Laboratory Exercise 8 – LIGHT AND OTHER ELECTROMAGNETIC WAVES

In the three parts of this exercise you will study some of the properties of electromagnetic waves. Whatever their wavelength, all e.m. waves travel at the same speed in a vacuum, can be reflected, refracted and scattered, and show interference effects. All these and more will be dealt with in detail in future physics courses; here we demonstrate some of them using two very different wavelengths, from 10^{-10} m (X-rays) through 10^{-6} m (visible light). The techniques used are different but the phenomena you will see are the same, although on quite different scales.

Part A: Refraction of light

Introduction

Newton found that a glass prism separated white light into its spectral colours. In this part you will carry out a refined version of Newton's experiments using a **prism spectrometer** to measure the deviation of light of different wavelengths when it passes through a prism. The property of bending light is called **refraction**, and is measured by the **refractive index**, μ. (The refractive index is actually the ratio of the speed of light in a vacuum to the speed in the transparent medium.) When μ, and hence the angle of bend, varies with the wavelength λ the effect is called **dispersion**. Every transparent material exhibits dispersion. In some optical instruments this can be a nuisance, producing coloured effects when white light is used, but in the prism spectrometer a large dispersion is useful because it improves the instrument's **resolution**, or ability to separate two close wavelengths.

The refractive index of colourless transparent materials decreases as the wavelength increases. This behaviour is called **normal dispersion**, although we now know that the opposite behaviour, naturally called anomalous dispersion, is just as common. In 1836 the mathematician Cauchy suggested that normal dispersion was well described by the expression:

$$\mu = A + \frac{B}{\lambda^2}$$

Here A and B are constants for the material. In fact Cauchy added a third term on the right, C/λ^4, but this is very small and usually ignored.

In this part you will check Cauchy's formula using a number of spectral lines whose wavelengths are accurately known, and you will do it for glass prisms of both low and high dispersion. Although there are some exceptions, denser glass has higher refraction, and greater dispersion, than less dense glass and the commercial names for different glasses (Light Crown, Extra Dense Flint and so on) reflect this. The usual way to describe the optical properties of spectrometer glass is to list not the Cauchy constants A and B, but some combination of the refractive indices at certain specified wavelengths. By measuring these you should be able to use a table of glass types to find out what type of glass your prisms are made of.

The refractive index is measured as follows. Light passing through the prism is deviated by an angle D. As the prism is rotated some position is found at which D has a minimum value, $D_{\min}$, the **angle of minimum deviation**. Textbooks on optics show that

$$\mu = \frac{\sin \tfrac{1}{2}\left(A + D_{\min}\right)}{\sin \tfrac{1}{2} A}$$

where A is the angle of the refracting edge of the prism — see figure 1. Light passes symmetrically through the prism when it is refracted through the angle of minimum deviation.

Setting up the spectrometer

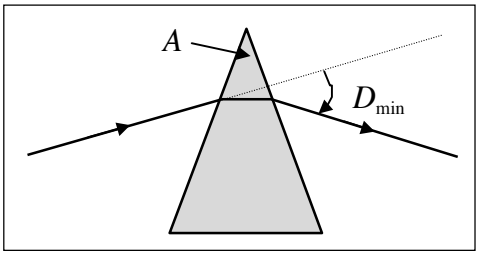

Figure 1 Definitions of A and D_{min}

This instrument is capable of considerable precision when properly adjusted, so it is worth spending a little time doing it carefully. The aim is to make light from the slit parallel as it passes through the **collimator** and to bring this parallel light to a focus on the cross-wires of the **telescope**'s eyepiece. The prism on its table must refract this parallel light in a plane perpendicular to the common rotation axis of table, collimator and telescope. Follow each step of the following simplified procedure in sequence — refer to figures 1, 2 and 3.

• Looking through the telescope eyepiece against a bright white background, bring the cross-wires into sharp focus.

• Carefully take the instrument into the main lab. Adjust the objective lens of the telescope relative to the eyepiece/cross-wire combination so that the image of a distant object on the horizon is sharply focused on the cross-wires. The telescope is now adjusted.

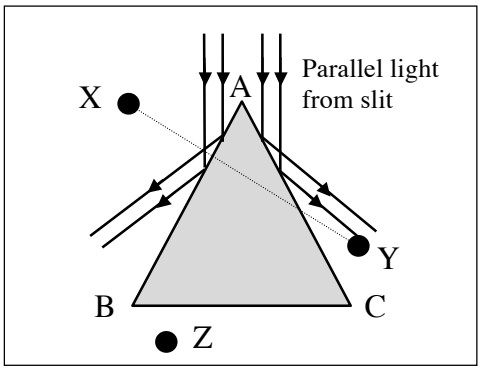

Figure 2 Setting up the spectrometer

• Illuminate the slit with a sodium lamp and view it through the telescope in the straight-through position, without a prism. Adjust the collimator to give a sharp image of the slit, and make this as narrow as possible while still passing light along its whole length. Make sure the slit is precisely vertical. The collimator is now adjusted.

• Place the prism on the table with its refracting edge pointing to the collimator and one of its refracting faces AB perpendicular to the line XY joining two of the levelling screws, as in figure 2. View the slit by reflection in AB, and by adjusting X and Y centre the image on the telescope's cross-wires. Repeat this for reflections in the face AC but this time adjust *only* screw Z. Check the reflection in AB and make small adjustments to X and Y, going through the sequence again if necessary until both faces reflect the light centrally down the telescope. The prism on its table is now adjusted.

NOTE: *The prisms are precision optical pieces. Handle them only by the top and bottom triangular surfaces. Do not touch the refracting faces. If they get finger-marked, ask for an optical wipe and clean them carefully.*

Hint: In what follows you need to view an image of the slit after the light has reflected off, or passed through, the prism. **Always first** push the telescope out of the way and use your naked eye to see the image, **then** when you know what to look for and roughly where it is, look through the telescope. *The commonest cause of frustration in optical measurements is squinting through an instrument when it's pointing in the wrong direction.*

Measuring the refracting angle *A*

• With the prism as shown in figure 2 and the prism table securely clamped, set the image of the slit, reflected in face AB, as accurately as you can across the intersection of the cross-wires, clamping the telescope and using the fine movement screw for the last delicate adjustment. Record *both* vernier scales which register the telescope's angular position. Unclamp the telescope and set it equally carefully to view the slit reflected in face AC, and again record its angular position on both scales. The *difference* between the two settings is $2A$. Take the average of your two determinations.

Measuring the angle of minimum deviation

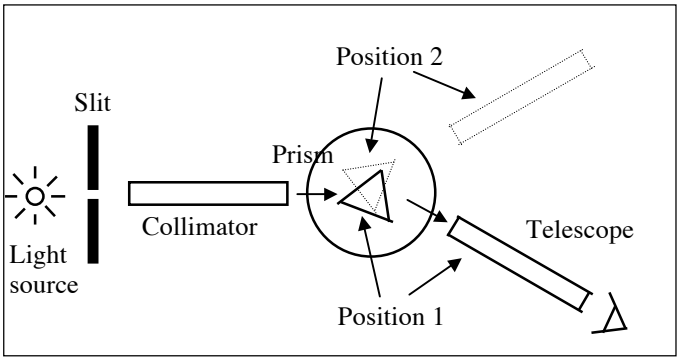

Figure 3 Measuring angle of minimum deviation

• Rotate the prism table to a position like position 1 in figure 3, where you judge the light to be passing roughly symmetrically through it. Use your naked eye to see the image of the slit — the yellow spectral lines of sodium will be prominent but you will also see fainter lines of other colours. Using the telescope, view the yellow lines (you may be able to resolve the two lines — if so choose the one of shorter wavelength, which is deviated more than the other) while you rotate the prism table back and forth; the image will move towards and then away from the straight-through position. It is at its most forward position when the light undergoes minimum deviation.

• Clamp the table and use the fine adjustment screw to set the precise position of minimum deviation. Set the cross-wires exactly on the image and check again that the deviation is a minimum. Record the vernier readings of both telescope scales.

• Now rotate the prism table to position 2, where the deviation is in the opposite direction, and repeat the measurements on this side.

• The angle turned through by the telescope between positions 1 and 2 is $2D_{min}$. Take the average of your determinations and, with your knowledge of A, find the refractive index for the yellow sodium lines.

• Replace the sodium lamp with mercury, cadmium and hydrogen lamps, and repeat your measurements of D_{min} for the brighter spectral lines from each. The wavelengths can be obtained from the laboratory technicians. Plot μ versus λ ***as you take your measurements***. [Students who do this later and find that a point lies well off a smooth curve because they have misidentified a spectral line have no excuse. *Take longer in the lab* to calculate and plot graphs.]

• By means of another suitable graph, attempt to verify Cauchy's relationship.

Dispersion in different glass

• Replace the first prism by the second, made of a different, denser, glass. Repeat the measurements you have just made so as to obtain values for the refractive index for the following spectral lines: sodium at 589.0 nm (the D line), hydrogen at 486.1 nm (the F line), hydrogen at 656.3 nm (the C line). Hence calculate the **reciprocal dispersive power** V:

$$V = \frac{\mu_D - 1}{\mu_F - \mu_C}$$

for both prisms. With this information on the optical properties of the two types of glass, attempt to find a closely similar type in the table in Kaye and Laby.

Part B: The velocity of light

Introduction

In this exercise you will measure the velocity of a beam of light of wavelength λ about 500 nm, both in a vacuum and when travelling through transparent plastic. The methods used in exercise 7 are no use here. As you know, the speed of light, c, is about 3×10^8 m/s, so substitution in $c = f\lambda$ yields a frequency f of about 6×10^{14} Hz, which is too high to be measured by an electronic frequency meter even if the very short wavelength could be measured accurately. The trick here is to vary the intensity of the light source at a frequency very much less than f. The change in intensity is carried along by the light wave, travelling at the same speed c and appearing as a sinusoidal variation of amplitude with a very much longer wavelength than that of the **carrier wave** itself. In other words, the carrier wave is **modulated** by the lower frequency signal — see figure 4. [This is the way that AM (amplitude modulation) radio works. The signal to be transmitted is impressed on a constant *frequency* carrier wave as a change in *amplitude*. In FM (frequency modulation) radio the *amplitude* of the carrier wave stays the same but its *frequency* is modulated.]

The modulation frequency used here is 50 MHz or 60 Mhz, depending on which apparatus you are using — if in doubt, check with a demonstrator. **Deduce** the corresponding modulation wavelength λ_m. The light is received by a photodiode detector whose response follows the modulation at 50 or 60 MHz. Even this reduced frequency is too high to be displayed on the oscilloscopes we use, so another electronic technique is used to reduce the frequency still further. A separate oscillator unit is tuned to a frequency slightly different from 50 or 60 MHz, e.g. 59.9 MHz. When this signal is mixed with a 60 MHz signal from the light source or photodiode the two 'beat' together producing a combined signal whose frequency is the *difference* of the two — this is called the **heterodyne** technique. The mixed signal has a frequency of ~100 kHz and can easily be displayed. Figure 4 shows that a 50 or 60 MHz signal which has travelled only a few metres to the receiver will be considerably out of phase with the signal from the source. A feature of the heterodyne technique is that it preserves this phase relation, so a measurement of the phase difference between the mixed low-frequency signal from the source and the mixed low-frequency signal from the receiver is a direct measure of the phase lag in the 50 or 60 MHz signal. Converting this phase difference to a fraction of the modulation period gives the time for the wave to travel a known distance to the receiver, and hence the wave velocity.

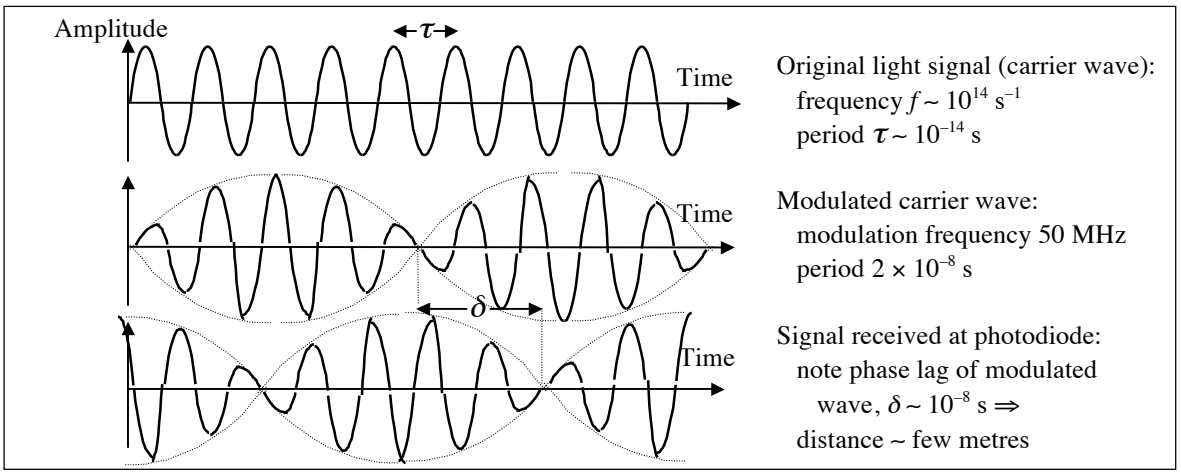

Figure 4 Amplitude modulation of carrier wave

Apparatus

This consists of a box containing the light source, modulator, receiver, and mixer, and a there-and-back path of a few metres for the light. The source is a light-emitting semiconductor diode (LED) which emits red light; a 50 or 60 MHz oscillator modulates the voltage across the LED. The light passes through a lens L1 (see figure 5) whose function is to produce a wide parallel beam which is sent down one arm of an optical bench and back along the other arm after reflection in two 45° mirrors M1 and M2. Another lens L2 focuses the parallel returning light onto a light-sensitive photodiode whose output signal oscillates at 50 or 60 MHz in phase with the returning light. Correct optical alignment is essential in this experiment, and it is worth spending some time getting it right.

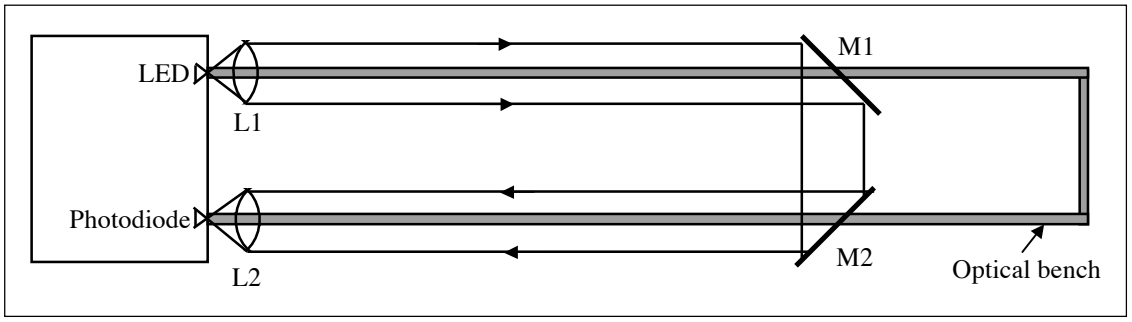

Figure 5 Apparatus

• Place L1 with its centre accurately level with the LED. Let the light that has passed through L1 fall on the screen carrying a circle of the same diameter as L1 and L2. This screen is used to trace the path of the light to the mirrors and back to L2. A truly parallel and aligned beam will exactly fill the circle and remain at the same height all the way along the path. This needs to be done in a darkened lab. Move the screen along the rail to M1 and adjust L1 as necessary to keep the beam parallel and on axis. Repeat with the screen on the return rail, adjusting only M1 and M2 by the screws on their back face (L1 should not need further adjustment if you have been careful) to bring the beam centrally onto L2, which should then be adjusted so as to focus the beam onto the photodiode. Final adjustments can be made by displaying the signals from the heterodyne mixer on the oscilloscope and maximising the response of the receiver relative to the transmitter. Ideally, the received signal should be the same size for all distances of the mirrors.

Measurements

• Use the *XY* mode of the 'scope to display the Lissajous figures formed by the outgoing and the incoming signal. Each mirror distance corresponds to a different phase lag between the two waves, so the figures in general are elliptical. If the sine waves have the same amplitude and are exactly in phase the ellipse becomes a straight line at 45° to the *X* axis; if they are 180° out-of-phase the line slopes the other way. A circle is produced at 90° phase difference. By adjustment of the channel gains you should be able to get 'scope traces of roughly the same amplitude in both *X* and *Y*. There is a phase control knob on the supply unit which allows you to select to some extent the light path which gives no apparent difference in phase (clearly this is necessary since the phase difference is actually zero only when the light path is zero, which is experimentally most inconvenient!). Check that the phase can be changed from 0° to 180° as the mirrors are moved along the length of the bench.

• When you are satisfied, make careful measurements of the spacing between the two mirror positions corresponding to 0° and to 180° phase difference. The extra distance travelled by the light between these two positions is a half-wavelength of the 50 or 60 MHz modulation wave, so the distance the mirrors move is one-half of this, that is $\lambda_m/4$.

• Repeat this measurement with a number of different settings of the phase control knob so as to get a good idea of the variability of the readings and hence their statistical error. Deduce the velocity of light in air. The uncertainty in your value will include a contribution from the statistical error just mentioned, and also a systematic error due to uncertainty about the precise frequency of the 50 or 60 MHz oscillator.

Speed of light in acrylic plastic

Two transparent acrylic rods (Perspex, Lucite and Plexiglas are trade names for this plastic) are to be placed on the outward and return rails close to the lenses. The mirror unit is brought close behind them and the phase control knob adjusted for a straight line display, whether 0° or 180° phase difference is immaterial. The rods are removed and the mirrors moved away until the *same* straight-line Lissajous figure is obtained. The distance between the two mirror positions represents the extra time taken for the light to travel the combined length L of the perspex rods in comparison with the same length of air.

To the light wave, the rods appear to be longer than the same actual length of air. This apparent length due to the slower light speed is called the **optical path**. It is equal to the actual length multiplied by the ratio of light speeds in air (almost the same as vacuum) and plastic, a ratio that as you know is the **refractive index** μ. So the movement S of the mirrors introduces an extra distance $2S$ equal to the difference between the optical path in plastic, μL, and the optical path in the same length of air which is the actual length L:

$$2S = (\mu - 1)L$$

• Make several measurements of the mirror movement needed, using slightly different initial settings, and deduce a value for μ. With enough measurements, ten or more, you will be able to assign a standard error to μ with some confidence. Hence find the velocity of light in the plastic.

The invariance of *c*

Suppose there are two sets of apparatus like this in the lab, at right angles to one another. Careful measurements of the speed of light in a vacuum were carried out by Michaelson and Morley using such a geometrical set-up (but a quite different technique). They had expected to find a difference because of the motion of the Earth, just as the measured sound speed on a windy day depends on the direction of the wind. The motion of the Earth through the 'æther' in which the light waves were thought to be travelling should have produced a similar effect. Michaelson and Morley found no evidence at all for this 'æther wind' effect; your measurements will not be precise enough for you to make such a claim. Independently, Einstein had concluded that the 'æther' is unreal, and that the speed of light is a universal constant independent of the motion of source or observer — a conclusion that led directly to his theory of relativity.

Part C: X-ray diffraction

Introduction

In this part we study what happens when electromagnetic waves pass through a regular three-dimensional array of small scatterers. The effects were first studied by the Braggs, father and son, who shone X-rays on crystals and found strong reflections in some directions but nothing elsewhere. They interpreted this as the effect of scattering not just from individual atoms but from whole sheets of atoms lying in parallel planes a distance d apart (see figure 6). When the glancing angle θ is such that the extra distance travelled by the X-rays between successive sheets of atoms, $2d\sin\theta$, is $n\lambda$, a whole number of wavelengths, all the scattered X-rays are in phase

and interfere constructively. Thus a large signal is seen in that direction as if the sheets of atoms had, like mirrors, reflected the X-rays. But this is not specular reflection — an enhanced signal is seen *only* in those special directions for which the so-called **Bragg condition**

$$2d\sin\theta = n\lambda$$

is satisfied. Knowing λ, the atomic spacing d can be found from these **X-ray diffraction** studies. This is the basis of much modern crystallography and molecular biology.

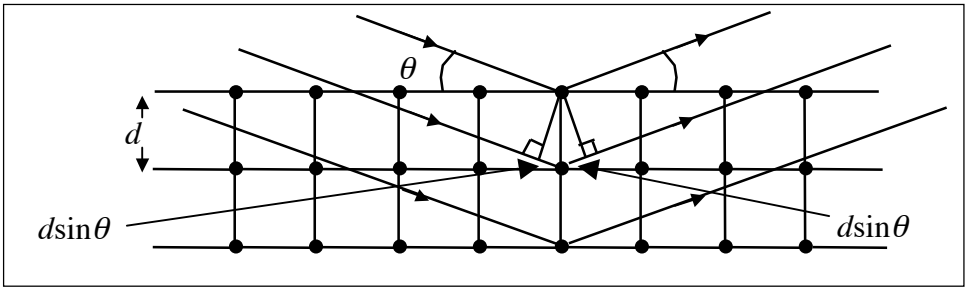

Figure 6 Scattering by a crystal

In this part you are given an X-ray diffraction pattern that has been recorded photographically. The pattern is produced by shining a narrow and tightly parallel (within 0.1°) X-ray beam on a thin wire of (in this case) tungsten, comprised of millions of tiny crystals lying in completely jumbled and random orientations (figure 7). The majority of these crystals are not in any special position and the beam passes through them unscattered. By chance, though, a few will be within about 0.1° of one of the Bragg angles, θ, relative to the direction of the beam, and these will give Bragg reflections at an angle of 2θ to the beam. So these reflected X-rays will travel outwards along a cone of apex angle 4θ, and will be recorded where they strike a strip of film wrapped around a cylindrical tube. The X-ray beam enters and leaves this tube through holes, and the whole arrangement is an **X-ray camera** for **powder diffraction** (the word 'powder' simply meaning that the target is not a single crystal but is made up of many tiny crystals). You can see on your film that the images are actually curved, since they are sections of a conical surface.

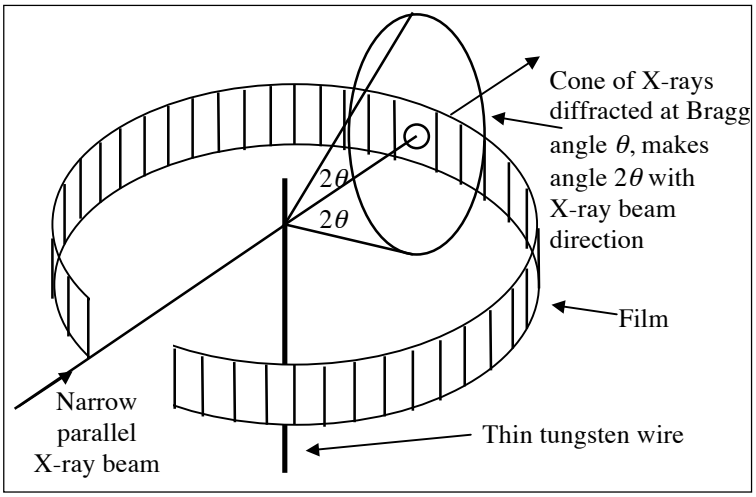

Figure 7 Powder diffraction

Measurements

The diameter of the camera used to take these photographs was 57.30 ± 0.02 mm. A measurement of the spacing between corresponding images to left and right of the beam hole is a measurement of the arc length around the original cylinder which, divided by the camera's radius, will give the apex angle 4θ in radians. Note that the diameter of the camera is carefully chosen so that exactly *half* of the left-right separation, measured in mm, is numerically equal to the angle θ in degrees (check that you understand this).

• ***Do not remove the film*** from its envelope. Tape it down onto a sheet of paper, fix an accurate ruler along the equatorial line of the film, and measure the coordinates of corresponding lines to the left and to the right. Try to estimate distances to one-fifth of a millimetre division. Tabulate your readings and evaluate the angle θ for each line. Note that the *sum* of the left and right

readings for each line should be constant — the extent to which it varies gives you an idea of the measurement inaccuracy and hence the error in the results.

Evaluation

You will have about eight values of θ, each corresponding to Bragg reflection. All these are first-order, i.e. $n = 1$. Each corresponds to a different value of d, the atomic spacing. To see this study figure 8, which shows a square crystal lattice of side a in just two dimensions. Besides the lines of atoms, a apart, running vertically and horizontally, there are other lines, less densely packed with atoms, running at angles as shown. A little geometry using Pythagoras' theorem will convince you that the spacings between these various sets of lines are all of the form

$$d = a/\sqrt{h^2 + k^2}$$

where h and k are small integers — they are the number of repetitions of the simple **unit cell** that defines the direction of the lines. In the examples shown:

$h = 1$ and $k = 0 \Rightarrow d_1 = a/1 = a$
$h = 1$ and $k = 1 \Rightarrow d_2 = a/\sqrt{2}$
$h = 1$ and $k = 2 \Rightarrow d_3 = a/\sqrt{5}$
and so on.

A similar rule applies in three dimensions. The spacings between sheets of atoms responsible for each of the Bragg reflections are:

$$d = a/\sqrt{h^2 + k^2 + l^2}$$

where h, k and l are all small integers. Inserting this expression into the Bragg equation with $n = 1$, squaring and rearranging gives

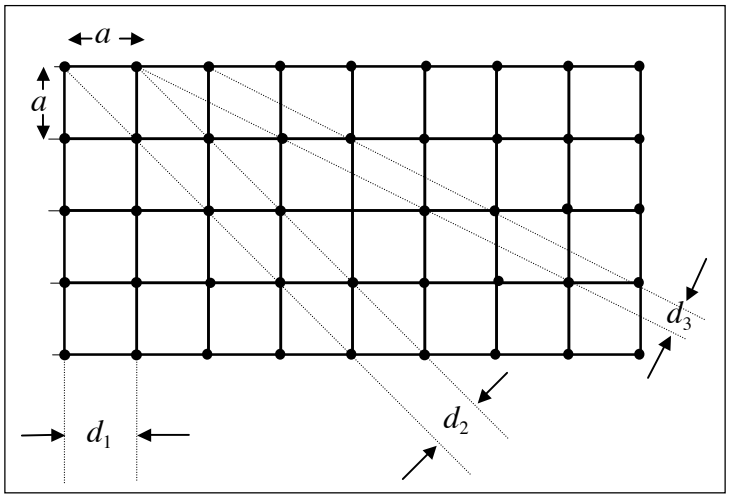

Figure 8 Unit cells

$$\sin^2\theta/\left(h^2 + k^2 + l^2\right) = \lambda^2/4a^2$$

That is, when divided by the appropriate small integer $(h^2 + k^2 + l^2)$, each value of $\sin^2\theta$ gives the same value, namely $\lambda^2/4a^2$. Hence knowing λ, the atomic spacing a can be found.

• You can do this by trial and error, finding what sequence of integers gives the same value, within errors, for each diffraction image. Or you can seek, again by inspection, the **highest common factor** of your $\sin^2\theta$ values — make an informed guess at the number which will divide each of them an (almost) exact whole number of times. Having found the highest common factor roughly, divide each entry in your table by the *exact* whole number, which is the value of $(h^2 + k^2 + l^2)$ for that line, and average the quotients. (For reasons which do not concern us here, not all such numbers are possible.)

• The X-rays used to take this photograph had wavelengths of 1.540562×10^{-10} m and 1.544390×10^{-10} m, one being rather more intense than the other. You may be able to distinguish separate Bragg reflections from the two wavelengths. Deduce the atomic spacing a in tungsten.