MSci 4261 ELECTROMAGNETISM

LECTURE NOTES XI

Most of the course so far has focussed on aspects of electromagnetism which have been known for some time. In these final notes, we will discuss briefly two areas in which electromagnetism arises in a fundamental way in modern physics. The corresponding lectures will go into more depth and explanation.

11.1 The Standard Model

When electromagnetic plane waves were discussed in an earlier lecture, it was noted that their energy and momentum transfer were consistent with light being made from photons with momentum p and energy pc. The explanation of the photoelectric effect in terms of light being made up of photons was a crucial advance in physics around the turn of the century. One might be tempted to believe that these photons are fundamental particles. However, it has emerged in the past few decades that in a certain sense this is false, with the photon being a mixture of two different fundamental particles and further having a "partner" particle which is a different mixture. This is one of the surprising consequences of the *electroweak* theory, which unifies electromagnetism with the weak nuclear force.

Before the electroweak theory, the weak nuclear force, responsible for radioactive beta decay, was understood in terms of a "four-fermi" interaction, involving an interaction of a proton, neutron, electron and neutrino. However, it came to be understood that this interaction was only an effective one, and that if one looked more closely there was a new set of particles which were mediating the interaction. In this way, the four particle interaction becomes two three-particle interactions linked by the exchange of a force particle (in the same way as the scattering of electrons can be understood as due to photon intermediation through three vertex interactions of a photon and incoming and outgoing electrons).

This had one benefit, in that a class of theories with these type of interactions through force particles have much better properties in terms of being able to calculate amplitudes. These are *gauge theories*, just like electromagnetism. But there was a significant problem with this formulation of the weak force - this is that the weak force is short-range (unlike the electromagnetic force), and thus the intermediate force particles involved must be massive, with masses around $100GeV$ to account for the known range of the force. Gauge theories with massive force particles have many problems in their formulation, which people were not able to solve.

The breakthrough came with the application of some ideas from outside high-energy physics. In particular, the important idea that the symmetries of a physical state of a system need not be the same as the symmetries of the system itself came into play here. In terms of the fields of high energy physics, this meant that one could have a theory with certain symmetries whose vacuum (lowest energy) states had less symmetry - ie some of the symmetry could be *broken*. Applied to gauge theories, this had the consequence that the massless force particles in the symmetric formulation of the theory became *massive* - just as required for the weak nuclear force theory.

It proved to be possible to put these ideas into practice in a way consistent with nature by combining the weak and electromagnetic forces into an *electroweak* theory, with a gauge symmetry like the electromagnetic one, plus another larger gauge symmetry. The gauge group involved is $U(1) \times SU(2)$. This symmetry is then broken to give the electromagnetic $U(1)$ symmetry which is observed, and the gauge particle is the photon. The three other gauge bosons become massive, one neutral and two oppositely charged. Finally, there are neutral and charged *Higgs* particles which are required as part of the symmetry breaking process. This theory was confirmed in the mid 1980's when these three massive gauge bosons were discovered, with masses exactly in the region predicted. Weinberg and Salam received Nobel prizes for their work in formulating the theory, and Rubbia was similarily rewarded for his work at CERN where the discoveries were made. In the electroweak model, the photon emerges as a mixture of the two neutral vector bosons in the unbroken theory, whilst the other independent mixture yields its partner - the neutral, massive Z boson.

This unification of electromagnetism and the weak nuclear force has many consequences - and all predictions of the theory which have been tested so far have been confirmed. One consequence, for example, concerns the force between electrons. Classically, this is of course just the Coulomb repulsion. When one takes quantum mechanics and special relativity into account, one must describe the interaction between electrons using Quantum Electrodynamics or QED, and look at the scattering amplitude. QED predicts that the interaction between electrons is altered at short distances or high energies, with consequences such as the Lamb shift in atomic spectra. This alteration can be intuitively thought of in terms of the interaction of the virtual photon which mediates the force, with fluctuations in the vacuum, where further virtual particles appear and can interact with the photon before disappearing again. In a sense, the photon is moving in an effective medium. Moving to the electroweak theory, we have further interactions which are possible, this time with the massive weak intermediate vector bosons (the neutral Z boson and the charged W bosons), as well as the Higgs particles. These extra allowed interactions, which become most important as the centre of mass energy of the interacting electrons reaches the electroweak scale (ie around 90 GeV), change the nature of the force between the electrons, and give predictable alterations to the scattering amplitudes. These have been repeatedly confirmed in experiments over the past fifteen years. Very recent results point to the discovery of the Higgs bosons themselves, the last remaining missing piece of the theory.

The strong nuclear force has also come to be understood as a gauge theory. This time the gauge group is larger, $SU(3)$. This has eight force particles, or vector bosons, which act between the three different types (or "colours") of quark. There are six such vector bosons which change quark colour (and hence carry colour charge themselves), and two neutral vector bosons. The theory of the strong nuclear force, or quantum chromodynamics (QCD), has its own special properties - for example, it explains why free quarks are not seen. This is a consequence of the result that the force between quarks actually increases with distance, so that at some point it is energetically favourable to create new bound states of quarks (eg protons, neutrons, etc) rather than continue separating the original quarks. Much study has been made of QCD and its predictions and tests. Suffice it to say here that its formal structure is analogous to the electroweak theory in that it is a gauge theory (although here without symmetry breaking). Thus the gauge structure which we first met in electromagnetism, and which at first sight may have appeared a mathematical curiousity, turns out to be a fundamental guide to the formulation of realistic theories of the forces of nature. The combination of QCD with the electroweak theory, into a gauge theory with gauge group $SU(3) \times SU(2) \times U(1)$, and symmetry breaking, gives the so-called *Standard Model*, which so far has passed all experimental tests.

Although there are various puzzles in the Standard Model (eg CP violation), perhaps one of the most striking puzzles is its general structure. It really has three gauge groups, plus a somewhat ad hoc symmetry breaking structure. Early attempts were made to unify the three gauge groups into one larger one - $SU(5)$ for example. However, one problem with much of this work is that it predicts proton decay with lifetimes at levels which have not been seen in experiments. These "Grand Unified Theories" (GUTs) are nevertheless attractive in principle and continue to be explored. One added ingredient of potential importance is *supersymmetry* - a new sort of symmetry which relates bosons to fermions, and which improves the formulation of quantum field theories in that it reduces the need for renormalisation of divergences. There are even classes of theories where all the infinities cancel. Supersymmetric GUTs have been actively studied in the past two decades. Although no evidence for supersymmetry has yet been found, it is quite commonly felt that, given that it exists formally and has very nice physical properties, it should exist in nature.

11.2 Duality, Gravity and M-Theory

The above-mentioned success in describing three of the fundamental forces of nature in terms of gauge theories naturally leads one to focus on the fourth force, that of gravity, and consider a similar description. Unfortunately, straightforward approaches to this have led to insuperable difficulties. We have a classical theory of gravity due to Einstein. One can therefore seek to formulate a quantum theory, using similar techniques to those used for the other forces. However, gravity is not a gauge theory in the same way as QED for example. The action is different and the gauge group acts in a different way for a start. Furthermore, simple applications of quantum ideas to gravity founder on the problem that the divergences which appear in quantum graphs cannot be renormalised away in gravity, and these infinities render the consequent amplitudes meaningless. Supersymmetric gravity theories, or supergravities, have been found, however they suffer from the same basic problems in general. Although progress continues to be made in the study of gravity (the Ashtekar formulation of the theory is an example), much attention has turned to string theory of late as a candidate theory which could include a consistent quantum gravity as well as unifying it with gauge theories.

It should be mentioned that the original formulations of string theories were used to describe the spectra of hadronic particles, by understanding them as quarks held together by gluonic string. It was then realised that one could also think of the strings as fundamental, and thus modern string theory was born. The attractive feature of this fundamental string theory is that it naturally contains both Yang-Mills (ie gauge) fields and gravity - the open strings, those with ends, include a desription of gauge fields, whilst the closed strings, those which are loops, describe gravity. In the early 1980's, at Queen Mary, London and at Caltech, USA, it was shown that supersymmetric string theories exist. These theories live in ten dimensions (nine space and one time). This work has since spawned a large and diverse industry throughout the world, ranging from applications to outstanding mathematical problems, to attempts to link the theories directly to observed results from high energy experiments and astrohysical observations. As with supersymmetry, there is no experimental evidence for string theory. However, it is generally felt that string theories provide our current best ideas for creating models of unified field theories containing gravity and the Standard Model. This subject is too large, and much of it too advanced, to discuss here. However, one aspect which is currently under great scrutiny has links to some properties of electromagnetism which we have mentioned in previous lectures. This is the issue of *duality.*

Returning to the subject of electromagnetism then, we have seen that whilst the field strength tensor $F_{\mu\nu}$ contains the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, the dual field strength tensor $*F_{\mu\nu}$ contains the same fields, but their roles are interchanged (see Lecture Notes 8). This can be viewed as a 90^0 rotation $\mathbf{E} \to \mathbf{B} \to -\mathbf{E}$. In earlier Exercises, we saw that the vacuum Maxwell equations are invariant under rotations by an arbitrary angle, which mix the electric and magnetic fields. In the presence of sources, this duality symmetry is preserved provided that there are both electric and magnetic sources (and that these transform in a similar way). (Quantum mechanical considerations, as first given by Dirac, however restrict the allowed electric and magnetic charges, e and g, carried by a particle. This condition is that $eg/2\pi$ must be an integer. This also follows by consideration of the quantisation of angular momentum.)

If one had a theory with a duality symmetry of the type just discussed, then each particle in the theory would have a dual partner, with electric and magnetic charges interchanged. For example, in a duality invariant theory with electrically charged vector bosons, there would also be partners, magnetically charged monopoles. Just this situation arises in an early model of the weak interactions, due to Georgi and Glashow. In this model one has so-called BPS monopoles which are dual to the vector bosons in the theory. The proposal was made that in fact there would exist a totally dual description of the theory, where the monopoles were fundamental and the vector bosons were nonperturbative solutions. This dual theory would describe the strong coupling regime of the theory (since the duality exchanges strong and weak coupling as well). However, this proposal founders on the fact that the vector bosons have spin one whereas the monopoles have spin zero, and thus a simple symmetry between them is problematic. The resolution to this is to consider a supersymmetric version of the theory, where the particles come in multiplets of fermions and bosons, and for the so-called $N = 2$ model, this duality can be made explicit. Study of this has recently led to major progress in our understanding of Yang-Mills theory away from the perturbative regime, with proposals made for the spectra of non-perturbative states for example.

In the mid 1990's, it was realised that this strong-weak coupling duality was also a feature of string theories in a surprising way, in that previously unrelated theories were shown to be intimately connected by duality symmetries. This so-called second superstring revolution has led to the conjecture that there is a fundamental theory in eleven dimensions which underlies the whole subject. This has been called M-theory. The five known string theories are related to this underlying theory by duality transformations. Unfortunately, at this point in time it is not known how to formulate M-theory. It appears to be the case that a conventional formulation in terms of fields is not possible, and that a fundamental change in thinking might be required. As an example, proposals for a noncommutative structure for spacetime have been made. Alternatively, M-theory may be some sort of membrane theory, and the fundamental objects may be surfaces rather than particles. It is too early to know what the correct formulation will be.

Since then, a very interesting new development has shown that duality relates gravity theories to Yang-Mills theories. In particular, the initial proposal was that a limit of a certain supersymmetric Yang-Mills theory was equivalent to a supergravity theory. (This was a precise proposal of an older idea that Yang-Mills theories appear like string theories in a certain limit.) Evidence has since accumulated for this sort of duality, which has the additional feature that the gravity theory becomes completely determined by its properties on the boundary of spacetime, where the Yang-Mills theory lives. This is a sort of holography, which was also conjectured on more general grounds some time before. Investigation of this new holographic duality is continuing. Very recently, new sorts of duality, such as one involving different gravity theories and gravity without gravitons, have emerged. Where all these new developments will lead is unclear, but it is likely that they will have a great impact on our understanding of Yang-Mills theories and gravity and their possible unification.

Further Reading:

1. Papers announcing the possible discovery of the Higgs particle (Nov. 2000):
   `http://www.elsevier.nl/gej-ng/10/36/12/57/380/summary.html`
   `http://www.elsevier.nl/gej-ng/10/36/12/57/369/summary.html`.
2. `http://superstringtheory.com/index.html`.
   This site contains popular material on string theory for a general audience.
3. `http://www.physics.ucsb.edu/ jpierre/strings/index.htm`,
   `http://www.lassp.cornell.edu/GraduateAdmissions/greene/greene.html`,
   `http://turing.wins.uva.nl/ rhd/string_theory.html`.
   These sites contain reasonably detailed discussions without mathematics, plus links to more advanced reviews.
4. *Electromagnetic Duality for Children*, JM Figueroa-O'Farrill, Edinburgh University, available at
   `http://www.maths.ed.ac.uk/ jmf/EDC.html`.
   Despite the title, this contains much advanced material.
5. An example of a modern research paper in M-theory: (this concerns duality in gravitational theories) (Nov. 2000)
   `http://uk.arXiv.org/abs/hep-th/0011215`.