy-momentum conservation by amount ΔE is permitted for a time $\Delta t \sim \frac{1}{2}\hbar/\Delta E$

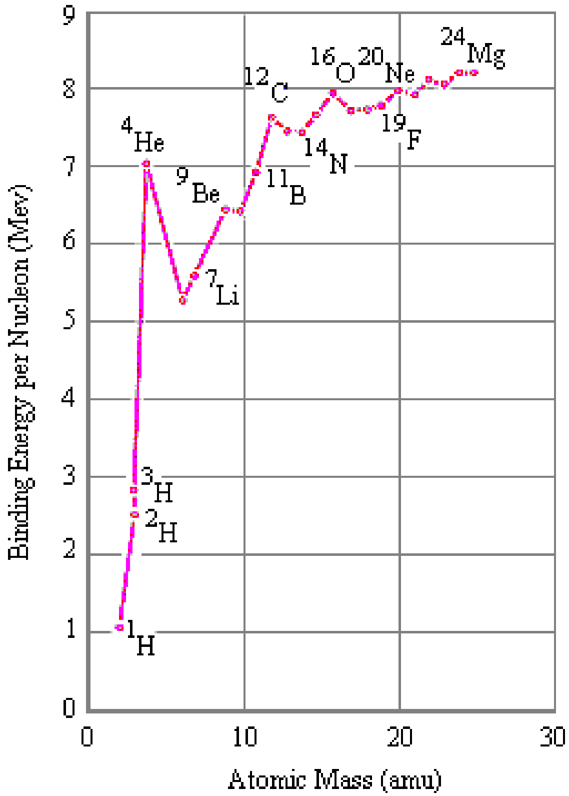
$\Delta E \sim$ Mass of exchange particle

$c\Delta t \sim$ Range of force

Shell Model of Nucleus







Sharp discontinuities occur when N or Z are:

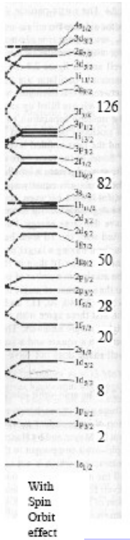
2, 8, 20, 28, 50, 82, 126

Nuclei with N,Z = 'magic number' have higher BE

These same magic numbers occur repeatedly

smooth transitions within shell
sharp discontinuities across shell boundary

In nuclear physics potential is created by nucleons themselves - no external agent



Max number in Shell	Total Angular Momentum Levels in Shell
126	$1h_{9/2}, 2f_{1/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2}$
82	$1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$
50	$2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$
28	$1f_{7/2}$
20	$1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$
8	$1p_{3/2}$, $1p_{1/2}$
2	$1s_{1/2}$

l	0	1	2	3	4
symbol	s	p	d	f	g

Consider shell level $1p_{3/2}$

quantum numbers:

principal: $n = 1$

orbital ang. mom $l = 1$

total ang. mom $j = 3/2$

degeneracy = $(2j+1) = 4$

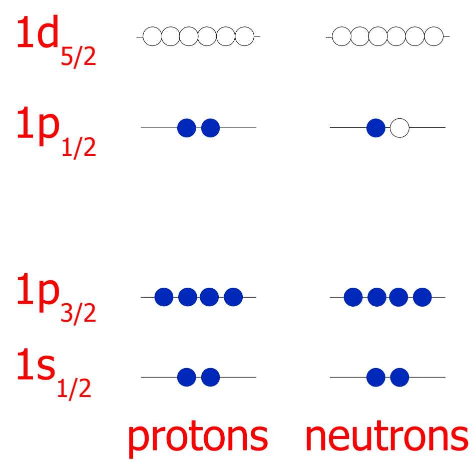
- Even-odd or odd-even nuclei have spin/parity of single unpaired nucleon
- Even-even nuclei have spin 0 (all neutrons / protons are paired)
- Odd-odd nuclei have $\mathbf{J = J_n + J_p}$ of last unpaired n and p

$^{15}_{8}O_7$

spin=½

$\pi = (-1)^l = -$

$J^\pi = ½^-$

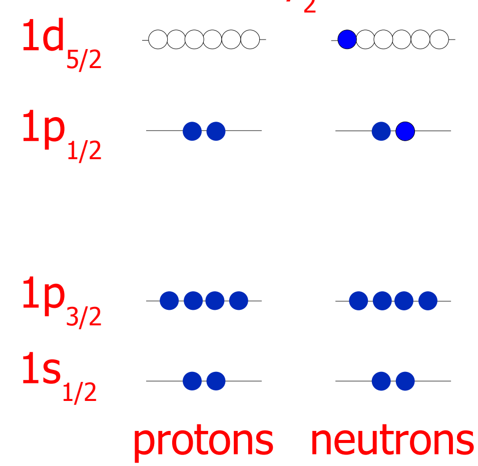


$^{17}_{8}O_9$

spin=$^5/_2$

$\pi = (-1)^l = +$

$J^\pi = {}^5/_2{}^+$



8 protons fill a shell

7^{th} neutron in $p_{1/2}$ shell ($j=1/2$ $l=1$)

Expect $^{15}_{8}O_7$ to have

spin ½

odd parity $P=(-1)^l$

8 protons fill a shell

9^{th} neutron in $d_{5/2}$ shell ($j=5/2$ $l=2$)

Expect $^{17}_{8}O_9$ to have

spin 5/2

even parity $P=(-1)^l$

Alpha Decay

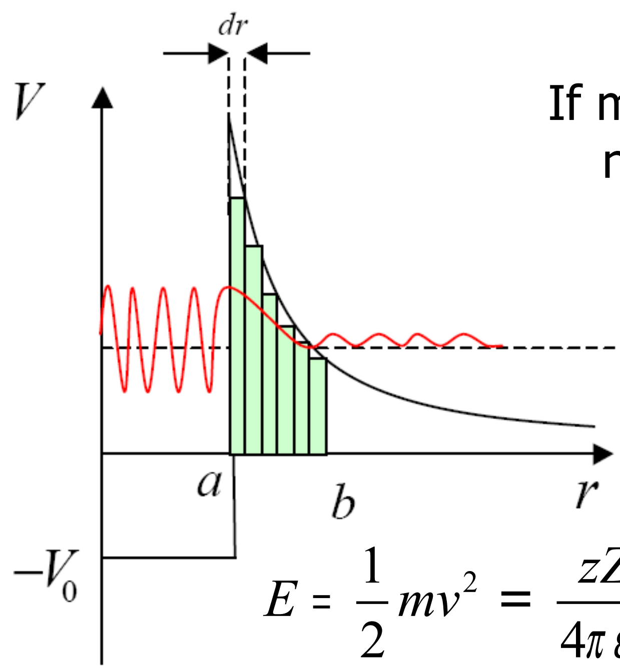
$(A,Z) \rightarrow (A\text{-}4, Z\text{-}2) + \alpha$

Alpha decay is spontaneous emission via QM tunneling of ${}^4_2He$ from nucleus

Least penetrating of all emissions - stopped by ~ few centimeters air

charge = +2 $\rightarrow$ strongly ionising, loses energy quickly in Coulomb scatters

Probablity to penetrate barrier is $P = e^{-2G}$ G is gamov factor $G \propto \frac{Z}{\sqrt{E}}$



If mother is at rest neglecting recoil

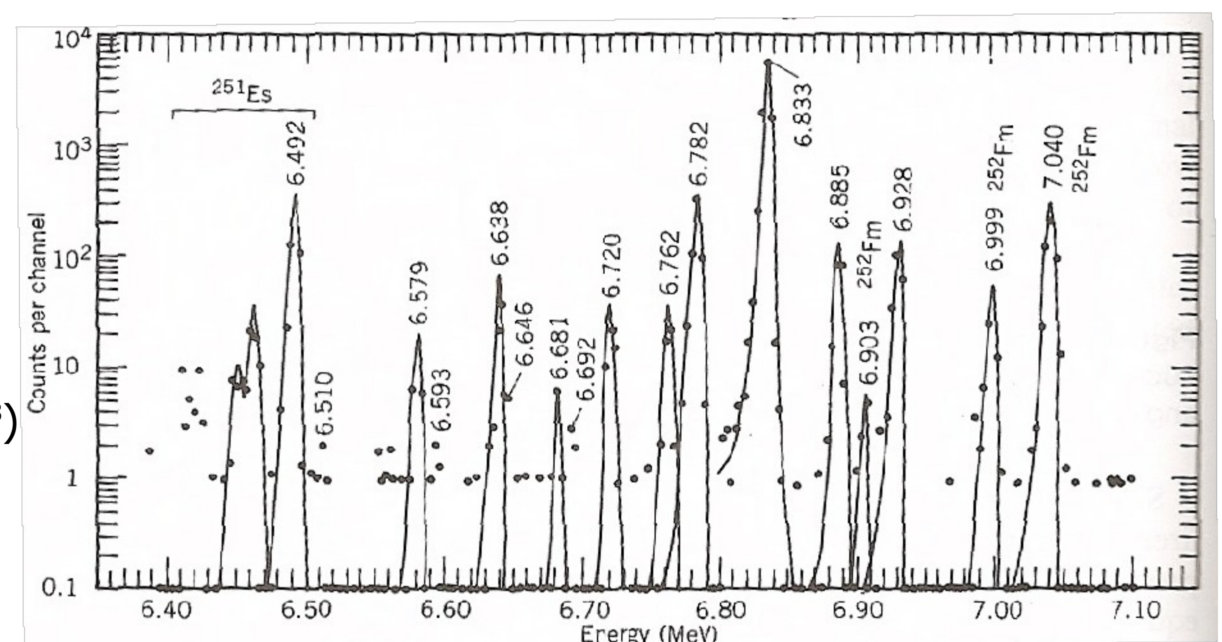
$$Q = \sum T_{fin} - \sum T_{init}$$

$$Q = T_\alpha$$

$$E = \frac{1}{2}mv^2 = \frac{zZe^2}{4\pi\varepsilon_0 R}$$

R is nuclear radius ($R \sim R_0 A^{1/3}$)

$$G \approx \frac{zZe^2}{4\pi\varepsilon_0 \hbar v}$$



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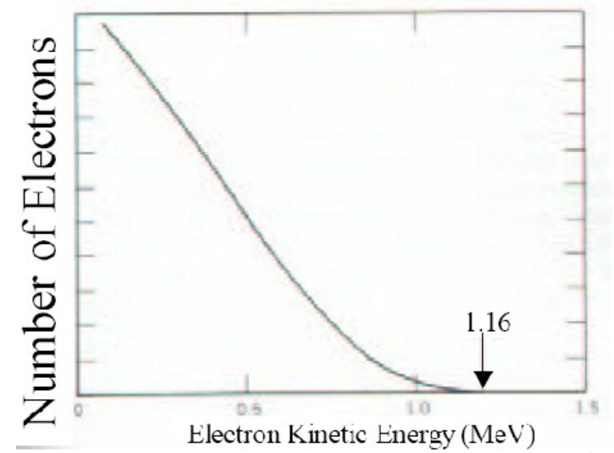
Beta Decay

smooth spectrum: with max T_e



$(A,Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}$ OR $(A,Z) \rightarrow (A, Z-1) + e^+ + \nu$

Process is interpreted as nucleon conversion

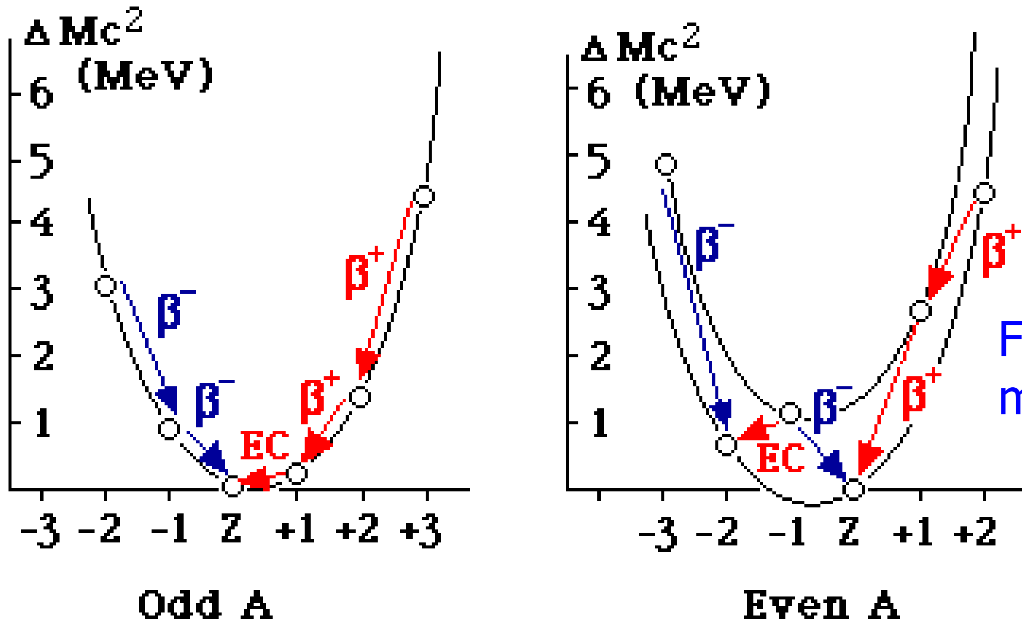
$n \rightarrow p + e^- + \bar{\nu}_e$ negative β^- decay

$p \rightarrow n + e^+ + \nu_e$ positive β^+ decay

$p + e^- \rightarrow n + \nu_e$ electron capture

Fixed A

For odd A only lowest mass state is β stable

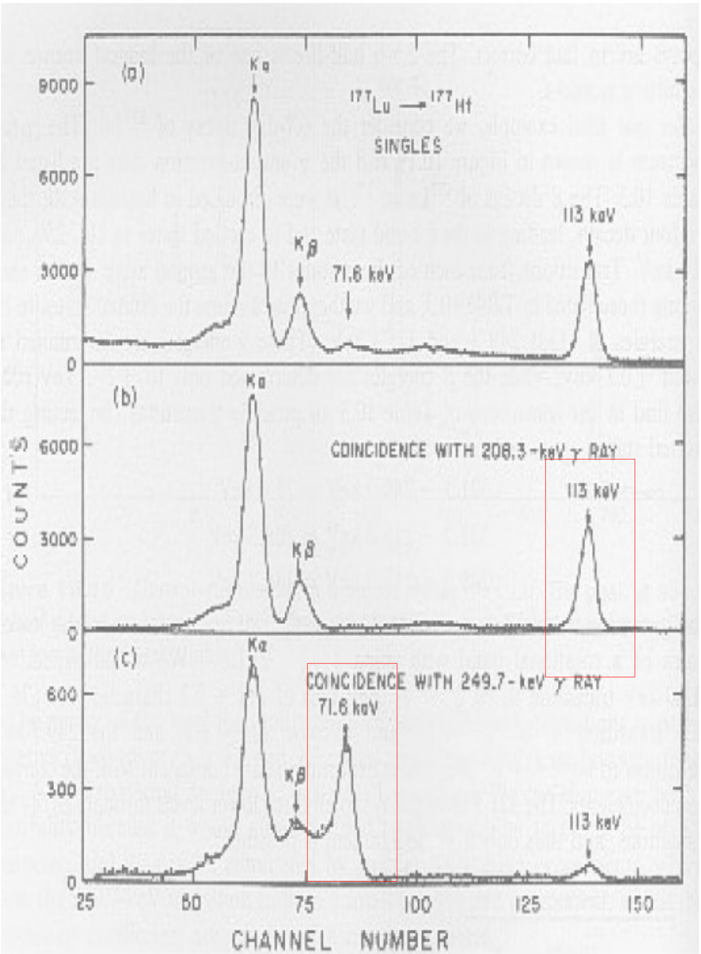


For even A two lowest mass states are β stable

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)$$

Gamma Decay

$$(A,Z)^* \to (A,Z) + \gamma$$

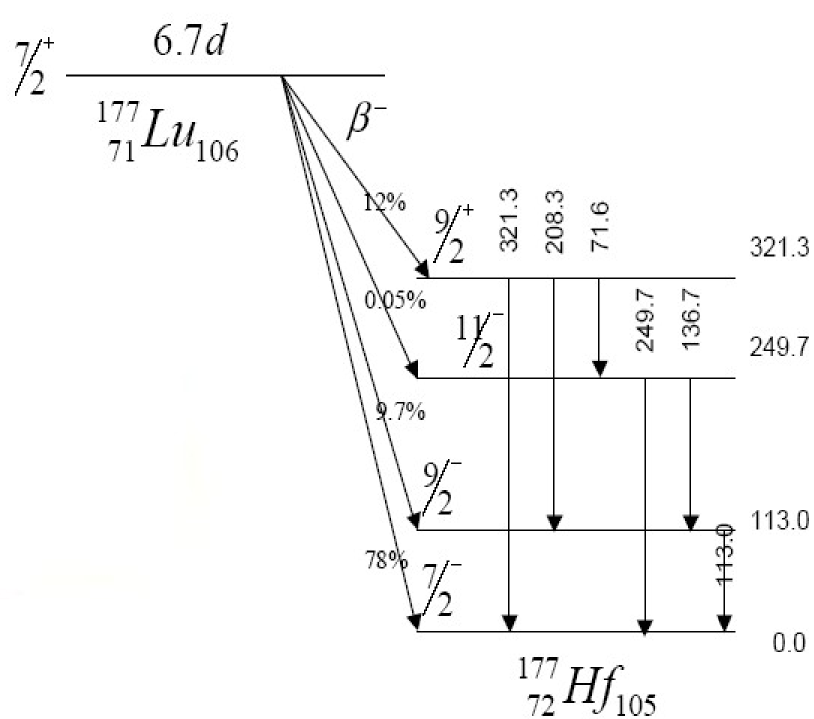


All gamma rays

coincidence with 208 KeV

coincidence with 249 KeV

- Gamma decays usually occur after alpha/beta has left daughter in excited state
- Can get chain of gamma decays to ground state



Coincidence technique helps determine structure

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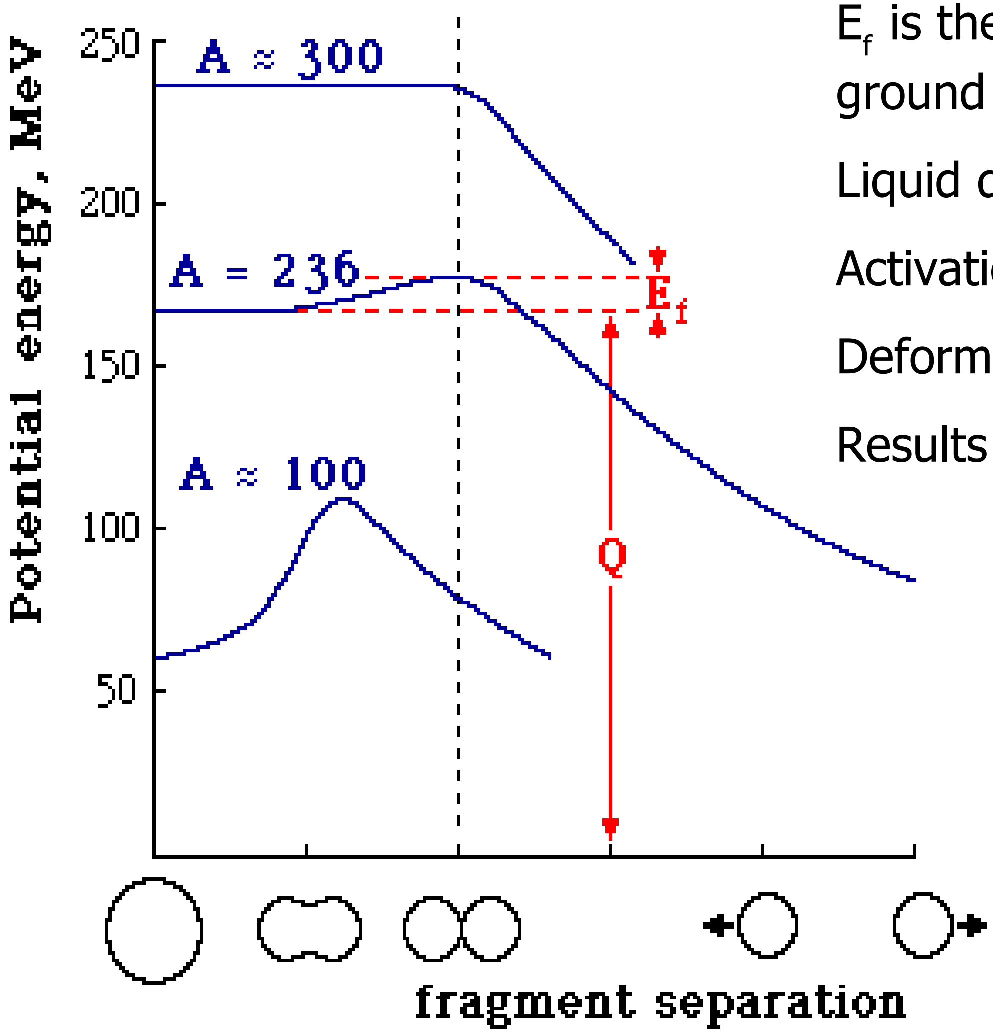
Nuclear Fission

More detailed calculation of fission process using liquid drop model:



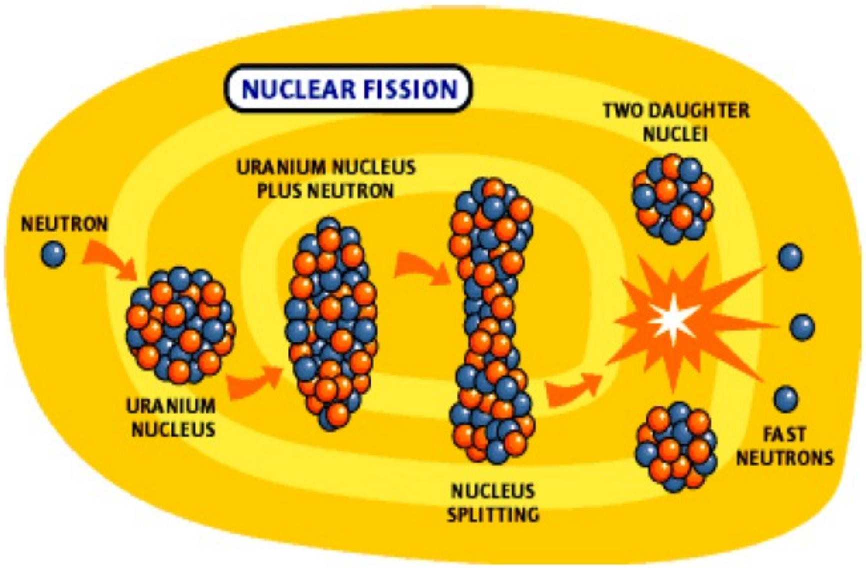
E_f is the **activation energy:** height of barrier above ground state

Liquid drop models provides intuitive picture of fission

Activation energy creates a deformation of the nucleus

Deformation becomes extreme

Results in nucleus splitting into 2



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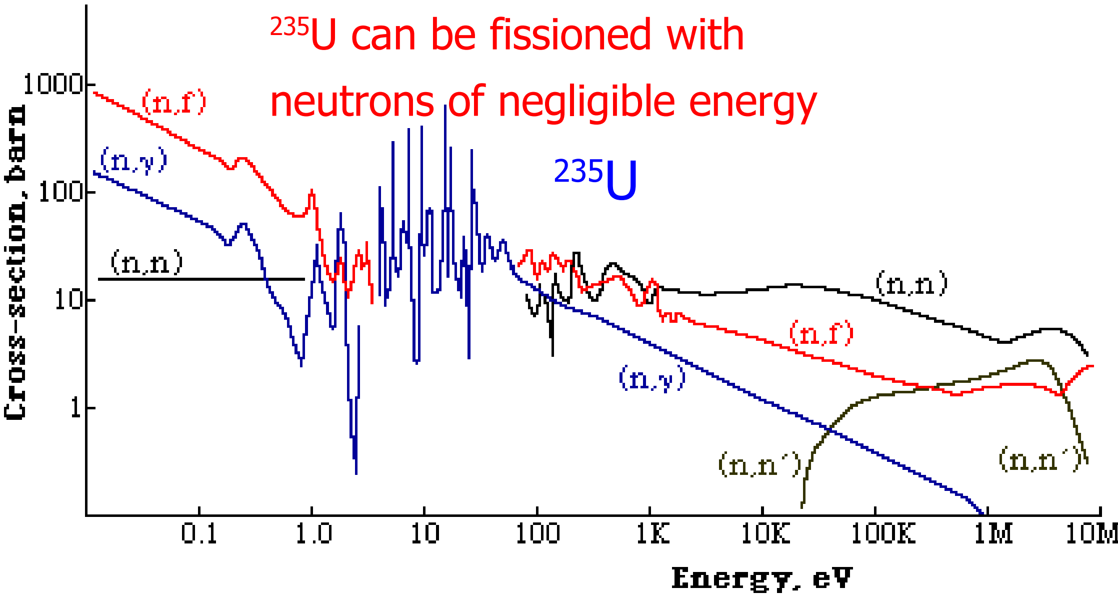
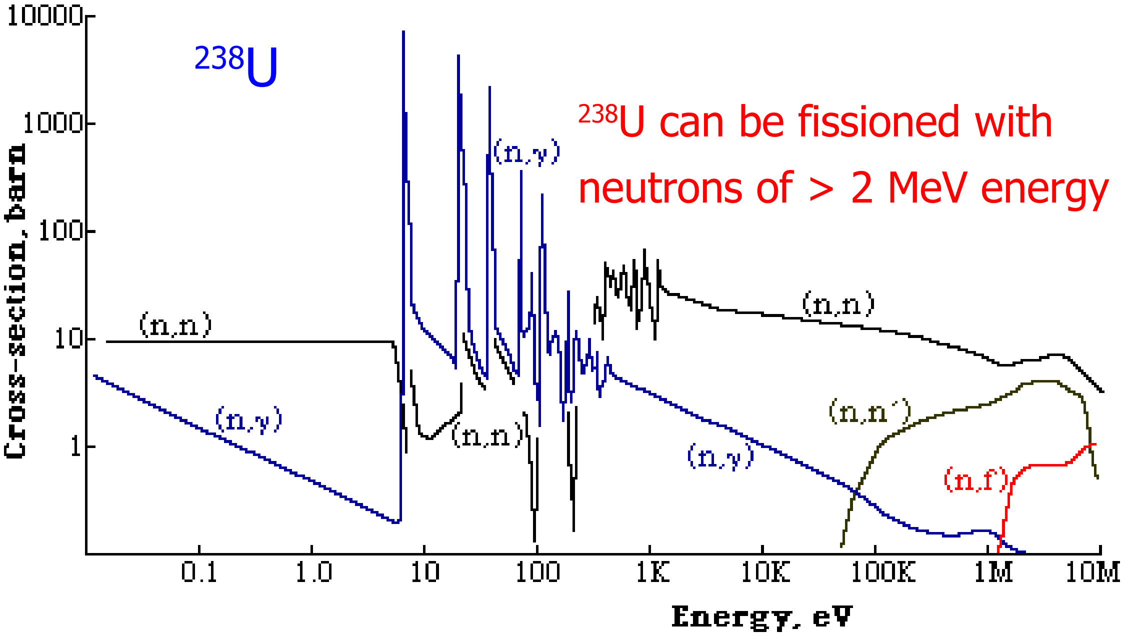
The Uranium System





fission cross section largest at low energy

fissile at all neutron energies

large neutron capture cross section ~10 eV

prompt neutron: E ~ 2 MeV

fission produces ν (~ 2.5) n^0s per fission

k_∞ = neutron reproduction factor

= #thermal n^0s produced per fission

prompt n^0s moderated by C rods in pile

η: # fission n^0s per orig thermal neutron

$$\eta = \upsilon \frac{\sigma_f}{\sigma_f + \sigma_a}$$

ε = fast fission factor: additional

n^0s gained from high E ^{238}U fission

p = fraction n^0s surviving ^{235}U(n,γ)^{236}U etc

f = thermal utilisation factor

= fraction surviving thermal C absorbtion

Fusion

Energy released can be exploited: provide power

advantages over fission energy production:

- initial light nuclei are plentiful
- final products are also light stable nuclei (not heavy radioactive nuclei)

disadvantage:

- nuclei must overcome Coulomb repulsion

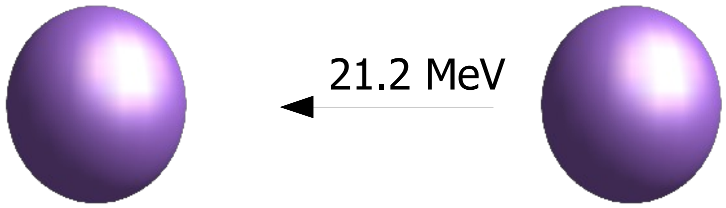
Consider $^{20}Ne + ^{20}Ne \rightarrow ^{40}Ca$

Q = 20.7 MeV

= 0.5 MeV per nucleon (comparable to fission ~200MeV/240 = 0.8)

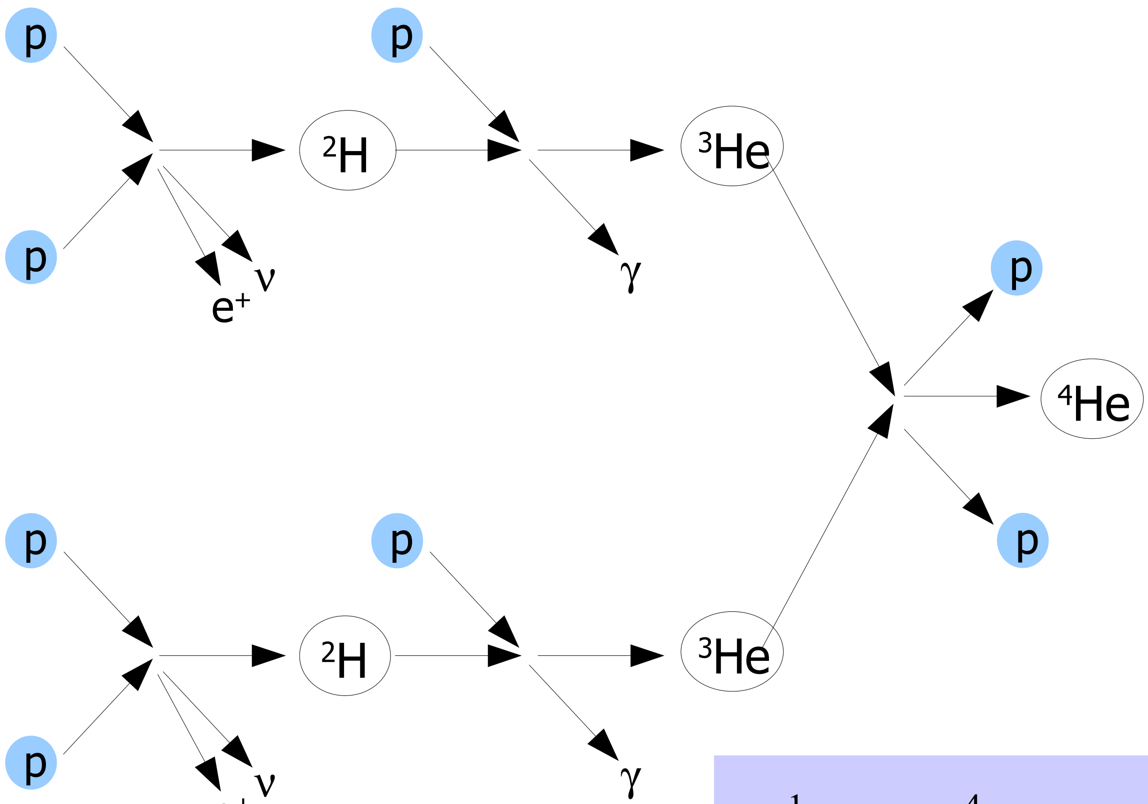
with nuclear surfaces touching - Coulomb repulsion = 21.2 MeV

collision with 21.2 MeV initial kinetic energy yields 21.2+20.7 = 42 MeV energy

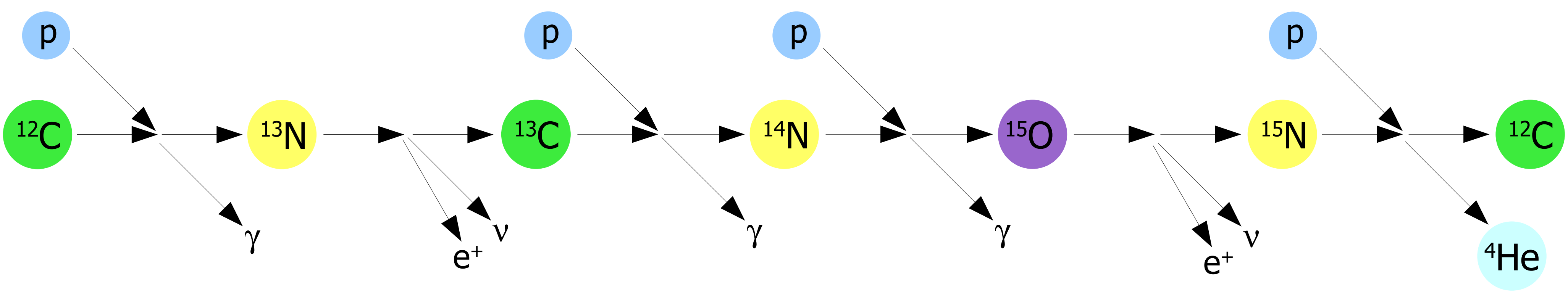


Thus energy gain is factor of 2

Proton-proton And CNO Cycles



$$4\,{}^1H \rightarrow {}^4He + 2e^+ + 2\nu_e + 2\gamma \qquad Q=26.7\ \text{MeV}$$



Nucleosynthesis

A 'resonance' is needed to explain abundance of ^{12}C
cross section has peaks for ^{8}Be and ^{12}C production

This is the net triple alpha process
Net reaction is simply: $3\,^{4}He \rightarrow {}^{12}C$

After production of ^{12}C further reaction chains are available:

$$^{12}C + {}^{4}He \rightarrow {}^{16}O + \gamma \qquad E_B = 3.57\text{ MeV}$$

$$^{16}O + {}^{4}He \rightarrow {}^{20}Ne + \gamma \qquad E_B = 4.47\text{ MeV}$$

$$^{20}Ne + {}^{4}He \rightarrow {}^{24}Mg + \gamma \qquad E_B = 5.36\text{ MeV}$$

After Helium supply is consumed:
carbon & oxygen burning

$$^{12}C + {}^{12}C \rightarrow {}^{20}Ne + {}^{4}He$$

$$^{16}O + {}^{16}O \rightarrow {}^{28}Si + {}^{4}He$$

At silicon burning stage photodissociation becomes important

$$^{28}Si + \gamma \rightarrow {}^{24}Mg + {}^{4}He$$

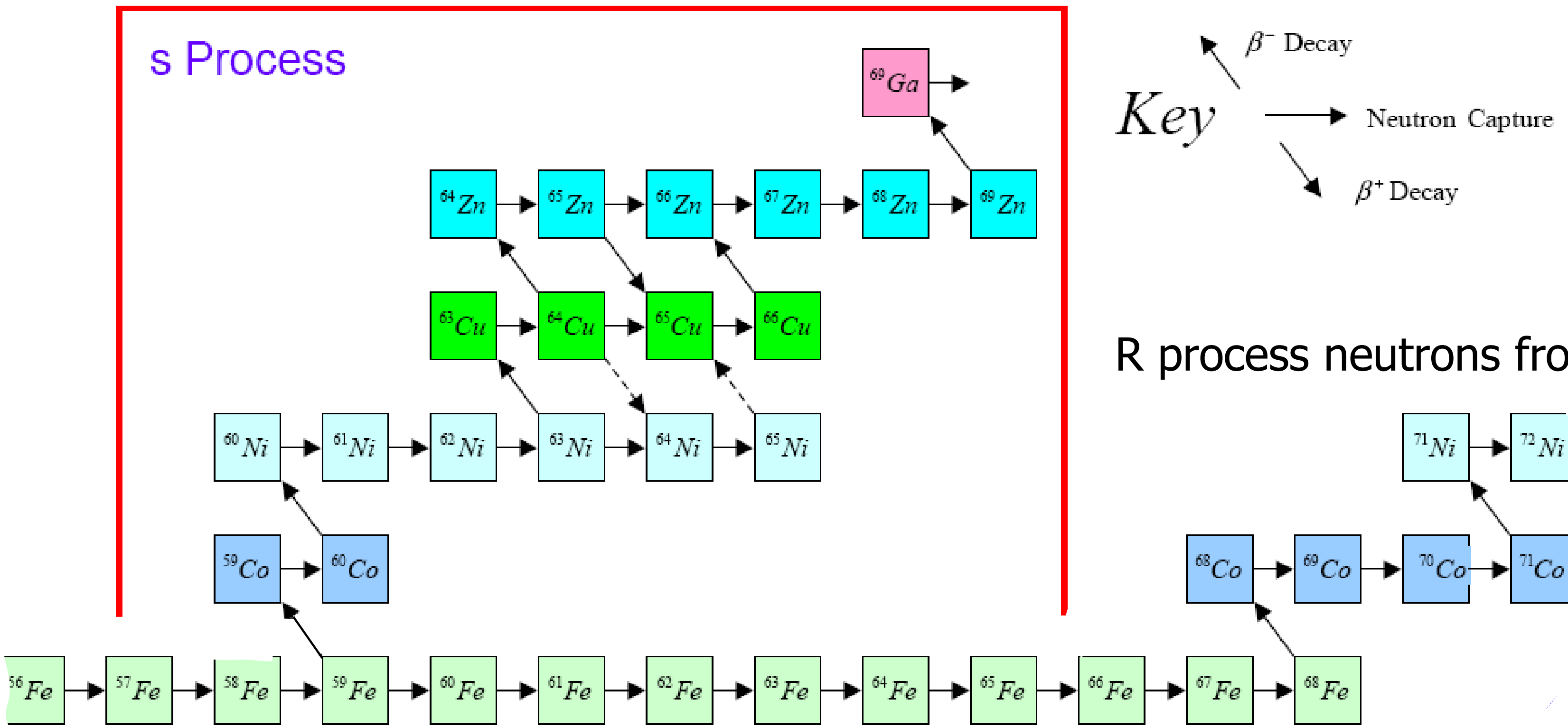
$$^{28}Si + {}^{4}He \rightarrow {}^{32}S + \gamma$$

Chains of these reactions produce
elements A~56 (Ni, Co, Fe)

Nucleosynthesis: R and S Process

Nucleosynthesis of heavier elements (Z > Fe) can be divided into 2 processes:

- s process: neutron capture very slow - allowing β decays to occur
- r process: neutron capture rapid - no time for β decays



R process neutrons from supernova!

S process neutrons come from spallation reactions from neutron rich isotopes:

$^{13}C\ (\alpha,n)\,^{16}O$ Q=2.2 MeV

$^{22}Ne\ (\alpha,n)\ ^{25}Mg$ Q=-0.48 MeV

Electron degeneracy pressure cannot support star

Increased density → e^- capture cross section occurs:

$p + e \rightarrow n + \nu_e$ protons convert to neutrons

No e^- to support core's gravitational collapse!

The Standard Model

Worlds most successful theory to date - Describes fundamental constituents of matter

Three generations of increasing mass

u d s c b t quarks: strong, weak, electromagnetic

e μ τ charged leptons: weak, electromagnetic

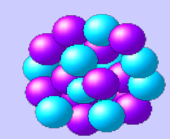
ν_e ν_μ ν_τ neutral leptons: neutrinos: weak

All matter made up of these 12 fermions (spin ½ particles) - 1st generation only!

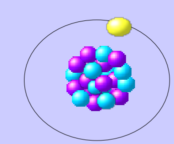
All are point-like particles (as far as we know!)

All forces of nature propagated by these 4 bosons (spin 1 particles)

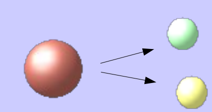
gluons — Strong: holds atomic nucleus together



photons — Electromagnetic: binds atom together



W and Z bosons — Weak: radioactive decay processes



Standard Model also requires existence of Higgs boson - as yet unobserved