Laboratory Exercise 7 – MEASUREMENTS IN ASTRONOMY

## Part A: The angular resolution of telescopes

Introduction

For astronomical observations, reflecting telescopes have replaced the refracting type of instrument. Large mirrors with great light-gathering power are lighter, more rigid and easier to control than equivalent lenses; lenses are also plagued by chromatic aberration which brings different colours to a focus at different points. One of the largest refractors made, the 28" at the Old Greenwich Observatory, is a beautiful instrument but suffers badly from chromatic aberration as you will find if you get an opportunity to use it. Both reflectors and refractors, however, have an intrinsic limit to their resolution, that is to their ability to distinguish two objects which appear very close together in the sky with a very small angular separation. This quality of a telescope, called its **angular limit of resolution**, or just its **resolution**, is determined by its aperture — the larger the aperture, the greater the resolution (and hence the smaller the angular separation that can be resolved). The object of this part of the exercise is to set up a reflecting telescope, to measure its resolution, and to compare this both with that of the unaided eye and also with an estimate from theory. An example of an angular measurement in astronomy completes this part.

Adjusting the telescope

The primary mirror of a reflecting telescope is a paraboloid, which brings parallel light from a distant object to a focus in front of it. It is therefore necessary to move the image out of the incident beam so that it can be observed without obscuring the incoming radiation. In the Newtonian arrangement this is achieved by reflecting the light off a small flat mirror set at 45° to the telescope axis and just in front of the focal plane (see figure 1). The image is then observed by an eyepiece which acts essentially as a microscope.

• The components of the telescope can be clamped to the triangular section of rigid steel, the 'optical bench'. Measure the **focal length** of the primary as accurately as you can (the error should not be greater than 1 cm) by using a distant light as a source and finding the distance of its image from the vertex of the mirror. View this image using a paper screen moving along the triangular beam.
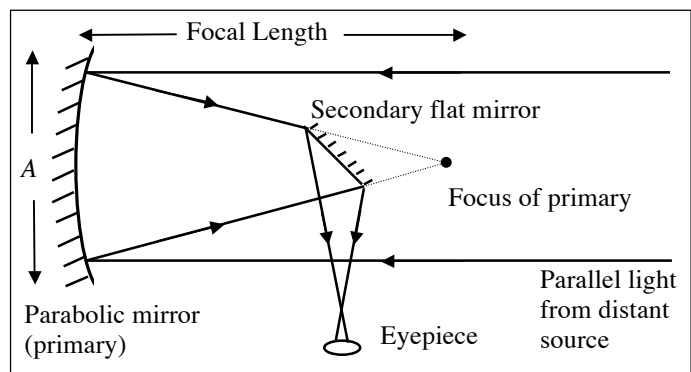
**Figure 1** Reflecting telescope

• Measure the diameter of the primary mirror (its **aperture**, A) and also the smaller diameter of the secondary mirror.

• Place the secondary at a position such that when it is at an angle of 45° the image is seen clearly in the eyepiece. (It may help to look first without the eyepiece and barrel and adjust so that your eye is seen reflected.) Ideally, the image of light reflected from the primary should completely fill the secondary.

• Make a scale drawing like figure 1 to see whether this is the case. Make any final adjustments while viewing the distant light source through the eyepiece.

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Measurements

• Use the resolution test card to measure the resolving power of both the telescope and your eye. Place the card in the laboratory as far as convenient from the telescope (at least 10 m if possible). View the sets of lines through the telescope and determine the set of smallest line spacing which can be resolved into individual lines. Use the ability to determine the direction of the lines correctly as evidence for resolution. Use the travelling microscope provided to measure the distance d between consecutive line centres in the set which can just be resolved. Measure also the distance D between the test card and the primary mirror. Hence calculate the small angle $\theta = d/D$ which is the experimental angular limit of resolution.

• By similar measurements, find the angular limit of resolution of your eye — you may wish to try this for left and right eyes separately as well as together. Do this wearing spectacles or contact lenses if you normally do so.

Note that this expression for θ gives the angle in units of **radians** rather than in degrees; recall that θ (radians) = (arc length)/radius (see figure 2). Thus 2π radians corresponds to a full circumference, or 360°, and so a radian is approximately 57.3°. A measure commonly used in astronomy in the **arcsecond**, or 1/3600th of a degree. Therefore, one radian is approximately 2.06×10^5 arcseconds.
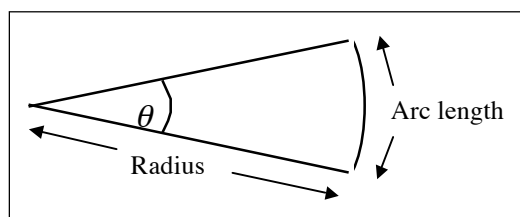
Figure 2 Definition of radian

Theoretical estimates of angular resolution

Light, being a wave motion, must be treated by a theory which includes the effects of **diffraction**, that is the bending of wave trains as they go around obstacles or as they pass through apertures (as you see on a large scale when sea waves spread out on entering a harbour mouth). The primary mirror is an obstacle to the incoming light waves so they are diffracted to some extent, thus smearing out and confusing the images of two sources having a small angular separation. The smallest angular separation, θ_{min}, which can just be distinguished (that is, the resolution) is determined by the ratio between λ, the wavelength of the light, and A, the diameter of the primary mirror. The ability to resolve overlapping images is a rather personal thing and there are several prescriptions (criteria) for the exact formula to use. One of them, the **Rayleigh criterion**, is based on the mathematical form of the diffraction patterns from circular apertures: it states that $\theta_{min} = 1.22\, \lambda/A$. Other prescriptions such as the **Abbé criterion** (which omits the factor 1.22) are sometimes used.

• Taking λ to be 550 nm, near the peak of the spectral curve for white light, use these criteria to calculate the theoretical angular resolution of the telescope and of the eye (for which the aperture is the diameter of the pupil, typically 3.0 mm). Compare the results with your experimental values.

Angular measure and the distance of astronomical objects

Astronomers measure angular positions in the sky. As the Earth circles the Sun the position of a nearby star shifts relative to that of more distant stars since its direction in the sky changes slightly. Figure 3 shows that in six months the position angle changes by an amount equal to the diameter of the Earth's orbit divided by the star's distance. Therefore, this distance equals the radius of the Earth's orbit divided by the deviation of the position angle from its average value (a quantity called the **parallax** of the star). The distance will be in kilometres if the Earth's orbital radius is also given in kilometres and the parallax is in radians. Astronomers find it more convenient not to bother with the exact value of the orbital radius but simply to call it an

Astronomical Unit (AU), and also to measure angles in arcseconds (the parallax of most stars is less than 1 arcsecond). Using these units the distance of a star is measured in **parsecs** (pc) — the distance in parsecs is the reciprocal of the parallax in arcseconds. Thus 1 pc = 2.06 × 10⁵ AU, or approximately 3 × 10¹³ km.
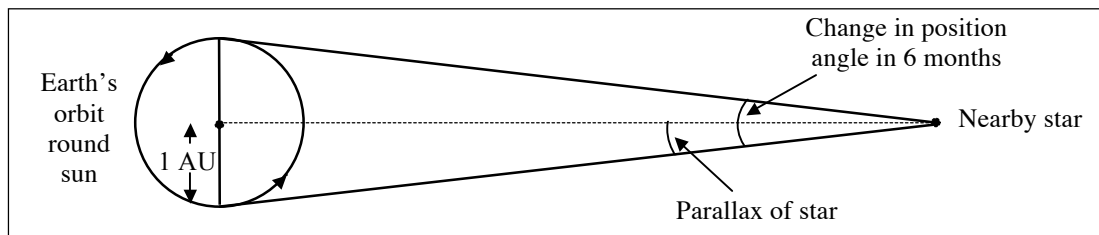
Figure 3 Parallax

• Besides their annual parallax, some stars show a real movement (**proper motion**) relative to more distant stars. Figure 4 shows two photographs of the same area of the sky, taken 10 years apart. One star, whose parallax is known to be 0.55 arcseconds, has moved appreciably during this time. Find it, measure the distance it has moved (estimate to a quarter of a millimetre using a good graduated rule), convert this distance to an angle using the fact that the photographs measure 40.5 arcminutes (1 arcminute = 60 arcseconds) in the horizontal direction, and calculate the velocity of the star across the line of sight.

## Part B: Line spectra, chromatic resolution and doppler shifts

Introduction

The spectrum of light from stars contains many sharp features, light or dark bands called **spectral lines**, that tell astronomers about the chemical composition and physical conditions on its surface. This is done by comparison with simple spectra produced in the laboratory. In this part you will use a diffraction grating to produce the line spectra of a number of elements, and will investigate how close in wavelength two lines can be before they become indistinguishable. This property is called the **chromatic resolution** of the instrument or sometimes, when there is no possibility of confusion with angular measurements (see part A) just the **resolution**. The better the resolution, the more closely one can investigate small wavelength changes caused, for example, by the Doppler effect. Finally, you will measure wavelength shifts in the spectrum of a star system that are actually due to relative motion of the stars.

The grating spectrometer

In this part you use a diffraction grating mounted on a precision rotating table. A parallel beam of light of wavelength λ striking one side of the grating will be **diffracted**, that is, deviated in all directions, as it passes through the apertures between the opaque lines of the grating. The transmitted light will interfere constructively (see exercise 1) at angles determined by λ and by the spacing d between the lines. In these directions (the **principal maxima**) the transmitted light intensity is greatest and a telescope set at these angles will see images of the source, a narrow slit parallel to the grating lines, in the colour of the wavelength selected. The expression for the angles of these principal maxima is:

$$\sin\theta_n = \frac{n\lambda}{d}$$

where the number $n = 1, 2, 3, \ldots$ is called the **order** of the principal maximum. The grating you will use has a small value of d so the right-hand side is large; since $\sin\theta_n$ must be less than one, only the first few orders will be visible.

40.5 arcminutes
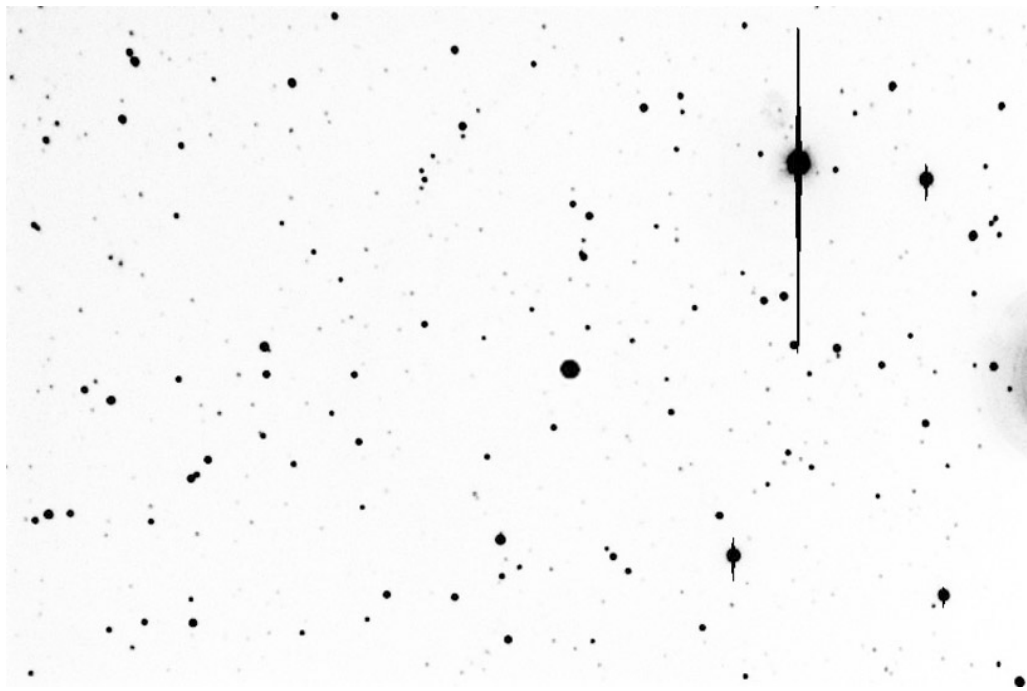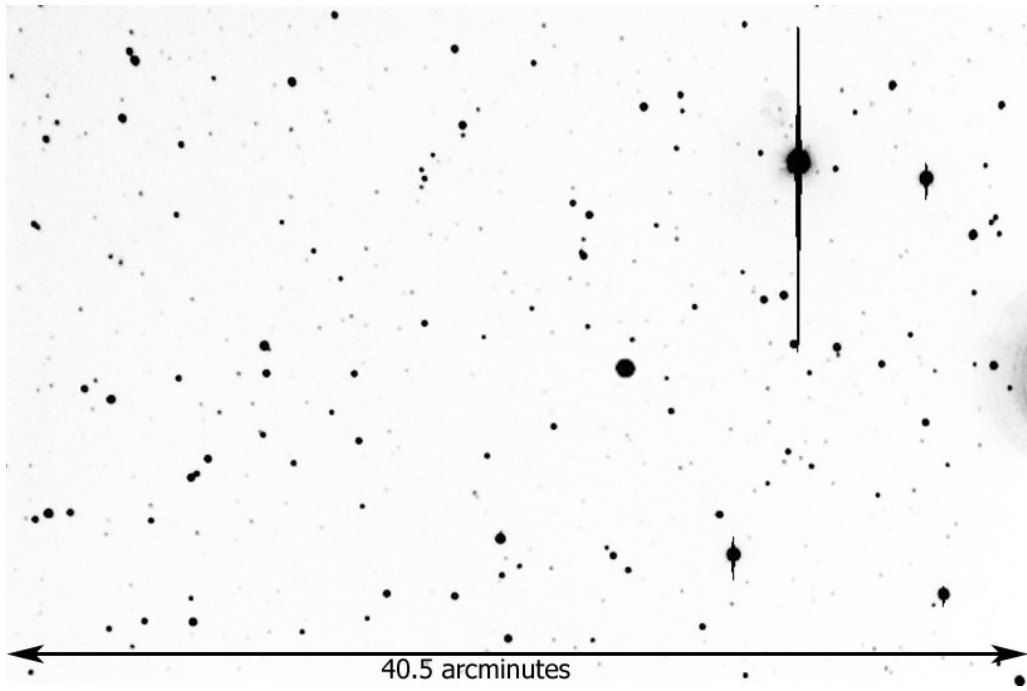
Figure 4 Proper motion of a star

The spectrometer is shown in figure 5. The collimator, which produces a beam of parallel light coming from the adjustable slit source, and the telescope can be rotated around the centre of the table. Vernier scales give accurate measurements of the angular position.

• Place the grating over the centre of the table and perpendicular to the collimator axis. Clamp the table and the collimator firmly, ensuring that the telescope can swing freely on both sides of the straight-through position.
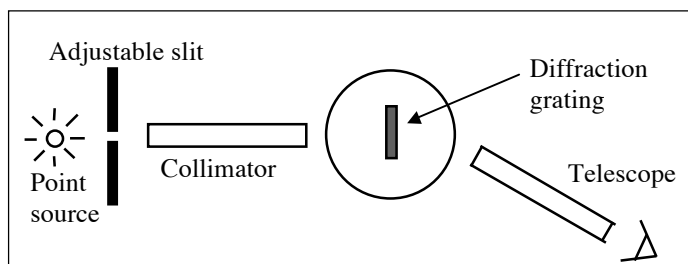
Figure 5 Grating spectrometer

• In this position observe the direct image of the slit, and adjust the slit, collimator and telescope as needed. You want the slit to be as **narrow as possible** while still appearing uniformly bright, and to be **vertical** (the lines of the grating are vertical). The focusing should be as sharp as you can get it. A useful **hint** is to take the spectrometer carefully into the main lab and adjust the telescope by focusing on a distant object on the horizon. Then adjust the collimator to focus the image of the slit. Record the angle of the telescope in the straight-through position, and subsequently always record the angles of a diffraction maximum on *both left and right sides*. Not only does this give you *two* independent measurements of the angle, but also since the average of left and right readings should be the initial straight-through reading, it also gives you a valuable *check* against blunders *and* an estimate of your measurement accuracy.

• At this point you should check that the grating is accurately perpendicular to the light falling on it from the collimator. Think of a way to do this.

Observation of spectral lines

• In order to calculate λ from measurements of the diffraction angle θ, we first need to know the grating line-spacing d. Calculate this from the number of lines per mm (or inch) which is engraved on the grating.

• Using the gas discharge tubes provided, measure a selection of prominent spectral lines of sodium, mercury, cadmium and hydrogen. Observe first, second and (where possible) third orders of principal maxima. Use the following list of prominent lines to *identify* them, and then *calculate their wavelengths*.

Element	Colour	Intensity
Sodium	green/yellow	weak
	green/yellow	weak
	yellow	very strong
	yellow	very strong
Mercury	violet	strong
	turquoise	weak
	green	very strong
	yellow	quite strong
	yellow	quite strong
Cadmium	blue	quite strong
	blue	strong
	green	strong
	red	quite strong
Hydrogen	blue	weak
	deep red	weak

• The two yellow lines of the sodium doublet near 589 nm, the famous 'D-lines' of sodium, should be well resolved if you have set up the instrument carefully. Be sure you identify the lines correctly, using the diffraction equation to decide whether the longer or the shorter wavelength is diffracted through the greater angle.

• Finally, look up the *true values* of the wavelengths of all the lines, and *compare* your results with these.

Chromatic resolution *[This section is optional; do it at the end if you have time]*

Diffraction theory shows that the angular separation of two nearby wavelengths is increased by (i) going to higher orders, as we have seen above, and (ii) increasing the *total* number of lines on the grating which contribute to the diffraction. If the Rayleigh criterion for angular resolution (see part A) is used, then it can be shown that the minimum wavelength difference that can be resolved is

$$\delta\lambda = \lambda/Nn$$

where N is the total number of lines and n the order. It is convenient to define the **resolving power** as the ratio $\lambda/\delta\lambda$ between the wavelength itself and the smallest difference that can be measured at that wavelength. The larger the resolving power the better the chromatic resolution. From the expression above, the resolving power equals Nn.

• The sodium D-lines should be well resolved when you use the full width of the grating. A simple way to vary the illuminated width is to clamp vernier callipers just in front of the grating and adjust the opening of the jaws. Find the narrowest opening that still allows you to resolve the D-lines with confidence. Evaluate the quantity Nn and compare with the value of $\lambda/\delta\lambda$. Repeat this several times for both orders. Do the experimental and the theoretical estimates of resolving power agree? Remember that the Rayleigh criterion is somewhat arbitrary.
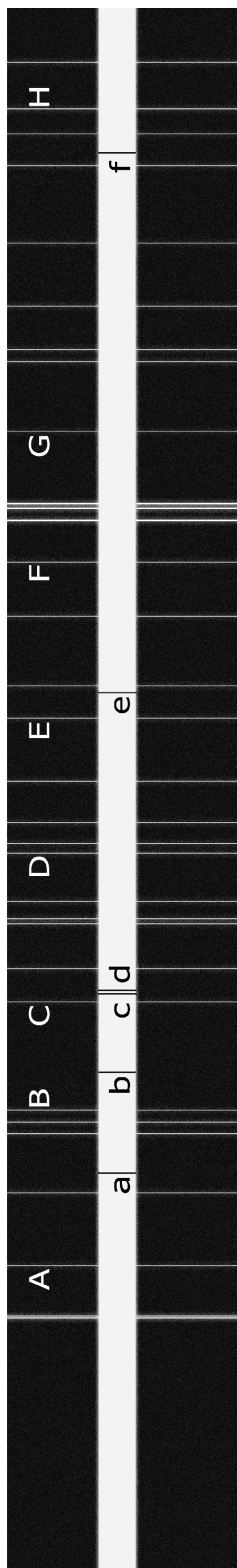
[End of optional section]

Doppler shift of spectral lines

A change in wavelength occurs when a source of light moves towards or away from an observer. The change, $\Delta\lambda = \lambda' - \lambda$, where λ is the normal wavelength and λ' is the changed wavelength, is proportional to the relative velocity v:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where c is the velocity of light. This is the Doppler shift. When source and observer are moving apart, v is positive, λ' is larger than λ and the light becomes redder. If they are approaching, v is negative and the light becomes bluer. The effect is very small but can be detected in the light from some stars and galaxies.

• Figure 6 is a photograph of part of the spectrum of iron, taken by focusing diffracted light from iron vapour onto a long strip of film. The principle maxima appear as bright lines, some of which are labelled *A–H*. Their wavelengths are well known, and are tabulated below. Superimposed across the centre of the spectrum of iron is that of a star, taken by directing the light from a telescope onto a spectrometer fitted with a camera. Some faint dark bands labelled *a–f* can be discerned; these are known to be lines of the elements hydrogen and calcium, whose wavelengths are also well known and tabulated below. However, because the star is moving relative to the Earth its light is Doppler shifted, so these stellar spectral lines do not appear at exactly the wavelengths measured in the laboratory. From the information below, and careful measurements of the positions of the stellar lines relative to the iron lines, find the apparent wavelengths of the lines *a-f*, deduce whether the star is moving towards or away from us, and estimate the relative velocity.

Stellar lines:			**Iron lines:**	
Line	**λ (nm)**	**Source**	**Line**	**λ (nm)**
a'	388.90 hydrogen	*A*	388.71
b'	393.38 calcium	*B*	395.67
c'	396.86 calcium	*C*	400.52
d'	397.01 hydrogen	*D*	407.17
e'	410.17 hydrogen	*E*	413.21
f'	434.05 hydrogen	*F*	420.20
		G	426.05
		H	440.48

The lines a'-f' given in the table above are unshifted and correspond to the shifted lines a-f in figure 6.

Figure 6 Doppler-shifted spectrum

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Part C: The expansion of the Universe

Introduction

In this part you will use the ideas and methods of parts A and B to measure the distances and the velocities of five distant galaxies, from photographs and spectra. A plot of distance versus velocity should then show that the two are related, a finding first made by the American astronomer Hubble in 1929. Hubble found that the spectra of many distant galaxies are shifted towards the red; he interpreted this as a Doppler shift and showed that the apparent velocity of recession increased linearly with the distance of the galaxy. Although there continue to be arguments about the value of the constant of proportionality (**Hubble's constant**) in this linear law, and even about the interpretation of the red shift as due to motion, there is no doubt about the observations. The quality of the material you will have to work with is considerably better than that available to Hubble.

The distance of the galaxies

Finding astronomical distances is difficult. This is the method we will use for these distant galaxies. Galaxies tend to form large clusters containing perhaps hundreds of members, some big and some small. It is found that in nearby clusters whose distances can be measured reasonably accurately by other means, the *brightest* elliptical galaxy is usually about 30,000 parsecs in diameter. (Galaxies are broadly classified as spiral or elliptical — the latter are often sufficiently close to spherical that it makes sense to refer to a single value for the diameter.) If we assume that this is true for all galactic clusters, then we can estimate the distance of a far-off cluster by measuring the apparent angular diameter θ of its brightest elliptical member and inferring the distance D from the expression (figure 7): $D = 30,000/\theta$ parsecs, where θ must be in radians.

- The scale of the photographs is given by the barred line, which is 150 arcseconds long. Measure the diameter of each galaxy's image, taking the mean of several measurements in different directions, and deduce the angular diameter of the galaxy in the sky. Convert this from arcseconds to radians, and deduce the distance D to the cluster.
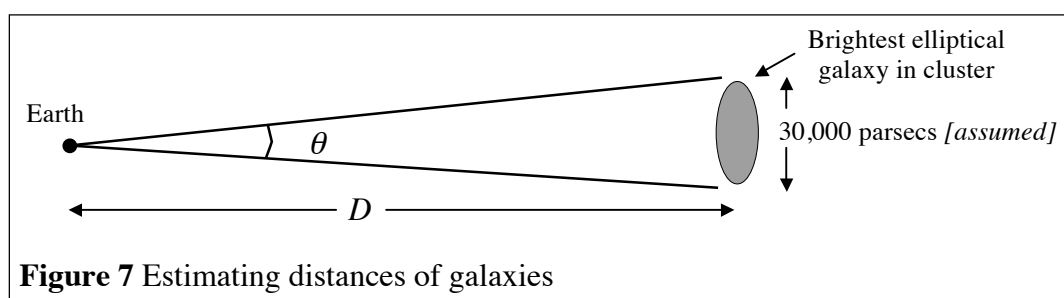
Figure 7 Estimating distances of galaxies

The velocity of the galaxies

The spectrum of each of the five galaxies shows two strong dark lines, prominent in elliptical galaxies and due to calcium. These, usually called the calcium H and K lines, are the lines b and c in the stellar spectrum you measured in part B, where their wavelengths are given. The arrows on the photographs show the extent to which these lines have been shifted towards the red. The comparison spectra above and below the galactic spectra are of helium. The lines marked $a–g$ in these spectra have the following wavelengths:

• By measurements similar to those in part B, find the **redshift** $\Delta\lambda/\lambda$ of the calcium lines. Hence calculate the recessional velocity of the galaxy.

Line λ **(nm)**

a	388.87
b	396.47
c	402.62
d	414.38
e	447.15
f	471.31
g	501.57

Hubble's law

• Plot a graph of recessional velocity *v* versus distance *D* for the five galaxies, and hence determine a value for *H* in the expression for Hubble's law:

$$v = HD$$

Give your value for *H*, which is **Hubble's constant**, in its usual units of $\text{km s}^{-1}\,\text{Mpc}^{-1}$.

• Use your value of *H* to estimate the age of the Universe.