

Laboratory Exercise 12 – THERMAL EFFICIENCY

Heat Engine

A heat engine uses the temperature difference between a hot reservoir and a cold reservoir to do work. Usually the reservoirs are assumed to be very large in size so the temperature of the reservoir remains constant regardless of the amount of heat extracted or delivered to the reservoir. This is accomplished in the Thermal Efficiency Apparatus by supplying heat to the hot side using a heating resistor and by extracting heat from the cold side using ice water. In the case of the Thermal Efficiency Apparatus, the heat engine does work by running a current through a load resistor. The work is ultimately converted into heat, which is dissipated by the load resistor (Joule heating).

A heat engine can be represented by a diagram (Figure 1). The law of Conservation of Energy (First Law of Thermodynamics) leads to the conclusion that $Q_H = W + Q_C$, the heat input to the engine equals the work done by the heat engine on its surroundings plus the heat exhausted to the cold reservoir.

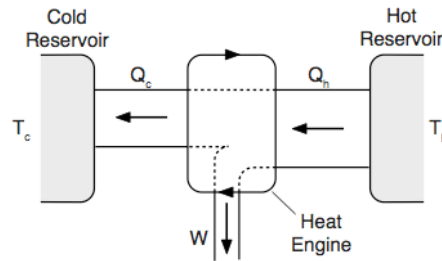


Figure 1: Heat Engine

The efficiency of the heat engine is defined to be the work done divided by the heat input

$$e = \frac{W}{Q_H}$$

So if all the heat input was converted to useful work, the engine would have an efficiency of one (100% efficient). Thus, the efficiency is always less than one.

NOTE: Since you will be measuring the rates at which energy is transferred or used by the Thermal Efficiency Apparatus all measurements will be power rather than energy. So $P_H = dQ_H/dt$ and then the equation $Q_H = W + Q_C$ becomes $P_H = P_W + P_C$ and the efficiency becomes

$$e = \frac{P_W}{P_H}$$

Carnot showed that the maximum efficiency of a heat engine depends only on the temperatures between which the engine operates, not on the type of engine.

$$e_{Carnot} = \frac{T_H - T_C}{T_H}$$

where the temperatures must be in Kelvin. The only engines which can be 100% efficient are ones which operate between T_H and absolute zero. The Carnot efficiency is the best a heat engine can do for a given pair of temperatures, assuming there are no energy losses due to friction, heat conduction, heat radiation, and Joule heating of the internal resistance of the device.

Adjusted Efficiency

Using the Thermal Efficiency Apparatus, you can account for the energy losses and add them back into the powers P_W and P_H . This shows that, as all losses are accounted for, the resulting adjusted efficiency approaches the Carnot efficiency, showing that the maximum efficiency possible is not 100%.

Heat Pump (Refrigerator)

A heat pump is a heat engine run in reverse. Normally, when left alone, heat will flow from hot to cold. But a heat pump does work to pump heat from the cold reservoir to the hot reservoir, just as a refrigerator pumps heat out of its cold interior into the warmer room or a heat pump in a house in winter pumps heat from the cold outdoors into the warmer house.

In the case of the Thermal Efficiency Apparatus, heat is pumped from the cold reservoir to the hot reservoir by running a current into the Peltier device in the direction opposite to the direction in which the Peltier device will produce a current. A heat pump is represented in a diagram such as Figure 2

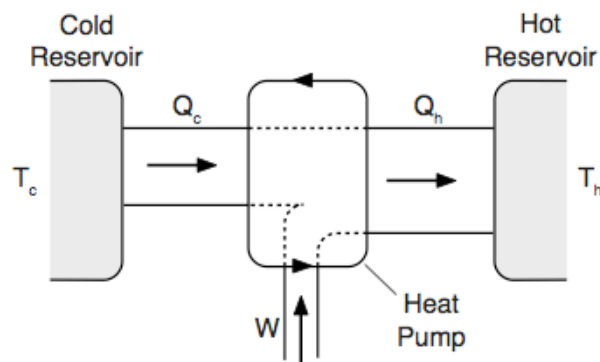


Figure 2: Heat Pump

NOTE: By conservation of energy, $Q_C + W = Q_H$, or in terms of power $P_C + P_W = P_H$.

Coefficient of Performance

Instead of defining an efficiency as is done for a heat engine, a coefficient of performance (COP) is defined for a heat pump. The COP is the heat pumped from the cold reservoir divided by the work required to pump it

$$\kappa = COP = \frac{P_C}{P_W}$$

This is similar to efficiency because it is the ratio of what is accomplished to how much energy was expended to do it. Notice that although the efficiency is always less than one, the COP is always greater than one. As with the maximum efficiency of a heat engine, the maximum COP of a heat pump is only dependent on the temperatures.

$$\kappa_{max} = \frac{T_C}{T_H - T_C}$$

where the temperatures are in Kelvin. If all losses due to friction, heat conduction, radiation, and Joule heating are accounted for, the actual COP can be adjusted so it approaches the maximum COP.

Measurements with the Thermal Apparatus

Three quantities may be directly measured with the Thermal Efficiency Apparatus: temperatures, the power delivered to the hot reservoir, and the power dissipated by the load resistors. The details of how these measurements are made follow.

Temperatures: The temperatures of the hot and cold reservoirs are determined by measuring the resistance of the thermistor imbedded in the hot or cold block. To do this, connect an ohmmeter to the terminals located as shown in Figure 4. The switch toggles between the hot side and the cold side. The thermistor reading can be converted to a temperature by using the chart located on the front of the Thermal Efficiency Apparatus. Notice that as the temperature increases, the thermistor resistance decreases (100 kΩ is a higher temperature than 200 kΩ). **NOTE:** To get the exact temperature reading you must interpolate between numbers on the chart.

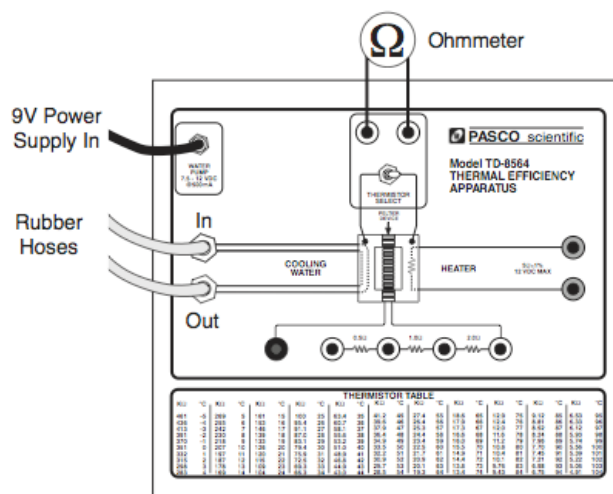


Figure 3: Thermal Efficiency Apparatus

Power Delivered to the Hot Reservoir (P_H): The hot reservoir is maintained at a constant temperature by running a current through a resistor. Since the resistance changes with temperature, it is necessary to measure the current and the voltage to obtain the power input. Then $P_H = I_H V_H$.

Power Dissipated by the Load Resistor (P_W)

The power dissipated by the load resistor is determined by measuring the voltage drop across the known load resistance and using the formula:

$$P_W = \frac{V^2}{R}$$

The load resistors have a tolerance of 1%. When the Thermal Efficiency Apparatus is operated as a heat pump rather than as a heat engine, the load resistors are not used so it is necessary to measure both the current and the voltage. So the current into the Peltier device is measured with an ammeter, and the voltage across the Peltier device is measured with a voltmeter and the power input is calculated with the formula $P_W = I_W V_W$.

Indirect Measurements: It will be necessary to know three additional quantities in the experiments: 1) The internal resistance of the Peltier device; 2) The amount of heat conducted through the device and the amount radiated away; 3) The amount of heat pumped from the cold reservoir. These quantities may be determined indirectly with the Thermal Efficiency Apparatus in the following ways.

Internal Resistance: Before the adjusted efficiency can be calculated, it is necessary to calculate the internal resistance. This is accomplished by measuring the voltage drop across the Peltier

device when an external load is applied. First run the Thermal Efficiency Apparatus with a load resistor (R) as in Figure 4.

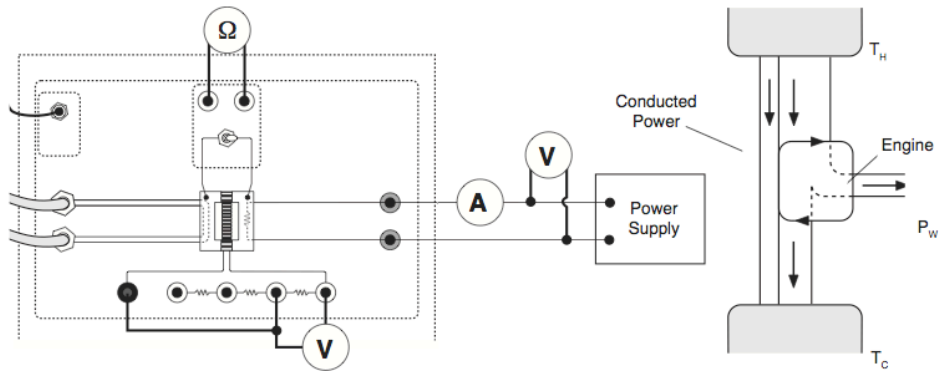


Figure 4: Heat Engine with a load

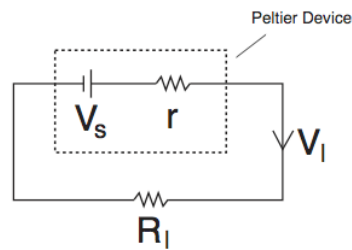


Figure 5: Procedure for finding internal resistance

The electrical equivalent of this setup is shown in Figure 5. Kirchoff's Loop Rule gives

$$V_S - Ir - IR = 0$$

Next, run the Thermal Efficiency Apparatus with no load, as in Figure 6.

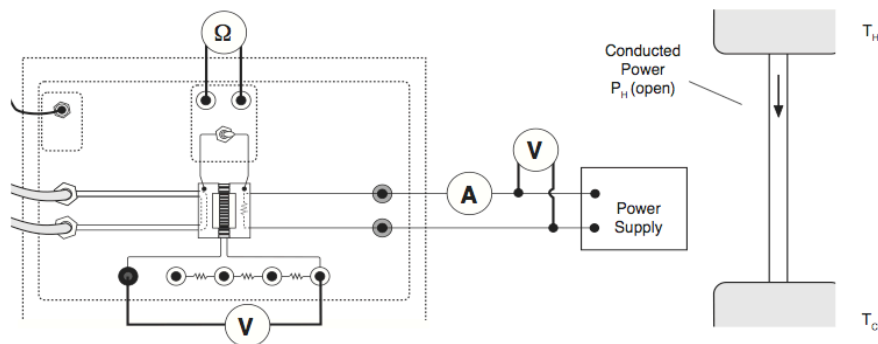


Figure 6: No load

Since there is no current flowing through the internal resistance of the Peltier Device, the voltage drop across the internal resistance is zero and the voltage measured will just be V_S . Since we have measured V_w rather than I in the heat engine mode, the equation above becomes

$$V_S - \left(\frac{V_w}{R} \right) r - V_w = 0$$

Solving this for the internal resistance gives us

$$r = \left(\frac{V_S - V_w}{V_w} \right) R$$

You may also find the resistance by measuring the currents for two different load resistors and then solving the resulting loop rule equations simultaneously.

Heat Conduction and Radiation: The heat that leaves the hot reservoir goes two places: part of it is actually available to be used by the heat engine to do work while the other part bypasses the engine either by being radiated away from the hot reservoir or by being conducted through the Peltier device to the cold side. The portion of the heat which bypasses the engine by radiation and conduction would be transferred in this same manner whether or not the device is connected to a load and the heat engine is doing work.

The Thermal Efficiency Apparatus is run with a load connected to measure P_H (Figure 4) and then the load is disconnected and the power input into the hot reservoir is adjusted to maintain the temperatures (less power is needed when there is no load since less heat is being drawn from the hot reservoir). See Figure 6. $P_{H(\text{open})}$ is the power input to the hot reservoir when no load is present. Since, while there is no load, the hot reservoir is maintained at an equilibrium temperature, the heat put into the hot reservoir by the heating resistor must equal the heat radiated and conducted away from the hot reservoir. So measuring the heat input when there is no load determines the heat loss due to radiation and conduction. It is assumed this loss is the same when there is a load and the heat engine is operating.

Heat Pumped from the Cold Reservoir: When the Thermal Efficiency Apparatus is operated as a heat pump, conservation of energy yields that the rate at which heat is pumped from the cold reservoir, P_C , is equal to the rate at which heat is delivered to the hot reservoir, P_H , minus the rate at which work is being done, P_W (Figure 2). The work can be measured directly but the heat delivered to the hot reservoir has to be measured indirectly. Notice that when the heat pump is operating, the temperature of the hot reservoir remains constant. Therefore, the hot reservoir must be in equilibrium and the heat delivered to it must equal the heat being conducted and radiated away. So a measurement of the heat conducted and radiated away at a given temperature difference will also be a measurement of the heat delivered to the hot reservoir. The heat conducted and radiated is measured by running the device with no load and measuring the heat input needed to maintain the temperature of the hot side (Figure 6).

Part A: Heat Engine Efficiency

In this experiment you will determine the actual efficiency and the Carnot efficiency of the heat engine as a function of the operating temperatures.

- Prepare the ice-water bath and immerse both rubber tubes from the Thermal Efficiency Apparatus into the bath.
- Plug the 9V transformer into the wall socket and into the pump on the Thermal Efficiency Apparatus. You should now hear the pump running and water should be coming out of the rubber hose marked “out”.
- Plug the ohmmeter into the thermistor terminals.
- Connect a DC power supply and a voltmeter and ammeter to the heater block terminals. Adjust the voltage to about 11V. **NOTE:** This is just a suggested value chosen to make the hot temperature nearly at the maximum allowed. Any voltage less than 12V is suitable. The Thermal Efficiency Apparatus should not be run for more than 5 minutes with the hot side above 80°C. A thermal switch will automatically shut off the current to the heater block if it exceeds 93°C to prevent damage to the device.
- Connect the 2Ω load resistor with a short patch cord as shown in Figure 4. Connect a voltmeter across the load resistor. The choice of the 2Ω load resistor is arbitrary. Any of the load resistances may be used.

Preliminary Procedure

Allow the system to come to equilibrium so that the hot and cold temperatures are constant. This may take 5 to 10 minutes, depending on the starting temperatures. To speed up the process, increase the voltage across the heating resistor momentarily and then return it to the original setting. If it is desired to cool the hot side, the voltage can be momentarily decreased. Remember that the thermistor resistance goes down as the temperature increases. Measure the temperature resistances of the hot side and the cold side by using the toggle switch to switch the ohmmeter to each side. Record the readings and convert the resistances to temperatures using the chart on the front of the device and record these temperatures. Record also the voltage (V_H) across the heating resistor, the current (I_H), and the voltage across the load resistor (V_W). Lower the voltage across the heating resistor by about 2 V and repeat the procedure until data for five different hot temperatures have been taken.

Analysis

For each of the data runs, calculate the power supplied to the hot reservoir, P_H , and the power used by the load resistor, P_W , and record these values in a Table. Calculate the temperature difference for each trial, the actual efficiencies from the powers, and the Carnot (maximum) efficiencies from the temperatures. Report all these values in a Table. To compare the actual efficiency to the Carnot efficiency, construct a graph. Plot the Carnot efficiency vs. ΔT and also plot the actual efficiency vs. ΔT . This may be done on the same graph.

Questions:

- The Carnot efficiency is the maximum efficiency possible for a given temperature difference. According to the graph, is the actual efficiency always less than the Carnot efficiency?
- Does the Carnot efficiency increase or decrease as the temperature difference increases?
- Does the actual efficiency increase or decrease as the temperature difference increases?
- The Carnot efficiency represents the best that a perfect heat engine can do. Since this heat engine is not perfect, the actual efficiency is a percentage of the Carnot efficiency. The overall (actual) efficiency of a real heat engine represents the combination of the engine's ability to use the available energy and the maximum energy available for use. From the data taken, what is the percentage of available energy used by this heat engine?
- The actual efficiency of this heat engine is very low and yet heat engines of this type are used extensively in remote areas to run things. How can such an inefficient device be of practical use?

Detailed Study:

You will now determine the actual efficiency and the Carnot efficiency of the heat engine and then compensate for the energy losses to show that the compensated actual efficiency approaches the Carnot efficiency.

- Prepare the ice-water bath and immerse both rubber tubes from the Thermal Efficiency Apparatus into the bath.
- Plug the 9V transformer into the wall socket and into the pump on the Thermal Efficiency Apparatus. You should now hear the pump running and water should be coming out of the rubber hose marked “out”.
- Plug the ohmmeter into the thermistor terminals.

To obtain all the necessary data for the heat engine it is necessary to run the Thermal Efficiency Apparatus in two different modes. The Heat Engine Mode determines the actual efficiency of the Peltier device. The Open Mode determines the losses due to conduction and radiation. Data from both modes is used to calculate internal resistance and the Carnot Efficiency.

Heat Engine Mode

Connect a DC power supply and a voltmeter and ammeter to the heater block terminals. Turn on the voltage to about 11 V. NOTE: This is just a suggested value chosen to make the hot temperature nearly at the maximum allowed. Any voltage less than 12V is suitable. The Thermal Efficiency Apparatus should not be run for more than 5 minutes with the hot side above 80°C. A thermal switch will automatically shut off the current to the heater block if it exceeds 93°C to prevent damage to the device.

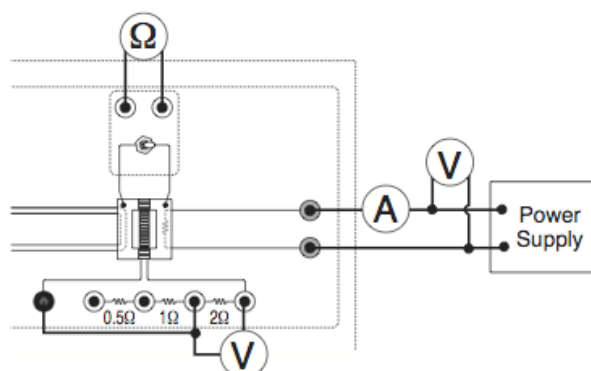


Figure 7

Connect the 2Ω load resistor with a short patch cord as shown in Figure 7. Connect a voltmeter across the load resistor. Allow the system to come to equilibrium so that the hot and cold temperatures are constant. This may take 5 to 10 minutes, depending on the starting temperatures. To speed up the process, increase the voltage across the heating resistor momentarily and then return it to 11V. If it is desired to cool the hot side, the voltage can be momentarily decreased. Remember that the thermistor resistance goes down as the temperature increases. Now measure the temperature resistances of the hot side and the cold side by using the toggle switch to switch the ohmmeter to each side. Record the readings in a Table. Convert the resistances to temperatures using the chart on the front of the device. Record the voltage (V_H) across the heating resistor, the current (I_H), and the voltage across the load resistor (V_W) in the Table.

Open Mode

- Disconnect the patch cord from the load resistor so no current is flowing through the load and thus no work is being done. Now all the power delivered to the heating resistor is either conducted to the cold side or radiated away. Leave the voltmeter attached so that the Seebeck voltage (V_S) can be measured. (see figure 6)
- Decrease the voltage applied to the hot side so that the system comes to equilibrium at the

same hot temperature as in the Heat Engine Mode. Since the temperature difference is the same as when the heat engine was doing work, the same amount of heat is now being conducted through the device when there is no load as when there is a load. (It may not be possible to exactly match the previous cold temperature.). Record the resistances in a Table and convert them to degrees. Also record V_H , I_H and V_s .

Calculations:

Actual Efficiency: Calculate the actual efficiency using

$$e = \frac{P_W}{P_H}$$

where $P_w = V_w^2/R$ and $P_H = I_H V_H$

Record the powers as in the example Tables shown below:

Internal Resistance = $r =$ _____

Mode	T_h (K)	T_c (K)	P_h	P_w	I_w
Engine (2Ω load)					
Open					

	Actual	Adjusted	Maximum (Carnot)	% Difference
Efficiency				

Maximum Efficiency: Convert the temperatures to Kelvin. Calculate the Carnot efficiency using the temperatures and record as shown above.

Adjusted Efficiency: The purpose of the following calculations is to account for all the energy losses and adjust the actual efficiency so that it matches the Carnot efficiency.

First, the work done in the actual efficiency calculation only includes V^2/R for the power dissipated by the load resistor R but, to account for total work done by the device, it should also include $I^2 r$ for the power dissipated by the internal resistance, r , of the device. This Joule heating of the Peltier device is not counted in the actual efficiency because it is not useful work. Thus, in the adjusted efficiency, the total work done in terms of power is

$$P'_W = P_W + I_W^2 r = \frac{V_W^2}{R} + I_W^2 r$$

where $I_w = V_w/R$. Calculate I_w for the 2Ω load and record it.

Second, the heat input must be adjusted. The heat that leaves the hot reservoir goes two places. Part of it is actually available to be used by the heat engine to do work while the other part bypasses the engine either by being radiated away from the hot reservoir or by being conducted through the Peltier device to the cold side. The portion of the heat which bypasses the engine by radiation and conduction would be transferred in this same manner whether or not the device is connected to a load and the heat engine is doing work. Therefore this heat can be considered to not be available to do work and should not be included in the heat input in the adjusted efficiency.

$$P'_H = \text{available heat} = P_H - P_{H(open)}$$

The Thermal Efficiency Apparatus is run with a load connected to measure P_H (Figure 4) and then the load is disconnected and the power input into the hot reservoir is adjusted to maintain the temperatures (less power is needed when there is no load since less heat is being drawn from the hot reservoir). See Figure 6. $P_H(\text{OPEN})$ is the power input to the hot reservoir when no load is present. Since, while there is no load, the hot reservoir is maintained at an equilibrium temperature, the heat put into the hot reservoir by the heating resistor must equal the heat radiated and conducted away from the hot reservoir. So measuring the heat input when there is no load determines the heat loss due to radiation and conduction. It is assumed this loss is the same when there is a load and the heat engine is operating.

Having accounted for the obvious energy losses, the adjusted efficiency should match the Carnot efficiency which assumes no energy loss. The adjusted efficiency is

$$e'_{adjusted} = \frac{P'_W}{P'_H} = \frac{P_W + I_W^2 r}{P_H - P_{H(open)}}$$

Calculate the internal resistance, r , using the equation

$$r = \left(\frac{V_p - V_W}{V_W} \right) R$$

Record this resistance, then calculate the adjusted efficiency and record the result. Calculate the percent difference between the adjusted efficiency and the Carnot (maximum) efficiency.

$$\%Difference = \frac{e_{max} - e_{adjusted}}{e_{max}} \times 100\%$$

Questions:

- If the difference between the temperature of the hot side and the cold side was decreased, would the maximum efficiency increase or decrease?
- The actual efficiency of this heat engine is very low and yet heat engines of this type are used extensively in remote areas to run things. How can such an inefficient device be of practical use?
- Calculate the rate of change in entropy for the system which includes the hot and cold reservoirs. Since the reservoirs are at constant temperature, the rate of change in entropy is

$$\frac{\Delta S}{\Delta t} = \frac{\Delta Q / \Delta t}{T} = \frac{P}{T}$$

for each reservoir. Is the total change in entropy positive or negative? Why?

Part B: Heat Pump Coefficient of Performance

Before doing this experiment, it is necessary to perform the HEAT ENGINE EFFICIENCY (Part A) experiment to get the data necessary to determine the internal resistance of the Peltier device. To complete the measurements for this experiment, use the following instructions to run the apparatus as a heat pump (pumping heat from the cold side to the hot side):

Prepare the ice-water bath and immerse both rubber tubes from the Thermal Efficiency Apparatus into the bath. Plug the 9V transformer into the wall socket and into the pump on the Thermal Efficiency Apparatus. You should now hear the pump running and water should be coming out of the rubber hose marked “out”. Disconnect the power supply to the hot side. Connect the power supply directly across the Peltier device with no load resistance. See Figure 8. Connect an ammeter and a voltmeter to the power supply.

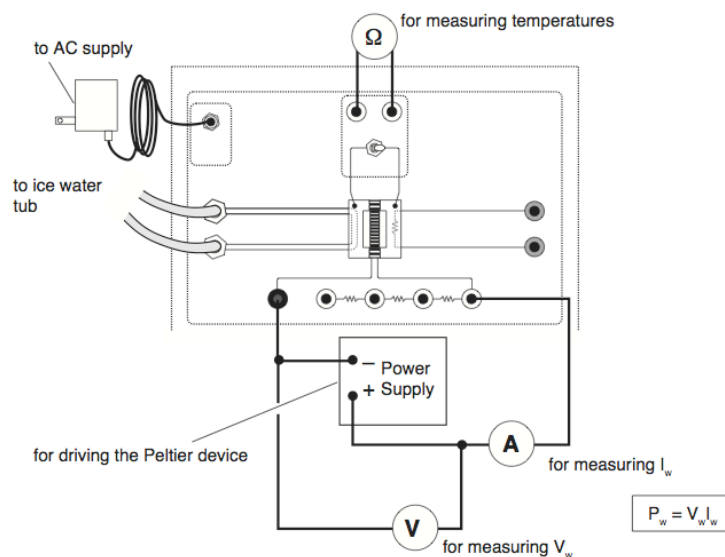


Figure 8: Heat pump mode

Procedure:

Increase the voltage until equilibrium is reached at the same hot temperature as in the previous experiment. The hot side is now being heated by heat pumped from the cold side rather than the heater resistor. Record the resistances and convert them to degrees. Also record the voltage (V_w) and the current (I_w) in a Table.

Analysis:

Actual Coefficient of Performance: Calculate the actual COP using the data taken in the Heat Engine experiment.

$$\kappa = \frac{P_C}{P_W} = \frac{P_H(OPEN) - P_W}{P_W}$$

Record this result in a Table. Maximum Coefficient of Performance: Calculate the maximum COP using

$$\kappa_{MAX} = \frac{T_C}{T_H - T_C}$$

Adjusted Coefficient of Performance: Part of the power being applied to the Peltier device is being dissipated in the Joule heating of the internal resistance of the device rather than being used to pump the heat from the cold reservoir. Therefore, to adjust for this, I^2r must be subtracted from the power input to the Peltier device. Then the COP becomes the heat pumped from the cold reservoir divided by work done to pump the heat, rather than dividing by the work done to pump the heat and heat the internal resistance. In terms of the power,

$$\kappa_{Adjusted} = \frac{P_{H(OPEN)} - P_W}{P_W - I_W^2 r}$$

Record this in the Table then calculate and record also the percent difference between the adjusted COP and maximum COP:

$$\% \text{ Difference} = \frac{\kappa_{MAX} - \kappa_{adjusted}}{\kappa_{max}} \times 100\%$$

Questions:

If the difference between the temperature of the hot side and the cold side was decreased, would the maximum COP increase or decrease?

Calculate the rate of change in entropy for the system which includes the hot and cold reservoirs. Since the reservoirs are at constant temperature, the rate of change in entropy is

$$\frac{\Delta S}{\Delta t} = \frac{\Delta Q / \Delta t}{T} = \frac{P}{T}$$

for each reservoir. Is the total change in entropy positive or negative? Why?

Part C: Load for optimum performance

This experiment finds the load resistor which maximizes the power output of the heat engine. The power delivered to the load resistor, R , is $P = I^2 R$. The amount of current that flows through the load resistor varies as the load is varied. From Figure 5, $V_S = I(r+R)$ where V_S is the Seebeck voltage and r is the internal resistance of the Peltier device.

So the power can be expressed in terms of the Seebeck voltage, the internal resistance, and the load resistance:

$$P = \left(\frac{V_S}{r + R} \right)^2 R$$

Assuming the Seebeck voltage remains constant if the temperatures of the hot and cold reservoirs are constant, the power can be maximized with respect to the load resistance by taking the derivative and setting it equal to zero:

$$\frac{dP}{dR} = \frac{V_S^2 (r - R)}{(r + R)^3} = 0$$

This shows that when the load resistance is equal to the internal resistance of the Peltier device, the power delivered to the load will be a maximum.

Procedure: Connect a DC power supply and a voltmeter and ammeter to the heater block terminals. Turn on the voltage to about 11V (Any voltage less than 12V is suitable. The Thermal Efficiency Apparatus should not be run for more than 5 minutes with the hot side above 80°C. A

thermal switch will automatically shut off the current to the heater block if it exceeds 93°C to prevent damage to the device.). Connect the 0.5 Ω load resistor with a short patch cord as shown in Figure 9. Connect a voltmeter across the load resistor.

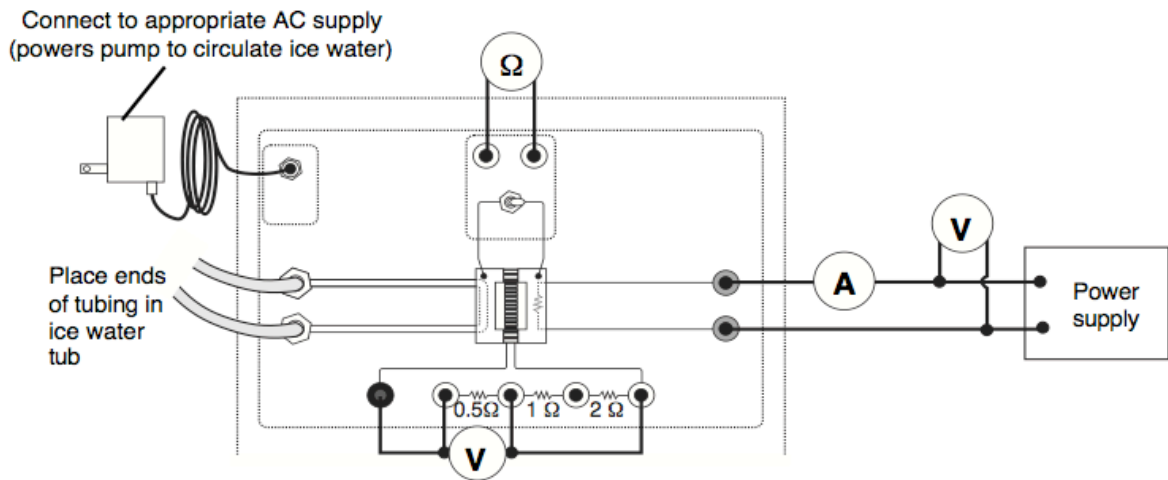


Figure 9: Connecting the 0.5Ω load resistor

NOTE: Alternatively, a variable power resistor (rheostat) may be used in place of the load resistors supplied with the Thermal Efficiency Apparatus. This has the advantage of being able to continuously vary the load resistance. However, it will be necessary to measure the resistance of the load.

Allow the system to come to equilibrium so that the hot and cold temperatures are constant. This may take 5 to 10 minutes, depending on the starting temperatures. To speed up the process, increase the voltage across the heating resistor momentarily and then return it to 11 V. If it is desired to cool the hot side, the voltage can be momentarily decreased. Remember that the thermistor resistance goes down as the temperature increases.

Measure the temperature resistances of the hot side and the cold side by using the toggle switch to switch the ohmmeter to each side. Record the readings in a Table. Convert the resistances to temperatures using the chart on the front of the device as explained earlier. Record the voltage (V_H) across the heating resistor, the current (I_H), and the voltage across the load resistor (V_W). Then calculate the power input to the hot side, $P_H = I_H V_H$, and the power dissipated by the load resistor, $P_L = V_W^2 / R$. Calculate the efficiency, $e = P_L / P_H$ and record all these values in a Table. Adjust the power input to the hot side to keep the temperature of the hot reservoir at the same temperature as it was for the 0.5 Ω resistor while repeating the steps above for the other possible load resistances: 1, 1.5, 2, 2.5, 3, and 3.5 ohms.

Questions

- For which load resistor is the efficiency a maximum?
- How does the load resistance for optimum efficiency compare with the internal resistance measured in Part A?