Power in the wind

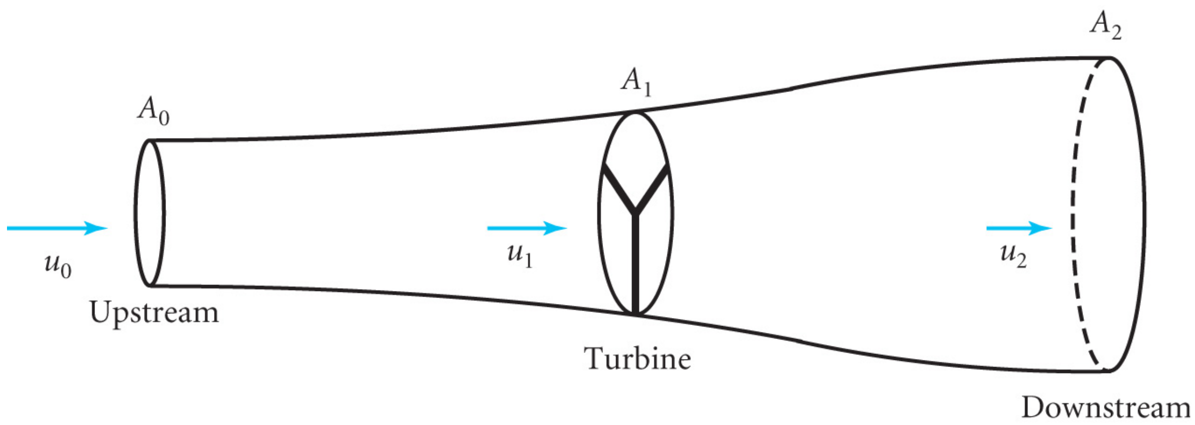
Energy per unit volume, $E = \frac{1}{2}\rho u^2$,

where u is the wind speed and ρ is the density of air.

Power, $P = EuA$, so

$$P = \frac{1}{2}A\rho u^3$$

Maximum extraction efficiency – the Betz limit



Thrust on turbine = rate of change of momentum

$$T = \frac{\mathrm{d}m}{\mathrm{d}t}(u_0 - u_2)$$

Where dm/dt is the mass of air flowing through the stream tube per second.

Power extracted, P = thrust × air speed at turbine

$$P = Tu_1 = \frac{\mathrm{d}m}{\mathrm{d}t}(u_0 - u_2)u_1 \qquad (1)$$

P can also be expressed as the rate of loss of kinetic energy of the wind,

$$P = \frac{1}{2}\frac{\mathrm{d}m}{\mathrm{d}t}(u_0^2 - u_2^2) \qquad (2)$$

Comparing (1) and (2), then

$$(u_0 - u_2)u_1 = \frac{1}{2}(u_0^2 - u_2^2) = \frac{1}{2}(u_0 - u_2)(u_0 + u_2)$$

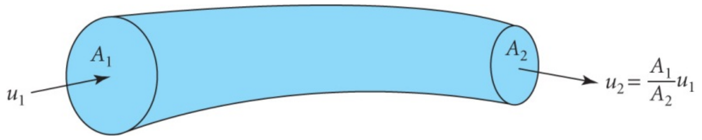
Hence,

$$u_1 = \frac{1}{2}(u_0 + u_2)$$

or

$$u_2 = (2u_1 - u_0) \quad (3)$$

By considering mass continuity (AJ Ch.3) i.e. ρuA=constant, where A=cross sectional area, ρ= density of air.



The mass flow per second dm/dt is given by,

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \rho uA = \rho u_1 A_1 \quad (4)$$

(pressure changes are low so ρ = constant)

Subs. u_2 from (3) and dm/dt from (4) into (1) gives,

$$P = 2\rho u_1^2 A_1 (u_0 - u_1).$$

Let u_1=(1-a)u_0, where a is the **induction factor**. Then

$$P = 2\rho u_0^3 A_1 [4a(1-a)^2]$$

Where the power coefficient C_P is the fraction of power in the wind that is extracted by the turbine,

$$C_P = \frac{P}{\left\{\frac{1}{2}\rho u_0^3 A_1\right\}} = 4a(1-a)^2$$

Setting dC_P/dt to zero gives the maximum power extracted P_{max} when a=1/3 equal to

$$P_{max} = \frac{1}{2}\rho u_0^3 A_1 \{16/27\}$$

Which is ~59% of the power in the incident wind passing through an area equal to that of the turbine, A_1. The limit for the power coefficient C_P of 16/27 of the incident wind power is called the **Betz limit** (or sometimes Lanchester-Betz limit)