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## BSc / MSci Examination

### PHY-231 NUCLEAR PHYSICS

Time Allowed: 2 hours 15 minutes

Date: 13th May 2003

Time: 14:30

**Instructions:** Answer **THREE** of six questions. All questions carry 25 marks.  
The total available marks for each part question are indicated to the right hand side of each question.

**Rubric:** The following data may be useful in answering some of the questions:

Charge of the electron	$e$	$= 1.602189 \times 10^{-19} \text{ C}$
Mass of the electron	$m_e$	$= 9.11 \times 10^{-31} \text{ kg} = 511 \text{ keV}/c^2$
Mass of the proton	$m_p$	$= 1.673 \times 10^{-27} \text{ kg} = 938.272 \text{ MeV}/c^2$
Mass of the neutron	$m_n$	$= 1.675 \times 10^{-27} \text{ kg} = 939.566 \text{ MeV}/c^2$
Atomic mass unit	$1 \text{ u}$	$= 931.502 \text{ MeV}/c^2$
Planck constant	$h$	$= 6.62618 \times 10^{-34} \text{ J s} = 4.13570 \times 10^{-15} \text{ eV s}$
Boltzmann constant	$k$	$= 1.38066 \times 10^{-23} \text{ J K}^{-1} = 8.6174 \times 10^{-5} \text{ eV K}^{-1}$
Speed of light in free space	$c$	$= 2.99792458 \times 10^8 \text{ m s}^{-1}$
Permittivity of free space	$\epsilon_0$	$= 8.85 \times 10^{-12} \text{ F m}^{-1}$
Permeability of free space	$\mu_0$	$= 4\pi \times 10^{-7} \text{ H m}^{-1}$
Avogadro's constant	$N_A$	$= 6.022045 \times 10^{26} \text{ kg-mol}^{-1}$
Rydberg constant	$R$	$= 1.097 \times 10^7 \text{ m}^{-1}$
Bohr magneton	$\mu_B$	$= 9.274 \times 10^{-24} \text{ J T}^{-1}$
Nuclear magneton	$\mu_N$	$= 5.05084 \times 10^{-27} \text{ J T}^{-1} = 3.1525 \times 10^{-14} \text{ MeV T}^{-1}$
Fine structure constant	$\alpha$	$= 1/137$

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- Q1. A nucleus of a radioactive element has a decay constant  $\lambda$ . Show that if there are initially  $N_0$  nuclei of the element the number remaining after time  $t$  is

$$N(t) = N_0 e^{-\lambda t} \quad [5]$$

Define the half-life and mean life of a radioactive nucleus in terms of the decay constant  $\lambda$ . State what is meant by a sequential decay.

[3]

In a sequential decay an initial number of  $N_0$  parent nuclei with decay constant  $\lambda_1$ , decays to a radioactive product nucleus with decay constant  $\lambda_2$ . Write down the differential equation describing the change in the number of product nuclei with time.

[5]

Show that the total number of product nuclei ( $N_2$ ) remaining after a given time,  $t$ , is:

$$N_2(t) = N_0 \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \quad [8]$$

Show that if  $\lambda_1 \gg \lambda_2$  then:

$$N_2(t) \approx N_0 \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t} \quad [4]$$

- Q2. The semi-empirical mass formula (SEMF) can be written as:

$$M(A, Z) = NM_n c^2 + ZM_p c^2 - a_v A + a_s A^{2/3} + \frac{a_c Z^2}{A^{1/3}} + \frac{a_a (A - 2Z)^2}{A} + \frac{\delta}{A^{1/2}}$$

Describe some of the main assumptions in the derivation of this formula and explain briefly the origin of the terms in this formula.

[11]

Using the semi-empirical mass formula show that the energy  $S_n$  required to separate a neutron from the nucleus ( $A, Z$ ) is given approximately by:

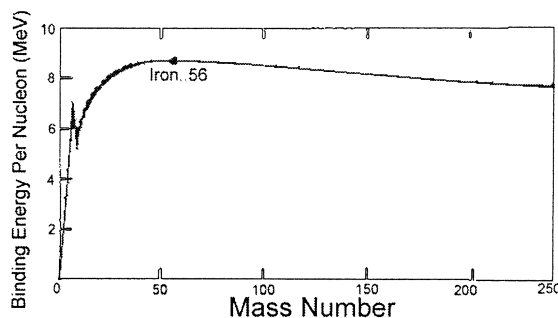
$$S_n \approx a_v - \frac{2}{3} a_s A^{-1/3} - a_a \left[ \frac{1 - 4Z^2}{A(A-1)} \right] \quad [7]$$

By considering values of  $A > 30$ , estimate the mass number of the  ${}_{11}\text{Na}$  nucleus that is just stable against neutron emission. The values of the coefficients in the SEMF are:

$a_v = 15.5$  MeV,  $a_s = 17.2$  MeV,  $a_c = 0.70$  MeV,  $a_a = 23.3$  MeV,  $\delta = \pm a_p$ ,  $a_p = 12.00$  MeV.

[7]

- Q3. A schematic of the binding energy per nucleon versus Atomic Mass Number is shown below.



Describe the process of nuclear fission and by considering the curve, indicate how it is possible that energy can be obtained from nuclear fission

[5]

Define the concept of criticality in a fission reactor and indicate why it is important. Give the factors on which the criticality component  $k$  depends on for a realistic reactor.

[10]

Explain the role of a moderator in a fission reactor and list the three main properties, which are important for a material to be a good moderator.

[5]

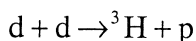
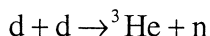
Most nuclear reactors are designed such that they operate sub-critically. Explain what is meant by this term and describe how the reactor can continue to run, operating in this mode.

[5]

- Q4. The elements, which we observe in the universe, were produced in three different ways. Some were produced in the immediate aftermath of the big bang and some are produced in stellar burning and some are produced when a star reaches the end of its life in a supernova explosion. List the main types of nuclei produced in each of the above stages.

[10]

Define what is meant by the  $Q$ -value for a reaction. Calculate the  $Q$  values in MeV for the fusion reactions (The masses,  $M$  of the nuclei are  $M(^2_1\text{H})=2.014102\text{u}$ ,  $M(^3_2\text{He})=3.016029\text{u}$ ,  $M(^3_1\text{H})=3.016049\text{u}$ )



[4]

Assuming the deuterons are at rest find the kinetic energies of the outgoing particles for both reactions.

[6]

Assuming that the deuterons have to come within 100 fm of each other for fusion to precede, calculate the energy that must be supplied to overcome the electrostatic repulsion. Approximately what temperature is this equivalent to?

[5]

Q5. Describe four pieces of evidence that support the idea of shell structure in nuclei.

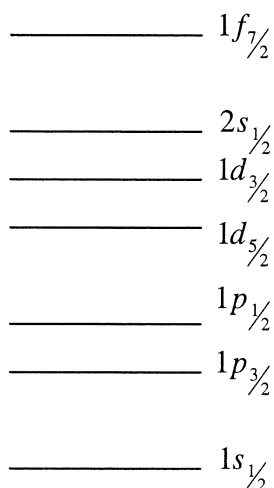
[4]

Give the main assumptions behind the development of the shell model.

[5]

Explain the notation in the diagram below and calculate the multiplicity of the sub-states up to and including the  $1f_{7/2}$  level.

[6]



Estimate the separation of the  $1p_{1/2}$  and  $1d_{5/2}$  energy levels for nuclei with mass number  $A \sim 16$  given the above ordering of the nuclear energy levels and the total binding energy for the oxygen ( ${}_8\text{O}$ ) isotope is

${}^{15}\text{O} \dots\dots 111.96 \text{ MeV}$

${}^{16}\text{O} \dots\dots 127.62 \text{ MeV}$

${}^{17}\text{O} \dots\dots 131.76 \text{ MeV}$

Give the spins and parities of each of the above oxygen isotopes

[10]

Q6. Which fundamental forces are responsible for  $\alpha$ ,  $\beta$  and  $\gamma$  radiations. What do these radiations consist of?

[6]

Considering a nucleus with atomic mass number  $A$  and atomic number  $Z$ , which decays into a daughter nucleus where the energy released, is  $Q$ .

- i) If the decay is via  $\alpha$  radiation, what is the atomic weight and atomic number of the daughter nucleus? Give the particle reaction formula and sketch the energy spectrum of the  $\alpha$  radiation. [4]
- ii) If the decay is via  $\beta^-$  radiation, what is the atomic weight and atomic number of the daughter nucleus. Give the particle reaction formula and sketch the energy spectrum of the  $\beta^-$  radiation. [4]
- iii) If the decay is via  $\gamma$  radiation, what is the atomic weight and atomic number of the daughter nucleus? Give the particle reaction formula and sketch the energy spectrum of the  $\gamma$  radiation. [4]

From knowledge of the strengths of the interactions involved what would you expect the range of relative lifetimes of these processes to be? Describe how the existence of the Coulomb barrier affects the lifetime for alpha-decay and hence explain one of the main reasons why alpha-decay lifetimes are shorter the higher the energy of the alpha-particle [7]