Laboratory Exercise 6 – MEASUREMENTS OF WAVE VELOCITY

Introduction

The theme of these experiments is to measure the speed of sound in different media: in air (6A), and in a copper rod (6C) and to investigate uses of wave phenomena in measurement techniques (6B). These are actually three separate exercises, each of which illustrates the use of the oscilloscope to display rapidly changing electrical signals.

Each part can be completed in a single laboratory afternoon if you are fairly efficient, using the built-in measurement capabilities of the 'scopes. These are precision measurements and are capable of considerable accuracy, so in each case we want you to *compare* your results with those found by experienced investigators and given in textbooks.

General principles

No detailed understanding of wave motion is needed to carry out this exercise. We simply use the relation **wave velocity = frequency × wavelength**, $v = f\lambda$, since in each part we measure the frequency and wavelength, and calculate the velocity. This is the usual way of finding wave velocities since it is extremely inconvenient to measure the time a wave takes to travel between a source and a receiver a long distance apart.

We also mention a relation that you will have to take on trust, namely $v_s = \sqrt{(K/\rho)}$; here v_s is the velocity of a sound wave (a compression or **longitudinal** wave, with to-and-fro motion of the molecules), ρ is the density of the solid, liquid or gas, and K is an **elastic modulus**, a quantity that measures the pressure needed to compress or deform the material by a given amount. There are different moduli depending on the material and how it deforms — you have probably met Young's modulus which measures the pressure needed to squeeze a solid in one direction. The relation enables the elastic modulus to be found from a measurement of sound velocity, and vice versa. Other types of waves (**transverse** waves involving side-to-side motion, as in a violin string) can also travel through solids, and for these there are similar but slightly more complicated relations between velocity and elastic modulus, which are mentioned and used in exercise 6C.

Exercise 6A: Sound waves in air

In this experiment you use several different methods to measure the wavelength of high frequency waves travelling through air. The sound waves are produced in a small transmitter driven by electrical signals from a sine wave oscillator, and are detected by a receiver which is a small microphone, similar to the transmitter, whose electrical output is displayed on the oscilloscope. Transmitter and receiver are placed facing each other on a graduated slide. Their response is sharpest near a frequency of 40 kHz, so your measurements relate to this frequency.

Frequency response

• Set the transmitter and receiver facing each other about 30 cm apart (figure 1). Vary the oscillator frequency and observe the response on the oscilloscope; there is a narrow band of frequencies for efficient reception. Set the oscillator to maximise the amplitude, measure the frequency with a laboratory frequency meter, and check it during the course of the experiment.

Figure 1 Basic set-up
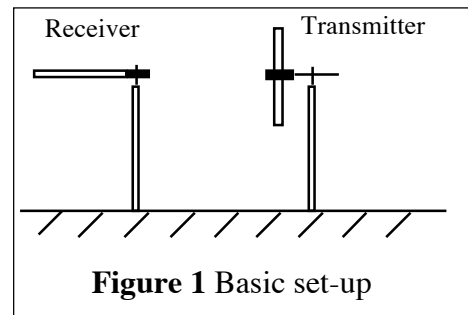
Direct measurement of wavelength

• Move the transmitter slowly towards the receiver. As you do so the relative phase of the transmitted and received signals changes, the wave trains shifting by one whole wavelength when you have moved the transmitter exactly this amount. There are millimetre scales on the shoeplates; measure the distance d that corresponds to a large number, N, of wavelengths, deduce the wavelength λ at this frequency, and so calculate the sound velocity v_s.

• Now recall the expression $v_s = \sqrt{(K/\rho)}$. In a gas the appropriate modulus K is the pressure, and in an ideal gas pressure divided by density is proportional to the absolute temperature T. So (show this for yourself) $v_s(T \,°C) = v_s(0 \,°C) \sqrt{(1 + (T \,°C)/273)}$. Find the temperature of the air in the lab, and so reduce your value for the speed of sound to the value at 0 °C. Compare this with tabulated information (e.g. Kaye and Laby). Estimate the errors in your measurement.

Standing waves

• Sound waves can bounce back and forth between the transmitter and receiver clamps. If the separation between transmitter and receiver is a whole number of half wavelengths, the there-and-back distance for one reflection is twice this, which is an exact whole number of wavelengths, and a **standing wave pattern** will be formed with **nodes** where the opposing waves cancel and **antinodes** where they reinforce. As you move the transmitter you can see the received signal increase in amplitude every half-wavelength as the standing wave pattern is formed. (In between there is a rather confusing variation of response because the travelling waves partially interfere). If this is difficult to observe try putting the aluminium discs on the receiver and transmitter, clamping them carefully so that the discs are flush with the fronts of the transmitter and receiver, and parallel to one another.

• Measure the change in distance corresponding to passing through a large number of nodes, deduce the wavelength, and compare with your earlier value.

• The change from one standing wave pattern to the next is accompanied by a change in the phase of the received and transmitted signals which you can see on the two-beam display of the 'scope. This phase change becomes more noticeable if you switch to *XY* display. In normal display mode the *horizontal* axis is controlled by a **timebase** circuit which drives the display in the horizontal (*X*) direction at a constant rate, adjustable by a front-panel knob at the right.

Selection of the *XY* display mode turns the timebase off and allows the display to be driven horizontally by one of the input voltage signals. Do this, and describe what you see.

Interferometry

• Place the perforated metal plates on the slide at right angles to the sound wave (figure 2). They let some of the wave through while reflecting part of it. Between these semi-reflecting plates the sound wave bounces back and forth, some of its energy escaping towards the receiver. There will therefore be a standing wave pattern between the plates when their separation is an integer number of half wavelengths, as we have seen. At these spacings the amplitude of the signal displayed on the 'scope will increase to a maximum, with weak minima at intermediate spacings.


Figure 2 Interferometry
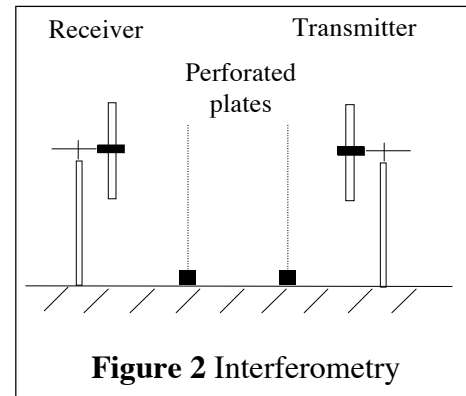
• Put the scope back into normal (not *XY*) two-beam operation, and move the perforated plates so as to measure the spacings of these maxima. This gives yet a third way of measuring the wavelength, so deduce it.

The technique of using semi-reflecting plates to form interference patterns of standing waves was developed by Fabry and Perot for use in optics, where it is widely used in spectrometers to measure, or select, different wavelengths. What you have here is a sonic analogue of the Fabry–Perot interferometer.

Figure 3 Phase-sensitive sonar

Phase-sensitive sonar

• Finally, take the transmitter off the slide and place it to one side with its ultrasonic waves directed at a perforated plate set at 45° (figure 3) so as to reflect the sound to the receiver. Viewing both traces on the 'scope, you will see that their relative phase changes as the plate is moved slightly towards or away from the receiver. A movement of one wavelength causes a complete phase shift of the same amount. The effect is seen more clearly if you switch to *XY* display, where the pattern is sensitive to quite small movements of the perforated plate.
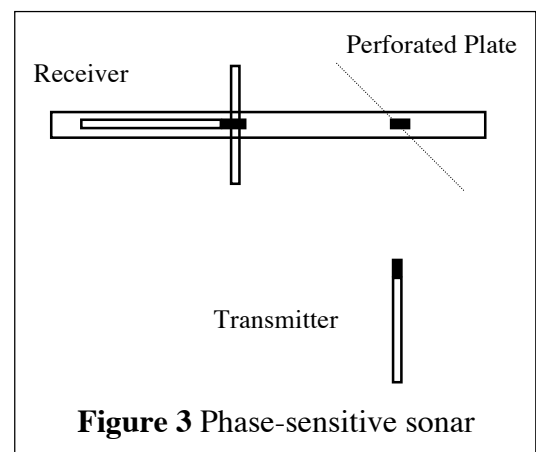
• To increase the sensitivity still further, switch the oscillator from sine wave to square wave operation. You will see, on dual-trace display, that neither transmitter nor receiver can respond to the rapid changes of the 'square' wave so a sine wave is still displayed. Switch back to *XY* and observe the effect of *combining* the square wave with the sine wave in *X* vs. *Y*. You have an oblong display whose height (or width — it depends which way round your input channels are connected) is *extremely* sensitive to movement of the plate. Blow gently on it and you will agree!

• Set your pattern to fill most of the screen, estimate how large a change in the oblong shape you can detect (perhaps a millimetre or so?), and deduce what movement of the plate this corresponds to. You have built a sensitive device for remote measurement of small displacements — a phase-sensitive sonar.

Excerise 6B: OPTICAL MEASUREMENTS

Introduction

Light is a form of wave motion, the colour being determined by the wavelength — about 400 nanometres for blue light and 700 nanometres for red (a nanometre, nm, is 10^{-9} metre). Suppose two light waves travelling in the same direction are brought together. If the crests and troughs of one coincide with the crests and troughs of the other the waves reinforce each other, whereas if the crests of one fall exactly on the troughs of the other the waves cancel one another out. The first case, leading to enhanced brightness, is called **constructive interference**, the second, **destructive interference**. If the two waves have exactly the same wavelength (therefore colour) and the same amplitude, the cancellation will be perfect yielding complete darkness. If white light is used cancellation can be perfect only for one wavelength.

A common way of producing interference effects is to take a single light wave and split it in two, say by reflecting part of it from each of two surfaces a small distance apart. If the waves are recombined they will interfere — this is the origin of the colours of a soap bubble, as shown in figure 4. (You may have noticed that these colours, being due to the *absence* of light near a certain frequency, are different from the usual spectral colours of the rainbow which are due to the *presence* of particular frequencies.) The colour of the bubble can be used to calculate the thickness of the water film which, being typically much less than 10^{-6} m, is otherwise very difficult to measure. The measurement is easier if

Figure 4 Reflection from bubble

light of a single wavelength (that is, colour, hence **monochromatic**) is used because complete destructive interference (total darkness) can then occur.
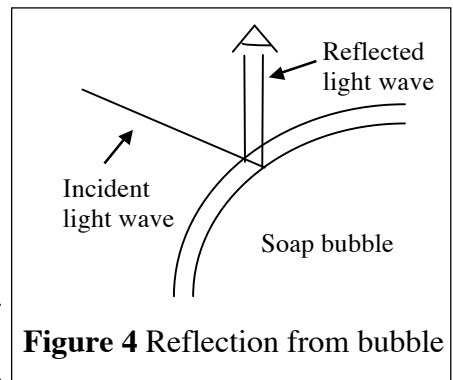
In this exercise you will observe optical interference of (nearly) monochromatic light, and use it to measure the thickness of an air gap between two glass surfaces, one of which, a simple convex optical lens, is curved. You can then calculate the curvature of the lens surface. Comparison with similar measurements made with a mechanical curvature gauge, or **spherometer**, should convince you that optical methods are *much* more precise than mechanical ones for measuring small distances. Many analytical and process control instruments utilise interference techniques; a simple example is given at the end of this exercise.

A useful mathematical result: the sagitta rule

Figure 5 shows an arc of a circle of radius R, its ends connected by a chord of length $2a$. If we look at either of the two right-angled triangles,

$$R^2 \quad (R \quad h)^2 \quad a^2 \quad R^2 \quad 2Rh \quad h^2 \quad a^2$$

$$a^2 \quad 2Rh \quad h^2$$

Since h is much less than R the term on the right is approximately $2Rh$, leading to the approximation known as the **sagitta rule:**
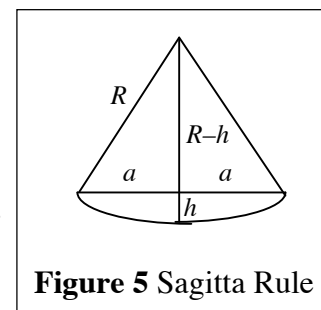
$$a^2 = 2Rh$$

Figure 5 Sagitta Rule

(so-called from a resemblance between the diagram and a bow and arrow, the Latin *sagitta* meaning arrow). It applies also to a sphere, with h being the distance between a tangent plane and a spherical surface such as that of a lens, and R being the radius of curvature of the surface. We shall use the sagitta rule in both the mechanical and the optical measurements of R.

Measurements: mechanical

The spherometer (figure 6) is an instrument for measuring the curvature of a spherical surface such as that of the lens you are given. Its tripod feet are set around a circle of radius a, corresponding to the chord length in the sagitta rule. In use, its centre point is raised (or lowered) a distance h above or below the plane of the feet, allowing the radius of a convex (or concave) surface to be calculated using the rule. One turn of the screw corresponds to a fixed change in h, which for your instrument is 1.0 mm. The edge of the attached disc is divided into a scale of 100 equal graduations, each corresponding to 0.01 mm. To obtain the correct measurement you will need to subtract your scale reading from 1.0

• To use the spherometer, balance it on the convex lens with the point raised, screw the point down until the tripod feet are just free to move, and note the scale reading against the vertical bar. This may be positive or negative depending on the instrument you have, but we are interested in measuring the **distance travelled** obtained by subtracting this measured value from the measurement of a flat surface…


Figure 6 Spherometer
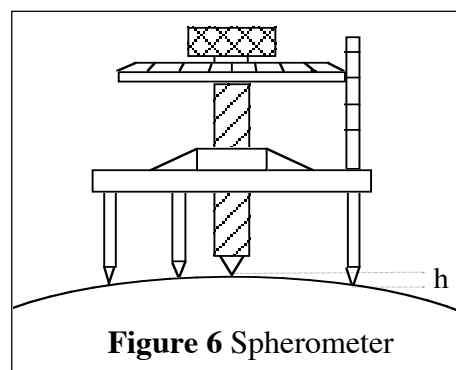
• Now stand the spherometer on the accurately flat *larger* sheet of glass, lower the point until it is again just touching the surface, and note the scale reading again. The difference between the two readings is distance h. **Repeat** the measurement of h several times.

• Find a by measuring the separation of the tripod feet which, by trigonometry, is equal to $a\sqrt{3}$. Check a by directly measuring the distance between any leg and the centre. Which of these measurements is more accurate, and why?

• Use your values of a and h to deduce R.

Your measurements of h will probably not agree to better than one scale division (0.01 mm), and anyway it is difficult to read the scale to better precision than this. So if h were, say, 0.25 mm, then your measurement would only be accurate to 1 part in 25, that is 4%. Also, there is an uncertainty on your measurement of a. Use the propagation of errors formula to calculate the uncertainty on R given your estimates of the uncertainty of a and of h. Remember, the **experimental error** or the **experimental uncertainty** in R, σ_R, can be expressed as either a percentage, $100 \times \sigma_R /R$, or as an actual value, $\pm \sigma_R$, using the symbol $\pm$ to indicate our uncertainty: $R \pm \sigma_R$.

Figure 7 Measurement of Newton's rings
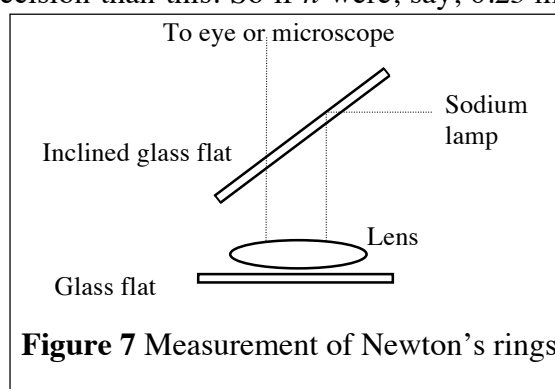
Measurements: optical

• Place the lens on the *smaller* flat sheet of glass underneath the inclined glass sheet in its holder, and put this on the stage of the travelling microscope. Turn on the yellow sodium lamp and shine the beam horizontally. The inclined glass sheet reflects some of the light down through the lens where part is reflected back from its bottom surface while the rest continues to the glass flat, is reflected, and retraces its path. The two returning light waves continue together back through the lens and up through the inclined sheet (figure 7).

• Look directly downwards. Your eye brings the two light waves to a focus on your retina where they interfere. If your vision is reasonably good you should be able to see a number of tiny concentric dark circles centred on the point where the lens touches the glass flat; these circles

$$a_0^2 = 2Rh_0$$
$$a_1^2 = 2Rh_1 = 2R(h_0 + \lambda/2) = 2Rh_0 + R\lambda$$
$$.$$
$$.$$
$$a_n^2 = 2Rh_0 + nR\lambda$$

are called **Newton's rings** (although they were actually studied first by Hooke). At a radius a from the centre there will be a dark ring if at that radius the distance h between the lens and the flat is exactly right for destructive interference between the reflected waves. At larger and smaller radii there are bright rings where constructive interference occurs. The sagitta rule relates a and h to the radius of curvature R of the lens surface. We move from one dark ring to the next whenever h increases by half a wavelength of light, since the extra distance travelled by one wave is $2h$, a whole wavelength. So starting with a dark ring at radius a_o and separation h_o, the sagitta rule gives for this and successive rings:

In these expressions λ is the wavelength of sodium light, which you can take to be 589 nm. A plot of a_n^2 versus n should yield a straight-line graph whose slope is $R\lambda$. Thus R can be found.

• Now for the measurements. The rings are much more easily seen through the microscope, and you can use the graduated scale on the horizontal bar to measure the position of the cross-wires as you move them from ring to ring. This scale has a **vernier** attachment to increase the reading accuracy (if you are not familiar with vernier scales ask a demonstrator). Practice moving the microscope horizontally by unclamping it, moving it to about the right position, clamping it firmly to the screw-drive, and then using the screw to move the microscope in a slow and controlled way.

• Set the cross-wire tangent to, say, the fifth dark ring to the left of the centre, and the same ring ($n = 5$) to the right, noting the scale readings at each position. Make sure that in going from left to right parts of the rings you pass through the *centre* of the rings. The difference between your readings is the diameter of the fifth ring. In a similar way measure the diameters of the 10th, 15th, 20th, …, rings. You should be able to measure out to about $n = 50$, though it is difficult to avoid losing count. The best procedure is to take all the readings to one side first, then repeat on the other side of the centre. *Be careful not to confuse millimetres and centimetres.*

• Use the propagation of errors formula to calculate the error on a^2 from your measurement of a.

• Plot your measurements as you go along, check that a straight line is a reasonable fit to your data, and find its gradient. **Do not forget to plot the error bars!** Does your line pass through the origin? Then use the computer to print a neat graph and to calculate the gradient, which should be close to your hand calculation. Deduce a value for R. The computer also calculates the error in the gradient. Use this to find the experimental error $\pm\sigma_R$ in your value of R. How does this compare with the error from the mechanical measurement?

A practical application

Suppose the gap between the lens and the flat plate were filled with something other than air — a liquid, say, with refractive index μ. The refractive index measures how much more slowly light travels in a medium than in a vacuum (or in air, whose refractive index can for most practical purposes be taken equal to unity). As the refractive index increases so the velocity, and the wavelength, of the light decreases in inverse proportion:

$$\lambda_{\mathrm{medium}} = \lambda_{\mathrm{vacuum}}\big/\mu$$

So the condition for destructive interference becomes:

$$a_n^2 = 2Rh_0 + nR\,\lambda\big/\mu$$

and if you repeat the experiment with a liquid instead of an air gap the slope of your graph will be μ times smaller.

• Try it. Place a small drop of distilled water from the squeeze bottle on the flat plate, lay the lens on top, repeat your measurements and, using the value of R you have already found deduce the refractive index of water. Plot your data on the same graph as before.

This is the basis of some commercial instruments for measuring refractive index. The reason for using distilled water, incidentally, is to keep mineral deposits off the optical glass — the refractive index, even of London tap water, is scarcely different!

A question of physics

Notice that the centre of the pattern is dark. You might expect that where the two reflecting surfaces are so close together that the difference in distance travelled by the two waves (the **optical path length**) is negligibly small, they would interfere constructively leading to enhanced, not diminished, brightness. On the other hand, you might argue that since at the centre there is no gap at all, but a continuous light path in glass, there can be no reflection from either surface, hence no reflected light, hence darkness! Which, if either, of these two conflicting arguments is right? You might care to ask your lab demonstrator.

Exercise 6C: The vibrations of a copper rod

Gases and liquids respond to pressure in a simple way. Their change of density depends only on the magnitude of the compressive force, not on its direction. That's why bubbles in water are spherical, not flattened. The response is described by a single elastic modulus, the **bulk modulus K,** and there is only one sound velocity, $v_s = \sqrt{(K/\rho)}$.

Solids are more complicated. Pull a rubber band and it becomes thinner as well as longer. The molecules respond to the direction as well as the magnitude of the force. A solid requires at least **two** elastic moduli, K which measures the stress needed to change its volume, and the **shear modulus** (or **rigidity modulus**) G which measures the stress that changes its shape. The word 'shear' tells us that molecules are sliding past each other, a motion strongly resisted in solids but not in fluids (which fill any shape of container). As it is difficult to compress a solid object without changing its shape, a more useful modulus than K is **Young's modulus E** which measures the stress that compresses a solid in one direction while allowing it to change shape as it pleases in other directions. K, G and E are not independent — the relation between them is:

$$K = \frac{EG}{9G - 3E}$$

This extra complexity allows solids to transmit several types of waves. For example, **seismic waves** in the Earth travel at different speeds depending on whether compression or shear is the dominant motion. In this exercise you will use thin copper rods to study the vibrations which they support. These vibrations would travel as waves along very long bars, but in these short bars the waves reflect back-and-forth from each end, setting up **standing wave** patterns when their wavelength (and hence frequency) is just right for reinforcement to occur. An electrical method is used to make the bar vibrate in the desired way, and the movement of the bar is also measured electrically. Even large forces produce rather small deformations of the rod, and the vibration amplitude will be small *unless* the frequency of the driving force is exactly equal to one of the natural vibrational frequencies of the rod. This is an example of **resonance**, the familiar situation in which the stimulus is applied at the same time in every cycle, as when you push a child higher and higher on a swing.

In this exercise you set up both **torsional** (twisting) and **bending** vibrations, as sketched in figure 8. Mathematical analysis shows that the speed at which torsional waves travel is determined solely by the shear modulus: $v^{\text{torsion}} = \sqrt{(G/\rho)}$. Standing waves are set up when a whole number, n, of half-wavelengths fit into the length l of the bar (the same criterion as for the fundamental frequency and harmonics of an organ pipe). This requirement and the relation $v = f\lambda$ lead to the following expression for the resonant frequencies of torsional vibration:

$$f_n^{\text{torsion}} = \frac{n}{2l}\sqrt{\frac{G}{\rho}}$$

The analysis for bending waves is rather more complex. Their velocity is determined by Young's modulus, and also depends on their wavelength. The final result is:

$$f_n^{\text{bending}} = \frac{\pi a}{16 l^2}(2n+1)^2\sqrt{\frac{E}{\rho}}$$

where a is the radius of the cylindrical rod. To be quite exact for $n = 1$ the factor $2n + 1 (= 3)$ must be replaced by 3.011.
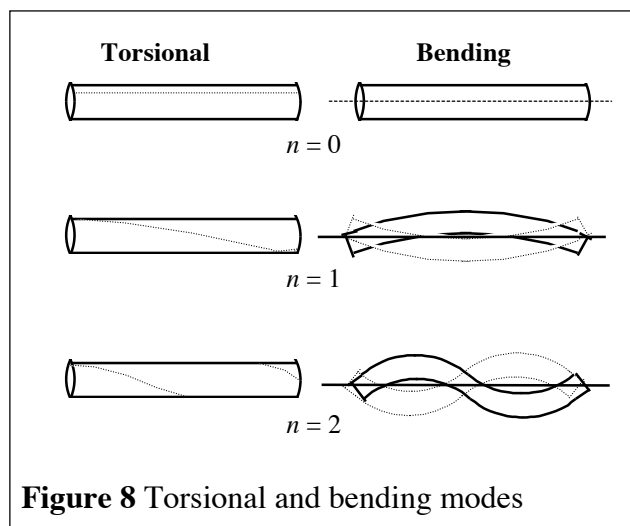

Figure 8 Torsional and bending modes

1. Bending vibrations

• Two copper rods are provided, one for bending and the other for torsional vibrations. Use the rod with circular coils wound near its end for the bending modes. Suspend it horizontally so that it can vibrate freely and connect one of the coils to the function generator via the black amplifier box. This box is required because of the poor impedance matching between the 50Ω output of the function generator and the 3Ω impedance of the coil. Arrange a permanent magnet to give a field which, by interaction with the current in the coil, will force the rod into bending vibrations (figure 9). Detect the vibrations by arranging a second magnet in a similar way near the other coil so as to induce a small voltage signal in that coil which, after amplification by the grey amplifier box, can be registered by the oscilloscope.

Figure 9 Arrangement of magnets
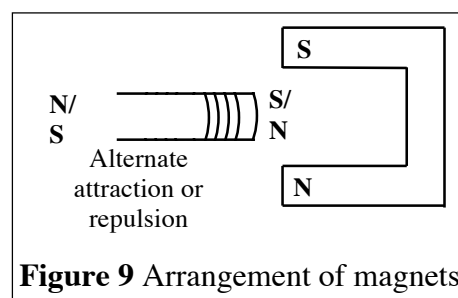
• Check that the apparatus is set up correctly by switching off the oscillator and striking the bar gently in the middle, when the fundamental vibration (i.e. $n = 1$, figure 8) should be excited and you should see its oscillation on the 'scope. From the trace, roughly estimate the frequency.

• Turn on the oscillator and look for the $n = 1$ resonance in this region, varying the oscillator frequency until a sharp maximum is obtained. Display both input and output signals on the 'scope, making the final adjustment of frequency so as to make the output as large as possible. Estimate the phase relationship at resonance. The frequency may not be indicated reliably by the oscillator dial, so use the oscilloscopes.

• Plot a suitable graph relating harmonic number n to frequency, measure the length and radius of the rod, and deduce a value for Young's modulus E of copper. The density may be found in Kaye and Laby, which also lists currently-accepted values for elastic moduli.

2. Torsional vibrations

• Use the other rod to set up torsional vibrations. The rectangular coil wound at the end of this rod produces a magnetic field in a different direction from the previous circular coil, so by arranging the permanent magnets appropriately (perpendicular to the coil, with the two ends different by 90°) you can both induce and detect twisting motion.

• Once again using the frequency generator set to generate a sine wave scan through and determine the resonant frequencies, and so deduce a value for the shear modulus G of copper. From your measurements of E and G, deduce a value for the bulk modulus K.

Resonant vibrations in quartz

Longitudinal sound waves, that is waves of compression running to-and-fro along the rod with velocity $v_s = \sqrt{(E/\rho)}$, are easy to produce mechanically. But there are some crystalline materials, for example quartz, in which these vibrations can be produced electrically. An applied voltage produces a small change in the crystal's length (the **piezo-electric** effect), so an AC voltage causes a wave of expansion and contraction to run through the crystal. The resonant frequency depends on the size of the crystal; a thin slice of quartz, for example, has a resonant frequency of hundreds of kilohertz. Only a tiny input of electric power is needed to sustain these resonant oscillations, whose frequency is determined by the size of the crystal and is thus extremely stable. These crystal oscillators are at the heart of every digital watch and computer clock.

• The speed of sound waves along such a quartz crystal is 5440 m/s and the density of quartz is 2600 kg/m^3. Calculate the value of Young's modulus, and find the thickness of a slice of quartz

whose fundamental resonant frequency is 1 MHz. Note that the requirement for resonance is the same as that for torsional vibrations.