Part B: The velocity of light

Introduction

In this exercise you will measure the velocity of a beam of light of wavelength λ about 500 nm, both in a vacuum and when travelling through transparent plastic. The methods used in exercise 7 are no use here. As you know, the speed of light, c, is about 3×10^8 m/s, so substitution in $c = f\lambda$ yields a frequency f of about 6×10^{14} Hz, which is too high to be measured by an electronic frequency meter even if the very short wavelength could be measured accurately. The trick here is to vary the intensity of the light source at a frequency very much less than f. The change in intensity is carried along by the light wave, travelling at the same speed c and appearing as a sinusoidal variation of amplitude with a very much longer wavelength than that of the **carrier wave** itself. In other words, the carrier wave is **modulated** by the lower frequency signal — see figure 4. [This is the way that AM (amplitude modulation) radio works. The signal to be transmitted is impressed on a constant *frequency* carrier wave as a change in *amplitude*. In FM (frequency modulation) radio the *amplitude* of the carrier wave stays the same but its *frequency* is modulated.]

The modulation frequency used here is 50 MHz or 60 Mhz, depending on which apparatus you are using — if in doubt, check with a demonstrator. **Deduce** the corresponding modulation wavelength λ_m. The light is received by a photodiode detector whose response follows the modulation at 50 or 60 MHz. Even this reduced frequency is too high to be displayed on the oscilloscopes we use, so another electronic technique is used to reduce the frequency still further. A separate oscillator unit is tuned to a frequency slightly different from 50 or 60 MHz, e.g. 59.9 MHz. When this signal is mixed with a 60 MHz signal from the light source or photodiode the two 'beat' together producing a combined signal whose frequency is the *difference* of the two — this is called the **heterodyne** technique. The mixed signal has a frequency of ~100 kHz and can easily be displayed. Figure 4 shows that a 50 or 60 MHz signal which has travelled only a few metres to the receiver will be considerably out of phase with the signal from the source. A feature of the heterodyne technique is that it preserves this phase relation, so a measurement of the phase difference between the mixed low-frequency signal from the source and the mixed low-frequency signal from the receiver is a direct measure of the phase lag in the 50 or 60 MHz signal. Converting this phase difference to a fraction of the modulation period gives the time for the wave to travel a known distance to the receiver, and hence the wave velocity.

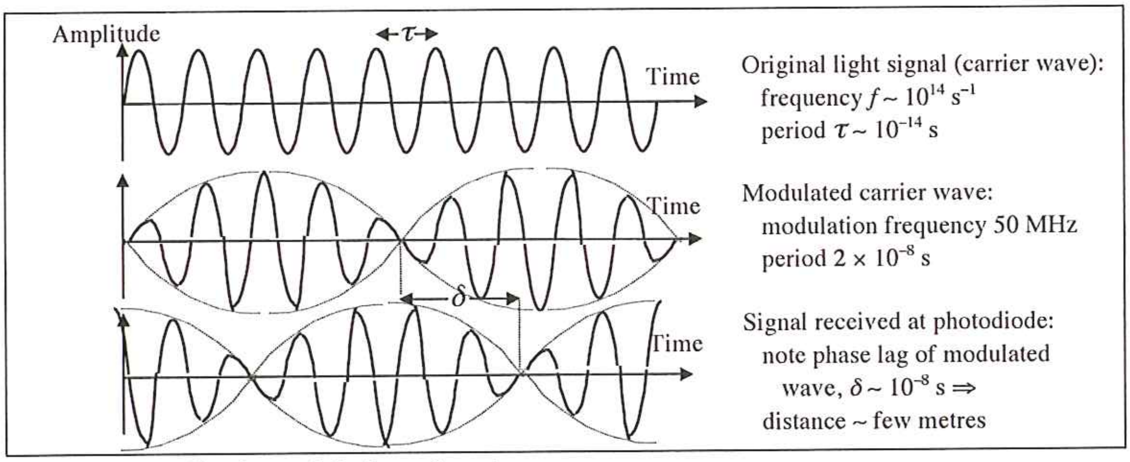

Figure 4 Amplitude modulation of carrier wave

Apparatus

This consists of a box containing the light source, modulator, receiver, and mixer, and a there-and-back path of a few metres for the light. The source is a light-emitting semiconductor diode (LED) which emits red light; a 50 or 60 MHz oscillator modulates the voltage across the LED. The light passes through a lens L1 (see figure 5) whose function is to produce a wide parallel beam which is sent down one arm of an optical bench and back along the other arm after reflection in two 45° mirrors M1 and M2. Another lens L2 focuses the parallel returning light onto a light-sensitive photodiode whose output signal oscillates at 50 or 60 MHz in phase with the returning light. Correct optical alignment is essential in this experiment, and it is worth spending some time getting it right.

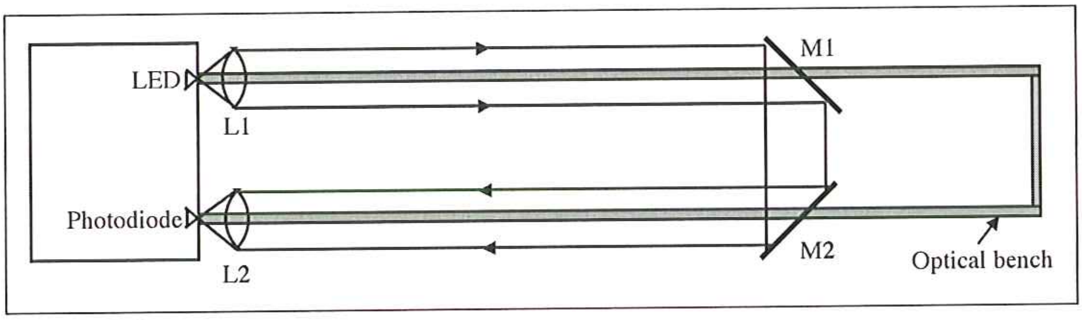

Figure 5 Apparatus

- Place L1 with its centre accurately level with the LED. Let the light that has passed through L1 fall on the screen carrying a circle of the same diameter as L1 and L2. This screen is used to trace the path of the light to the mirrors and back to L2. A truly parallel and aligned beam will exactly fill the circle and remain at the same height all the way along the path. This needs to be done in a darkened lab. Move the screen along the rail to M1 and adjust L1 as necessary to keep the beam parallel and on axis. Repeat with the screen on the return rail, adjusting only M1 and M2 by the screws on their back face (L1 should not need further adjustment if you have been careful) to bring the beam centrally onto L2, which should then be adjusted so as to focus the beam onto the photodiode. Final adjustments can be made by displaying the signals from the heterodyne mixer on the oscilloscope and maximising the response of the receiver relative to the transmitter. Ideally, the received signal should be the same size for all distances of the mirrors.

Measurements

- Use the *XY* mode of the 'scope to display the Lissajous figures formed by the outgoing and the incoming signal. Each mirror distance corresponds to a different phase lag between the two waves, so the figures in general are elliptical. If the sine waves have the same amplitude and are exactly in phase the ellipse becomes a straight line at 45° to the *X* axis; if they are 180° out-of-phase the line slopes the other way. A circle is produced at 90° phase difference. By adjustment of the channel gains you should be able to get 'scope traces of roughly the same amplitude in both *X* and *Y*. There is a phase control knob on the supply unit which allows you to select to some extent the light path which gives no apparent difference in phase (clearly this is necessary since the phase difference is actually zero only when the light path is zero, which is experimentally most inconvenient!). Check that the phase can be changed from 0° to 180° as the mirrors are moved along the length of the bench.

- When you are satisfied, make careful measurements of the spacing between the two mirror positions corresponding to 0° and to 180° phase difference. The extra distance travelled by the light between these two positions is a half-wavelength of the 50 or 60 MHz modulation wave, so the distance the mirrors move is one-half of this, that is $\lambda_m/4$.

- Repeat this measurement with a number of different settings of the phase control knob so as to get a good idea of the variability of the readings and hence their statistical error. Deduce the velocity of light in air. The uncertainty in your value will include a contribution from the statistical error just mentioned, and also a systematic error due to uncertainty about the precise frequency of the 50 or 60 MHz oscillator.

Speed of light in acrylic plastic

Two transparent acrylic rods (Perspex, Lucite and Plexiglas are trade names for this plastic) are to be placed on the outward and return rails close to the lenses. The mirror unit is brought close behind them and the phase control knob adjusted for a straight line display, whether 0° or 180° phase difference is immaterial. The rods are removed and the mirrors moved away until the *same* straight-line Lissajous figure is obtained. The distance between the two mirror positions represents the extra time taken for the light to travel the combined length L of the perspex rods in comparison with the same length of air.

To the light wave, the rods appear to be longer than the same actual length of air. This apparent length due to the slower light speed is called the **optical path**. It is equal to the actual length multiplied by the ratio of light speeds in air (almost the same as vacuum) and plastic, a ratio that as you know is the **refractive index** μ. So the movement S of the mirrors introduces an extra distance $2S$ equal to the difference between the optical path in plastic, μL, and the optical path in the same length of air which is the actual length L:

$$2S = (\mu - 1)L$$

- Make several measurements of the mirror movement needed, using slightly different initial settings, and deduce a value for μ. With enough measurements, ten or more, you will be able to assign a standard error to μ with some confidence. Hence find the velocity of light in the plastic.

The invariance of *c*

Suppose there are two sets of apparatus like this in the lab, at right angles to one another. Careful measurements of the speed of light in a vacuum were carried out by Michaelson and Morley using such a geometrical set-up (but a quite different technique). They had expected to find a difference because of the motion of the Earth, just as the measured sound speed on a windy day depends on the direction of the wind. The motion of the Earth through the 'æther' in which the light waves were thought to be travelling should have produced a similar effect. Michaelson and Morley found no evidence at all for this 'æther wind' effect; your measurements will not be precise enough for you to make such a claim. Independently, Einstein had concluded that the 'æther' is unreal, and that the speed of light is a universal constant independent of the motion of source or observer — a conclusion that led directly to his theory of relativity.