Laboratory Exercise 9 – FUNDAMENTAL and SUBATOMIC PHYSICS

The first and last parts of this exercise are concerned with two subatomic elementary particles — the **electron**, which is stable, and the **pion**, an unstable particle that lives fleetingly before it decays. Both measurements are based on the concept of momentum, so although the middle section on kinematics may seem incongruous, all three parts are linked by similar principles.

Part A: The charge-to-mass ratio of the electron, *e/m*

Introduction

A force acts on a charged particle moving in a magnetic field, tending to push the particle sideways. [The same force is responsible for the movement of a current-carrying conductor between the poles of a magnet, the basis of electric motors.] The force acting on an electron of charge e and mass m moving with velocity v in a field B is equal to Bev. Introducing the momentum $p = mv$, this becomes Bep/m. If the magnetic field is constant the electron undergoes a constant sideways acceleration. That corresponds to motion in a circle. Equating the centripetal force to the product of mass and inward acceleration v^2/r gives

$$\frac{Bep}{m} = \frac{mv^2}{r} = \frac{p^2}{mr}$$

so the momentum is

$$p = Bmr\left(\frac{e}{m}\right)$$

The kinetic energy of the electron comes from acceleration through a potential difference (voltage) V. Equating potential energy lost to kinetic energy gained gives

$$eV = \tfrac{1}{2}mv^2 = \frac{p^2}{2m}$$

Substituting for p we find

$$\left(\frac{e}{m}\right) = \frac{2V}{B^2r^2}$$

so the charge-to-mass ratio can be found by measuring the radius of the circular path, and knowing both the magnetic field and the accelerating voltage.

The apparatus comprises a spherical bulb or 'tube', with 'guns' to produce and control a narrow beam of electrons, surrounded by a pair of **Helmholtz coils** for producing the magnetic field within the bulb. There are separate power supplies for tube and coils.

Electron beam tube

The electrical features, shown in figure 1, are: a **cathode** which produces electrons, an **anode** which accelerates them through voltage V, and an additional **deflector** electrode which can steer the beam slightly. Connections are made through the base and neck of the tube. The tube is almost completely evacuated, allowing the electrons to travel freely with few collisions with gas atoms. A small quantity of helium is present; its atoms emit greenish light if ionised by collision, so making the beam visible when the ambient light level is low. Part of the glass bulb is coated with luminescent paint. There are two independent electron beams selected by a switch on the base of the tube, one directed across a diameter of the bulb, the other directed tangentially upwards. You will use the tangential beam, which can be bent into a complete circle by the magnetic field. Before switching anything on identify all the components, trace the circuit connections, and follow the instructions carefully.

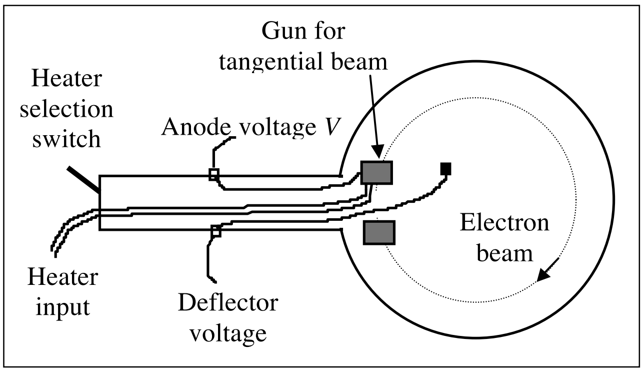

Figure 1 Electron beam tube

Each electron gun comprises an indirectly heated cathode and a conical anode with a small hole to allow the electrons to emerge; the deflector electrode nearby controls either beam. There are separate controls and meters for the anode (0–300 V) and the deflector electrode (0–50 V). When the power supply is switched on, current flows through the heater of whichever electron gun is selected. The heaters must *not* warm up while there is an accelerating voltage on the anode, so *before switching on make sure that the anode voltage is zero* by turning the control fully anticlockwise. ***Failure to do this can cause serious damage.*** Likewise, *reduce the anode and deflector voltages to zero before switching off.*

Helmholtz coils

A pair of coils arranged as in figure 2 and carrying equal currents I produce a substantially uniform magnetic field in the region between them, which is where the electron beam is produced. The essential feature of this **Helmholtz pair** geometry is that the separation of the coils is equal to their average radius R. If there are N turns of wire on each coil then the magnetic field intensity is

$$B = \frac{32\pi NI}{R\sqrt{125}} \times 10^{-7} \text{ teslas}$$

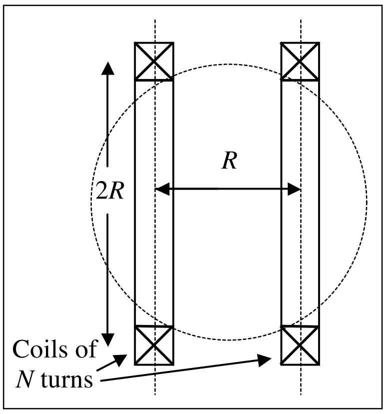

Figure 2 Helmholtz coils

These coils are wound with 320 turns of enamelled copper wire, and their mean diameter is 13.6 cm. Hence their radius is 6.8 cm. Make sure, using convenient spacers or in other ways, that the coils are parallel and that the separation between the middle of their coil windings is also 6.8 cm. Currents up to 1 A are provided from a separate power supply.

Measurements

• With the Helmholtz coil supply off and the anode voltage zero, switch on the electron tube supply. ***Wait one minute*** before applying voltage to the anode. Raise the voltage slowly. At about 50 V you will see the first sign of the beam, by 70 V it should have enough energy to cross the tube, and by 100 V it is a bright filament. Switch on the Helmholtz coil supply and increase the current. If the beam is a spiral rather than a circular arc, the coils and the tube may be slightly misaligned. This is difficult to correct (with this apparatus) but you might try turning the whole assembly around — the extra deflection may be due to the effect of a local magnetic field. A small spiral deflection doesn't matter. A voltage can be applied to the deflector plates to make sure that the electron beam leaves the anode in the right direction, but you will probably not need to use this control much.

• Measuring the diameter of the circular beam is difficult. You can hold a transparent ruler in front of the tube, or try placing a mirror behind the tube and lining up a ruler with its reflection, or even try a travelling microscope if the electron beam is narrow.

When combined, the expressions for the value of e/m and for B yield

$$\frac{1}{d} = \left(\frac{16\pi N}{5 \times 10^7 R} \sqrt{\frac{e}{10mV}} \right) I$$

where d is the diameter of the circular beam. Hence a graph of $1/d$ versus coil current I, for constant accelerating voltage V, should yield a straight line from whose slope e/m can be found. (Note that I should be plotted on the x-axis and $1/d$ on the y-axis because we know I quite precisely, while $1/d$ has substantial measurement errors.)

• Use at least two values of V, but preferably three or four, starting near 100 V and going no higher than about 270 V. For each value of V, increase I and measure d for each of about ten diameters, aiming for about 5 mm intervals. The largest value should have the beam skimming the phosphorescent screen, the smallest can be about 40 mm. The beam may be very blurred, so its centre line is uncertain. Keep a close check on V and correct it if it tends to drift.

• If you are having problems obtaining data as above because d is difficult to measure, try keeping d constant while varying V and I to give a sufficient number of data points.

Analysis

• Plot $1/d$ versus I for each value of V. From the gradient of each of your graphs deduce a value of e/m. Average these, and estimate the uncertainty of your answer. Considering the difficulty of the d measurement, this experiment can give results remarkably close to the accepted value of the electron e/m, which is 1.76×10^{11} C kg^{-1}. For example, from readings taken with anode voltages of 90 V and 180 V a Course Organiser obtained 1.78×10^{11} and 1.69×10^{11} C kg^{-1} for lines drawn through the origin.

• If all of the magnetic field comes from the Helmholtz coils, then your straight lines should pass through the origin. Consider whether or not this is the case. If it is not, then the electrons are being bent even when $I = 0$, possibly by a small and constant additional magnetic field B_0 due to the Earth's field or to iron in the lab. If I is such that it produces a field B equal and opposite to B_0 then there will be no bending, so d is infinitely large and $1/d$ is zero. This corresponds to where the straight line crosses the x-axis. To deduce B_0, use the value of this x-intercept: set B_0 equal to the value of B from the Helmholtz coils and so show that at this point

$$B_0 = -\frac{32\pi N}{\sqrt{125} \times 10^7 R} I$$

• Use this expression to deduce B_0 if your data appear to justify it.

Part B: Kinematics on a linear air track

Introduction

The linear air track has a number of holes through which air is blown to support metal riders or 'vehicles'. It provides an almost friction-free means of studying collisions and investigating the principles of classical mechanics as formulated by Newton. What we study here are the properties of objects in motion, that is **kinematics**. Kinematics is an extension of **statics** (bodies at rest) and is concerned with laws that apply whatever the forces that cause the motion. The study of the forces and their effects is **dynamics**.

This neat distinction works excellently in everyday life, but Einstein showed that it is an approximation only true in our sluggish low-speed world. In the relativity theory which has replaced Newtonian mechanics as an *exact* theory, statics does not exist because nothing is at rest in every frame of reference. Einstein's kinematics is more complicated than Newton's, but if the dynamics is formulated carefully it contains the same great fundamental laws of classical mechanics — the laws of conservation of energy and momentum. The linear air track is a good place to study these.

Setting up

The manufacturer's booklet has a description of the apparatus. For measuring the speed of the vehicles we use a lamp and a light-sensitive detector, arranged so that a card on the vehicle breaks the light beam for a time which is measured digitally to an accuracy of 1 ms. There are two of these systems per track so that the speed of two vehicles can be measured independently.

• Horizontal levelling is critical. After a rough adjustment using a spirit level, the air supply should be turned on and the final levelling carried out with an unloaded vehicle as an indicator. It is not enough simply to bring a moving vehicle to rest by altering the levelling, as this will over-compensate. After each adjustment the vehicle should be brought to rest by hand and any tendency for the vehicle to move one way or the other further compensated. Appendix 2 of the manufacturer's booklet shows that a difference in level of no more than 0.25 mm between the ends can be tolerated.

• The speed of the vehicles is measured by a photoelectric timing technique. Set up a lamp with a focusing lens on one side of the track and the light sensitive detector (photodiode) on the other, 10 to 15 cm away. The electronic timers are configured so that they start to count when the beam is broken by the card carried by a vehicle, and stop counting when the card has passed. To do this connect the light source to the supply and connect the START terminals together with a shorting lead, connect the photodiode to the STOP terminals, set MODE to 1, and select TIME. Adjust the light beam if necessary so that the timer is switched off when light falls on the photodiode, and on when it doesn't. Then push the vehicle, with card mounted, into the beam and note the positions where counting just starts and where it just stops. The distance the vehicle has moved is the *effective* length of the card. Do this for each vehicle.

Friction as a cause of speed loss

A moving vehicle bouncing back and forth between two tightly stretched rubber bands will eventually come to rest. It loses its kinetic energy both through viscous drag (friction) from the air pad supporting it, and also through loss of energy in each collision with the rubber bands. The second is not likely to be too important, especially as in most measurements the rebound is not recorded, but the first will cause the vehicle to slow down at a steady rate so it will introduce a definite uncertainty into most measurements. We must therefore study the speed loss in detail.

• Appendix 2 of the booklet shows that the effect of friction is to reduce the vehicle's speed after travelling a distance s from its initial speed v_0 to a lower speed $v = v_0 - Ks$, where K is a constant term involving the viscosity of air and the dimensions and mass of the vehicle. The effect of speed loss at rebounds is not dealt with in the Appendix, but you should try to **show** that if the same fraction of the vehicle's kinetic energy is lost at each rebound then the velocity after n rebounds will be $v = v_0 e^{-Cn}$, where C is another constant.

• Position a photodiode assembly about halfway along the track. Release a vehicle from one end by catapulting it at a moderate speed (less than 1 m/s) and measure its speed on each traverse. Plot speed versus n, the number of rebounds. If *friction* were the only cause of energy loss we would expect a linear relation between v and n (which is proportional to s). If the *rebounds* were the only cause of energy loss we would expect an exponential relation, in which case a graph of $\log v$ versus n would be a straight line. Probably both effects occur.

• Inspect your graph to decide whether it makes sense to treat the high-speed data as being dominated by rebound energy loss and the low-speed data by frictional energy loss. If so (experience suggests it does!) draw a straight line through the low-speed data, extrapolate it back to the origin, and find K from the gradient. The *difference* Δv between the measured and the extrapolated speed at each of these points represents the effect of rebound loss, which drops to virtually nothing after a few rebounds.

• Plot $\log(\Delta v)$ versus n to confirm this, and find C, the fraction of velocity lost in each rebound.

• The constant K is the loss in speed per metre of track, due to friction. You can use it either to correct your results, which might be quite difficult, or to estimate the error in speed measurements made over a given length of track.

Conversion of potential to kinetic energy

Your earlier work in physics, as well as exercise 2, has taught you that energy is needed to stretch a rubber band, that the energy stored is proportional to the area under the force/extension graph, and that if this graph is linear the area is proportional to the square of the total extension. On the air track this energy can be converted to kinetic energy $mv^2/2$, where m is the mass, allowing a check of these kinematic relations.

Here is an analysis of what to expect. Let x_0 be the unstretched length of rubber band, x its length when stretched between supports, and D be the difference between x and x_0. From Hooke's law the potential energy stored is proportional to D^2. When you pull back the band you extend it by an additional amount d, say, giving it a total extension of $(D + d)$. The potential energy stored is now proportional to $(D + d)^2 = (D^2 + 2Dd + d^2)$. So the difference should be proportional to the kinetic energy $mv^2/2$ transferred to the vehicle when it is released.

• If you substitute typical values for x and x_0, and use Pythagoras' theorem to find a typical value for d, you may find that the third term in this bracket, d^2, is small compared to the first two terms (show this!). This is convenient because if we drop the third term we find

$$\tfrac{1}{2}mv^2 \propto 2Dd$$

suggesting that a graph of v^2 against d should be a straight line. If on the other hand d^2 is not small enough, then it will be necessary to plot v^2 against $d^2 + 2Dd$. (This might be easiest using Excel.)

• Measure the velocity with which the vehicle is launched for various extensions of the rubber band, using the adjustable screw to position the band precisely. Take several readings at each extension to see how reproducible they are. Catapulting off-centre will introduce wobble; ignore any obviously wobbly launches. The extension of the band is calculated from the geometry of the triangle formed by the unextended band and the two halves of the extended band. Plot v^2 against d (or $d^2 + 2Dd$, see above) and comment.

• Couple two vehicles together, and confirm that for equal catapult extensions the kinetic energies $mv^2/2$ of all combinations are equal, in other words that v is proportional to $1/\sqrt{m}$.

• These measurements show, at best, that the kinetic energy acquired by the vehicle is *proportional* to the potential energy of the stretched band; they do *not* show equality. The value of the kinetic energy for a particular launch is easily calculated, given the mass of the vehicle. The potential energy can be measured by hanging a scale pan on a rubber band and finding the weight needed to produce the corresponding extension. Make this comparison for several extensions, and comment on the results.

Elastic collisions

This phrase is used to describe collisions with **no loss of energy**. Two vehicles fitted with repelling magnetic buffers will collide almost elastically as long as their relative speed is low enough — if not there will be an audible 'clink' as they collide.

• Place one large vehicle at rest in the centre of the track, launch the other towards it, and observe the sequence of collisions. If these, and the rebounds at the ends, were perfectly elastic the sequence would continue for ever. As it is, more energy will probably be lost in a rebound at one end than at the other, and the vehicles will eventually come to rest at that end of the track.

Your observations should clearly suggest that in a perfectly elastic collision between two equal masses, one of which is at rest, *all* the energy of the moving mass is transferred. The moving mass comes to a complete halt and the other rebounds at the speed of the mass which struck it. This is an example of the law of **conservation of momentum**: the initial momentum is mv and is concentrated in the moving mass, the other having no momentum. The conservation law states that the final momentum must also be mv. The law doesn't indicate how this momentum is shared between the two vehicles, but since their masses are equal they could share the speed v in any proportion as long as their velocities add up to v. *But* the total energy must still be $mv^2/2$, and this too can be shared in any proportion as long as the squares of the velocities add up to v^2. These two requirements, one from conservation of momentum and one from conservation of energy, are impossible to fulfil together unless one of the final velocities is zero. (Try it — convince yourself that if $a + b = c$ *and* $a^2 + b^2 = c^2$ then either a or b *must* be zero.)

• No actual measurements of speed were needed yet. Now replace the launch vehicle with the lighter one and catapult it towards the heavy vehicle at rest. Measure the speed v of the launch vehicle before collision and the speed V of the struck vehicle after collision. Use the laws of conservation of energy and momentum, as above, to calculate what the velocity V should be in terms of v and the masses of the two vehicles. Check the validity of this calculation for a range of initial velocities, making corrections for loss of speed due to friction if you think it necessary.

• Repeat, with the roles of the heavy and light vehicles reversed. These measurements are best done by two students working together.

Inelastic collisions

These involve some loss of kinetic energy, usually by heat and sound. Any deformation of the colliding vehicles will heat them up and cause loss of kinetic energy, which is why the phrase **rigid body** is often used in mechanics; it is an ideal non-deformable object that always collides perfectly elastically [hardened steel ball-bearings are close to perfectly elastic objects]. The easiest way to make the air-track vehicles collide inelastically is to fill the hollow end of one with plasticine and use a buffer with a pin on the other so that the two vehicles stick together when they collide. The squashy plasticine absorbs a lot of the kinetic energy. Add plasticine to each vehicle as necessary to keep it balanced when it slides, and so that the two heavy vehicles still have the same mass.

• Repeat the sequence of measurements under **elastic collisions**. Your data should show that although kinetic energy is not conserved in the collisions (we've made sure of that), *momentum is still conserved*. This is a very important result. Whereas energy can take many forms, is sometimes difficult to account for, and is eventually dissipated away as random heat (as you see even on the air track), momentum is recognisably the same quantity in all branches of physics and is not dissipated away. You might ask where the momentum of the catapulted vehicle came from. Can you provide a satisfactory answer?

Explosions

• The opposite of collisions, objects coming together, are explosions, objects flying apart when kinetic energy is released. Momentum is conserved in explosions, too. Hold two vehicles fitted with magnetic buffers close together so that they repel one another. With practice it is possible to release the vehicles from rest so that they fly apart without wobbling. When you can do this, measure their respective speeds for various ratios of mass. The ratios 1:1, 3:2, 2:1 and 4:1 are all possible. Check the conservation of momentum in each case.

Part C: The decay of the π-meson

Introduction

The **π-meson** is an unstable particle which exists for a few tens of nanoseconds before breaking up into two elementary particles. If the π-meson is electrically charged these particles are a **neutrino** and a **muon**, the neutrino being nearly massless and neutral (hence its name) and the muon having the same charge as the π-meson, either $+e$ or $-e$. In such transmutations of elementary particles the total energy is conserved, as always, but in calculations one has to be careful to include not only the kinetic energy but also the energy associated with the mass of the particles according to the Einstein relation $E = mc^2$. As we have seen in part B, momentum also is always conserved. In this exercise you will study a number of examples of break-up or **decay** of charged π-mesons; by applying the laws of conservation of energy and momentum the **mass** of the π-meson can be found.

The photographs

You are given some photographs of happenings in a bubble chamber, which is in a magnetic field so that charged particles move in spirals. The pictures show the gently curving tracks of a beam of positively charged π-mesons entering at the left, some of which stop in the chamber and then decay to positive muons (which are visible) and neutrinos (which are not). After travelling a short distance (due to receiving some kinetic energy from the decay of the π-mesons, which are heavier) the muons come to rest and also decay, after about 2 microseconds. The muons decay into three other elementary particles: two neutrinos, which are nearly massless, and a positron (the anti-particle of the electron) — of these only the positrons are visible in the chamber.

• Identify the decay sequences π-meson → muon → positron, which form unmistakable ‘hooked’ patterns in which the muons leave short tracks between the ends of the π-meson tracks and the start of the distinctive sharply-curving spiral tracks of the positrons. These curl up to smaller radius as they lose energy, and hence momentum, in the liquid filling of the chamber.

Applying the conservation laws

We now show that all that is needed to calculate the mass M of the π-meson is the kinetic energy T of the muon, together with a knowledge of the muon’s mass m. (In the same way only the velocity and mass of one of the air track vehicles was needed in order to calculate the complete kinematics of the ‘explosions’ in the last section of part B.)

First, however, we must modify the laws of classical mechanics so they work in the relativistic world of elementary particles travelling at nearly the speed of light. If a positron were travelling at a low speed v it would have kinetic energy $T = mv^2/2$ and momentum $p = mv$, and the relation between the two would be $T = p^2/2m$. But the positrons are travelling almost as fast as light, so their rest-mass energy must also be considered. The appropriate expression in relativistic mechanics for the total energy E of the positron is

$$E^2 \quad m^2c^4 \quad p^2c^2$$

which comes from applying Pythagoras' theorem to a triangle whose sides are the rest-mass energy mc^2 and the momentum times velocity of light, pc — see figure 3(a). [This expression is in fact *always* true, but in our sluggish non-relativistic world we usually don't realise it. Consider figure 3(b), where an arc corresponding to rest-mass energy mc^2 has been marked on the hypotenuse, the kinetic energy T making up the rest of the total energy. Then

$$\begin{aligned} T &= E - mc^2 \\ &= \sqrt{m^2c^4 + p^2c^2} - mc^2 \\ &= mc^2\sqrt{1 + p^2/m^2c^2} - mc^2 \\ &= mc^2\left(1 + p^2/2m^2c^2 + \ldots\right) - mc^2 \end{aligned}$$

by binomial expansion; if the momentum p is very small only the first term of the expansion is needed, and the expression simplifies to $T = p^2/2m$, which is the familiar result already quoted.]

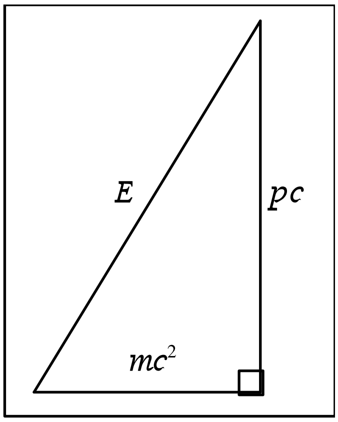

Figure 3(a)

The relation $E^2 = m^2c^4 + p^2c^2$ gives the *total* energy E, including the rest-mass energy, in terms of the momentum p of a particle, and its mass; c is the speed of light. But the total energy E of the muon is also equal to the sum of its kinetic and rest-mass energies, $T + mc^2$. Since the neutrino has nearly zero mass its total energy can be taken to equal pc, where p is its momentum. Equating the rest-mass energy of the π-meson to the sum of the muon's and neutrino's energies (conservation of energy) gives

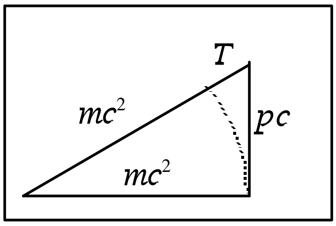

Figure 3(b)

$$Mc^2 \quad T \quad mc^2 \quad pc$$

The π-meson at rest has no momentum, so (conservation of momentum) the momentum of the neutrino must be equal (and opposite) to the momentum of the muon:

$$p = \frac{\sqrt{E^2 - m^2c^4}}{c}$$

After substituting $E = T + mc^2$ and eliminating p between the equations for conservation of energy and momentum we find

$$Mc^2 = mc^2 + T\left(1 + \sqrt{1 + \frac{2mc^2}{T}}\right)$$

showing as was stated that only T and m are needed to find M. The muon mass m is known from measurements to be 1.88×10^{-28} kg.

- You must find T from the photographs.

Finding the kinetic energy of the muon

A charged particle travelling in a bubble chamber loses energy by ionising the atoms it passes — the tracks you see are the trails of bubbles that form round each of these clusters of ionisation. The greater the rate of ionisation the faster the initial kinetic energy is lost and the shorter the track. Experiments have shown that the length L of the track of a particle with charge $\pm e$ and mass m is related to its initial kinetic energy T by the expression

$$T = 6\times10^{-12}\sqrt{L} \text{ joules}$$

Thus a measurement of the muon track length L, expressed in metres, gives T.

All the muons start off with the same kinetic energy and there is little variation in their rate of energy loss, so it should be clear that all the muon tracks should be the same length. But on your photographs they are obviously *not* all the same length. Each photograph shows only the projection of the track onto the plane of the film. The tracks of muons travelling towards or away from the camera will be foreshortened, so you have the task of reconstructing the actual track length from many projected tracks of different lengths. The actual track length is clearly longer than most of the tracks you measure. The Appendix shows that this problem has a simple mathematical solution. You measure a large number of track lengths L, find their average $\langle L\rangle$, and multiply by 1.273. The result is your best estimate of the actual track length.

Measurements

• Study about 15 photographs, which should yield about a hundred clear muon tracks. Use the scales printed on the clear acetate sheet to measure the lengths of as many as possible, taking care not to omit some very short tracks which are clearly extremely foreshortened; to do so would introduce a bias towards long tracks.

• Deduce a value for M, the mass of the π-meson. The accepted value is 2.49×10^{-28} kg.

APPENDIX

Consider a number of line segments each of length l, oriented at random in space as the muon tracks are. Suppose one of them makes an angle θ with the perpendicular to the plane of the film. Its projected length is $l\sin\theta$, which is the length L that you measured. There is an equal chance of the line segments pointing into any small element $\mathrm{d}\Omega$ of solid angle, so the mean value of $L = l\sin\theta$ is found by integrating it over all solid angles and dividing by the total solid angle:

$$\langle L\rangle = \langle l\sin\theta\rangle = \frac{\int l\sin\theta\,\mathrm{d}\Omega}{\int \mathrm{d}\Omega} = \frac{\pi}{4}\,l$$

Derive this result for yourself. Hence $\langle l\rangle = (4/\pi)\langle L\rangle = 1.273\langle L\rangle$, as stated above.