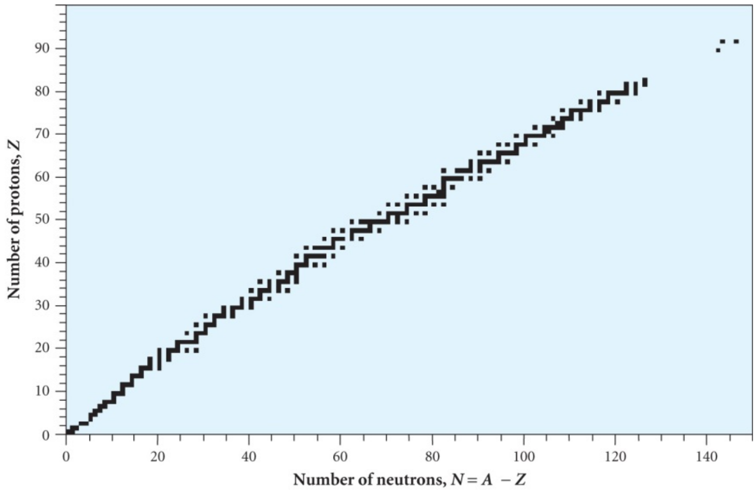
Energy from nuclear fission

Carbon-free
1 tonne uranium (U) delivers equivalent energy to 20,000 tonnes of coal
Average U concentration in Earth’s crust is 2.8 ppm
About 50 years U supply at present rate of consumption

Exploits decay of unstable nuclei, e.g.

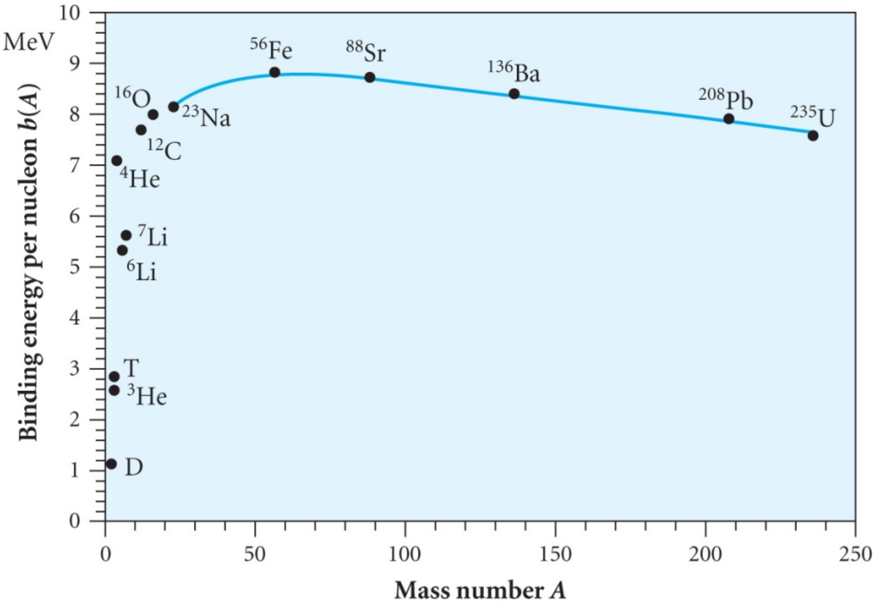
$$^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \mathrm{e}^{-} + \bar{\nu}$$

Beta-day, half-life=5730 years, used in radiocarbon dating.



Binding energy and stability factors: nucleon binding by short-rage attractive force, electrostatic repulsion of protons, neutron-proton pairing, nucleons on exterior of nucleus vs. number in interior,…

Expect binding energy of nucleus to be proportional to *A*, i.e. binding energy per nucleon *b*(*A*) constant. But find there is a maximum at *A*~60 (^{56}Fe).



Decrease for $A>60$ due to electrostatic repulsion between protons. Decrease for $A<60$ more nucleons on the surface of the nucleus therefore less bound.

Energy released by fission of a heavy nucleus or fusion of two light nuclei.

E.g. Uranium (mass number A_1) splits into two lighter nuclei (with mass numbers A_2 and A_3) giving off two neutrons. These nuclei will be neutron rich relative to stable nuclei with the same mass number so undergo beta-decays until stable nuclei are reached. The emitted neutron are absorbed by a nucleus resulting in an energy release equal to the binding energy of the neutron (approximately $b(A)$).

The nucleon in the lighter nuclei are more tightly bound than in the U nucleus so the total energy release E_R is approximately given by,

$$E_R=A_2[b(A_2)- b(A_1)]+ A_3[b(A_3)- b(A_1)]$$

In beta-decays, some energy is taken by the neutrinos, which interact very weakly with matter, the remainder is deposited in the surrounding material.

Ex. N1. Calculate the energy release when ^{235}U *fissions resulting in two lighter stable nuclei with mass numbers 140 and 93 (b(235)=7.6 MeV, b(140)=8.35 MeV, b(93)=8.7 MeV).*

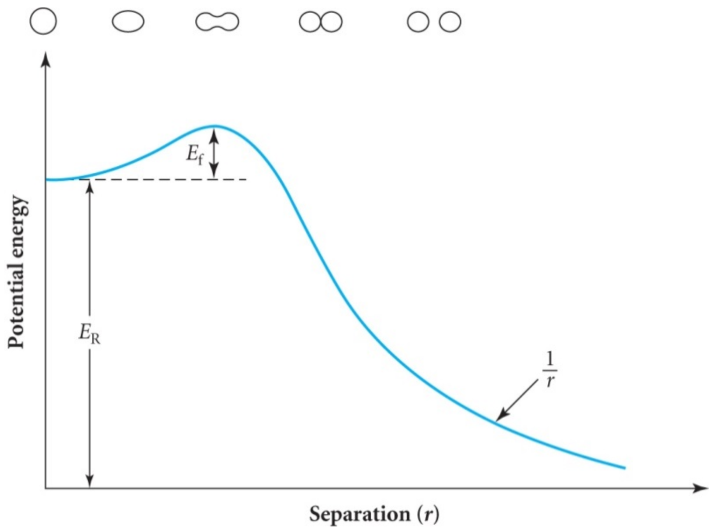
Ex. N2. Calculate the energy released when deuterium ^{2}H *and tritium* ^{3}H *fuse to form* ^{4}He *with the release of a neutron*

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow {}^{4}\mathrm{H} + \mathrm{n}$$

(b(2)=1.1 MeV, b(3)=2.6 MeV, b(4)=7.1 MeV).

Neutron-induced fission

Uranium is stable w.r.t. deformation on its equilibrium shape:



Probability of uranium fission is increased enormously when uranium nucleus captures a neutron

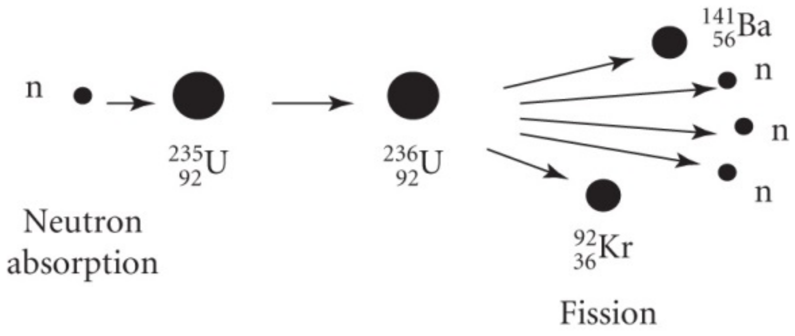
$$n + {}^{235}U \rightarrow {}^{236}U^*$$

The energy of the excited $^{236}U^*$ is above the height of the energy barrier E_f and can therefore fission promptly. By contrast,

$$n + {}^{238}U \rightarrow {}^{239}U^*$$

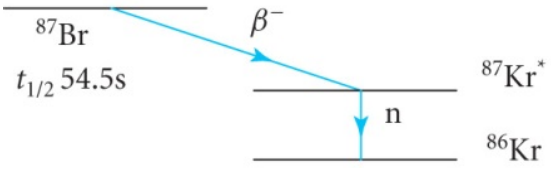
the energy of the excited $^{239}U^*$ is below the height of the energy barrier E_f and cannot therefore fission promptly. This is because the neutron is more tightly bound in ^{236}U than ^{239}U.

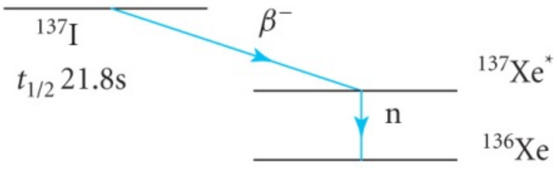
Example of neutron-induced fission of ^{235}U:



$$n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3n$$

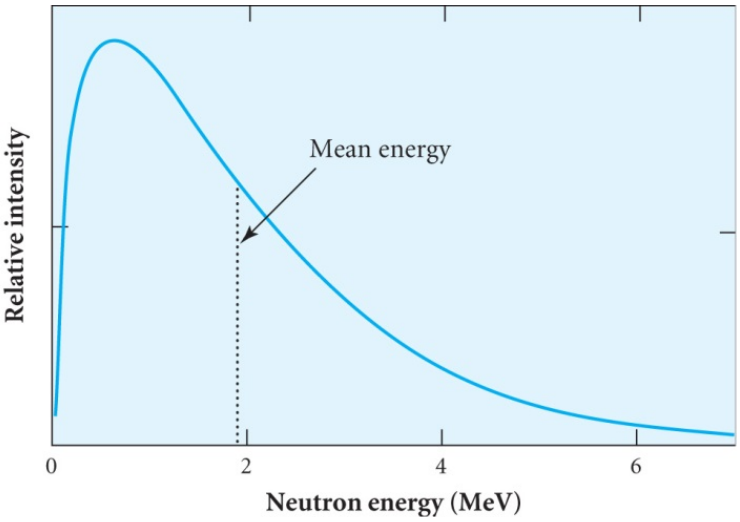
The energy from fission can be divided into **prompt release** and **delayed release** following beta-decay. Examples of beta-delayed neutron emission:





These are very important in controlling the chain reactions in a nuclear reactor.

On average 2.4 neutrons are emitted in the neutron-induced fission of ^{235}U with a broad range of energy (mean energy~2 MeV)



A **chain reaction** is possible if at least one of the released neutrons induces fission of other nuclei.

Ex. N3 How much energy is released when 1 kg of uranium enriched to 3% ^{235}*U is consumed in a nuclear reactor?*

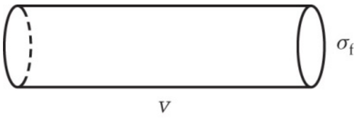
Chain reactions

The neutrons with energies in the range eV (thermal neutrons) - MeV undergo **scattering, capture, and induced fission** with probability expressed as a cross-section, σ (units: barns(b), 1 barn = 10^{-28} m^2). The cross-section of a uranium nucleus is ~2b [NB this is much greater than the cross-sectional area of the nucleus]

Neutron absorption cross-section is the sum of the capture and neutron-induced fission cross-sections,

$$\sigma_a = \sigma_c + \sigma_f$$

Neutron moving with speed v though uranium with n_f ^{235}U nuclei per unit volume then in one second the neutron will sweep out a volume $\sigma_f v$:



If there are n neutrons per unit volume then the fission reaction rate per unit volume is:

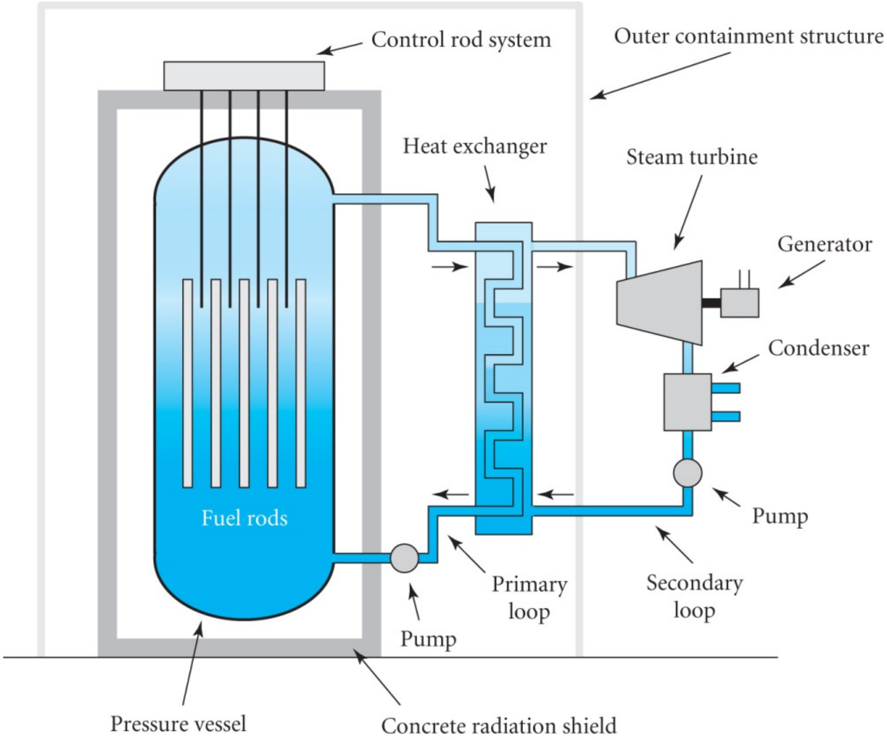
$$R_f = n_f \sigma_f v n = \Sigma_f \varphi$$

Where $\Sigma_f \varphi$ (units: m^{-1}) is called the **macroscoptic cross-section** for neutron induced fission. The **neutron flux** is given by the product nv (typival value=10^{17} $m^{-2}s^{-1}$)

Moderators

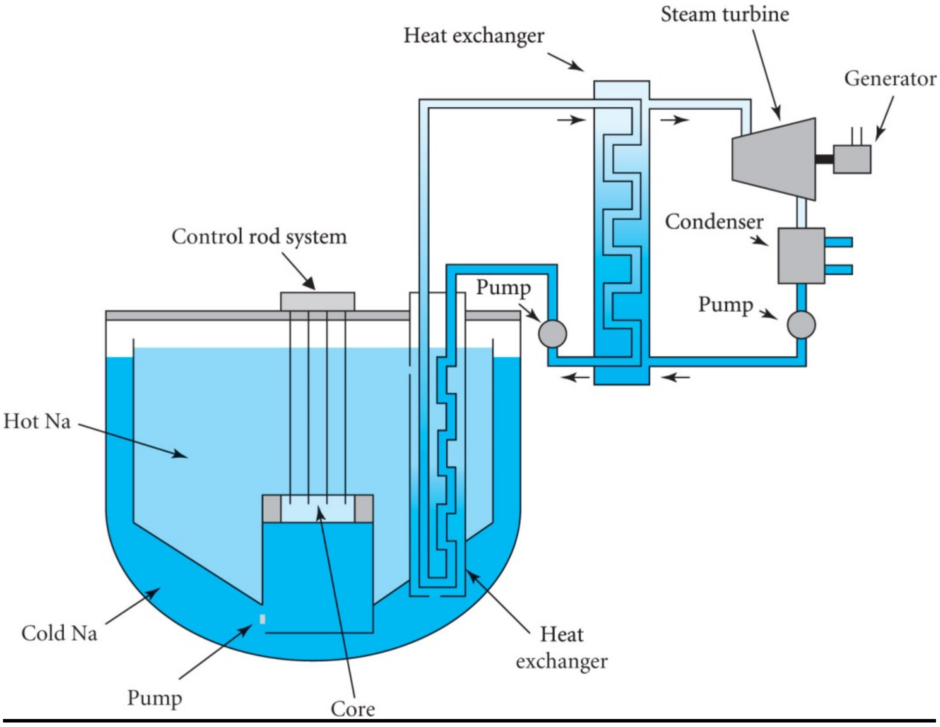
The average energy of neutrons produced by fission needs to reduced by many orders of magnitude to reach thermal energies (meV – eV). This is done by making the fuel in the form of rods and surrounding with a moderator material.

Pressurised water reactor (PWR)



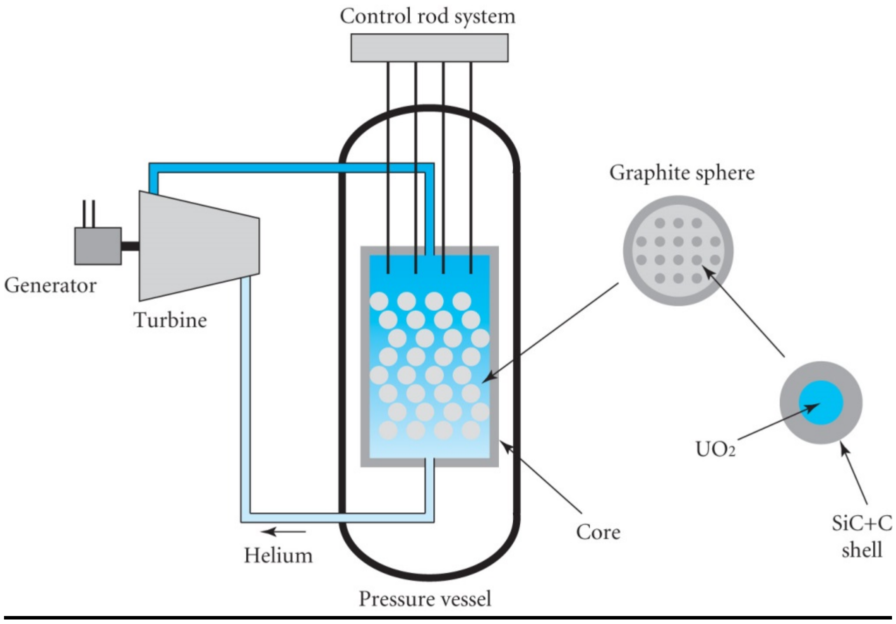
(From AJ, chapter 8)

Sodium-cooled fast reactor



(From AJ, chapter 8)

High temperature gas cooled reactor



(From AJ, chapter 8)

Nuclear waste

High level: waste generated in reactor core; half-life 10^5- 10^6 years; liquid high level waste stored underground for 50 years then vitrified; vitrified waste then sealed in steel containers and put in stable geological repositories.

Intermediate level: intense radioactivity; solidified and stored in containers with concrete shielding.

Low level: low intensity radioactivity; suitable for shallow burial.